MATLAB®

The Language of Technical Computing

Computation

Visualization

Programming

The MATH WORKS Inc.

Using MATLAB

Version 5

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Using MATLAB

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Contents

1 Introduction

2 MATLAB Working Environment

3 Debugger and Profiler

4 Matrices and Linear Algebra

5 Polynomials and Interpolation

6 Data Analysis and Statistics

7 Function Functions

8 Ordinary Differential Equations

9 Sparse Matrices

10 M-File Programming

15 File I/O

1

Introduction

About the Cover

The cover of this guide depicts a solution to a problem that has played a small, but interesting role in the history of numerical methods during the last 30 years. The problem involves finding the modes of vibration of a membrane supported by an L-shaped domain consisting of three unit squares. The nonconvex corner in the domain generates singularities in the solutions, thereby providing challenges for both the underlying mathematical theory and the computational algorithms. There are important applications, including wave guides, structures, and semiconductors.

Two of the founders of modern numerical analysis, George Forsythe and J.H. Wilkinson, worked on the problem in the 1950s. (See G.E. Forsythe and W.R. Wasow, *Finite-Difference Methods for Partial Differential Equations*, Wiley, 1960.) One of the authors of this guide (Moler) used finite differences by combinations of distinguished fundamental solutions to the underlying differential equation formed from Bessel and trigonometric functions. The idea is a generalization of the fact that the real and imaginary parts of complex analytic functions are solutions to Laplace's equation. In the early 1970s, new matrix algorithms, particularly Gene Golub's orthogonalization techinques for least squares problems, provided further algorithmic improvements.

Today, MATLAB allows us to express the entire algorithm in a few dozen lines, to compute the solution with great accuracy in a few minutes on a computer at home, and to readily manipulate color three-dimensional displays of the results. We have included our MATLAB program, `membrane.m`, with the M-files supplied along with MATLAB.

What Is MATLAB?

MATLAB® is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses include:

- Math and computation
- Algorithm development
- Modeling, simulation, and prototyping
- Data analysis, exploration, and visualization
- Scientific and engineering graphics
- Application development, including graphical user interface building

MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. This allows you to solve many technical computing problems, especially those with matrix and vector formulations, in a fraction of the time it would take to write a program in a scalar noninteractive language such as C or Fortran.

The name MATLAB stands for *matrix laboratory*. MATLAB was originally written to provide easy access to matrix software developed by the LINPACK and EISPACK projects, which together represent the state-of-the-art in software for matrix computation.

MATLAB has evolved over a period of years with input from many users. In university environments, it is the standard instructional tool for introductory and advanced courses in mathematics, engineering, and science. In industry, MATLAB is the tool of choice for high-productivity research, development, and analysis.

MATLAB features a family of application-specific solutions called *toolboxes*. Very important to most users of MATLAB, toolboxes allow you to *learn* and *apply* specialized technology. Toolboxes are comprehensive collections of MATLAB functions (M-files) that extend the MATLAB environment to solve particular classes of problems. Areas in which toolboxes are available include signal processing, control systems, neural networks, fuzzy logic, wavelets, simulation, and many others.

The MATLAB System

The MATLAB system consists of five main parts:

The MATLAB Language. This is a high-level matrix/array language with control flow statements, functions, data structures, input/output, and object-oriented programming features. It allows both “programming in the small” to rapidly create quick and dirty throw-away programs, and “programming in the large” to create complete large and complex application programs. The language features are organized into six directories in the MATLAB Toolbox:

`ops`	Operators and special characters.
`lang`	Programming language constructs.
`strfun`	Character strings.
`iofun`	File input/output.
`timefun`	Time and dates.
`datatypes`	Data types and structures.

The MATLAB Working Environment. This is the set of tools and facilities that you work with as the MATLAB user or programmer. It includes facilities for managing the variables in your workspace and importing and exporting data. It also includes tools for developing, managing, debugging, and profiling M-files, MATLAB’s applications. The working environment features are located in a single directory.

`general`	General purpose commands.

Handle Graphics®. This is the MATLAB graphics system. It includes high-level commands for two-dimensional and three-dimensional data visualization, image processing, animation, and presentation graphics. It also includes low-level commands that allow you to fully customize the appearance of graphics as well as to build complete graphical user interfaces (GUIs) for your

MATLAB applications. The graphics functions are organized into five directories in the MATLAB Toolbox.

`graph2d`	Two-dimensional graphs.
`graph3d`	Three-dimensional graphs.
`specgraph`	Specialized graphs.
`graphics`	Handle Graphics.
`uitools`	Graphical user interface tools.

The MATLAB Mathematical Function Library. This is a vast collection of computational algorithms ranging from elementary functions like sum, sine, cosine, and complex arithmetic, to more sophisticated functions like matrix inverse, matrix eigenvalues, Bessel functions, and fast Fourier transforms. The math and analytic functions are organized into eight directories in the MATLAB Toolbox.

`elmat`	Elementary matrices and matrix manipulation.
`elfun`	Elementary math functions.
`specfun`	Specialized math functions.
`matfun`	Matrix functions – numerical linear algebra.
`datafun`	Data analysis and Fourier transforms.
`polyfun`	Interpolation and polynomials.
`funfun`	Function functions and ODE solvers.
`sparfun`	Sparse matrices.

The MATLAB Application Program Interface (API). This is a library that allows you to write C and Fortran programs that interact with MATLAB. It includes facilities for calling routines from MATLAB (dynamic linking), calling MATLAB as a computational engine, and for reading and writing MAT-files.

How to Use the Documentation Set

MATLAB comes with an extensive set of documentation consisting of an online Help facility and online MATLAB Function Reference as well as printed manuals. The full set of printed documentation includes the following titles:

- The *MATLAB Installation Guide* describes how to install MATLAB on your platform.
- *Getting Started with MATLAB* explains how to get started with the fundamentals of MATLAB.
- *Using MATLAB* provides in depth material on the MATLAB language, working environment, and mathematical topics.
- *Using MATLAB Graphics* describes how to use MATLAB's graphics and visualization tools.
- The *MATLAB Application Program Interface Guide* explains how to write C or Fortran programs that interact with MATLAB.
- *MATLAB New Features* provides information useful in making the transition from the latest release of MATLAB.

What I Want	**What I Should Do**
I need to install MATLAB.	See the *Installation Guide* for your platform.
I'm new to MATLAB and want to learn it quickly.	Start by reading *Getting Started with MATLAB*. The most important things to learn are how to enter matrices, how to use the : (colon) operator, and how to invoke functions. After you master the basics, you can access the rest of the documentation as needed, or you can use online help and the demonstrations to learn other commands.
I'm upgrading from an earlier release.	Read the *New Features* document to find out about the new features in the latest release. Pay special attention to the section about upgrading for guidance on converting your M-files. You should then refer to *Using MATLAB* and *Using MATLAB Graphics* for specific details about the new features.

What I Want	What I Should Do
I want to know how to use a specific function.	Use the online Help facility. You can use the M-file help window to get brief online help or access the MATLAB Function Reference via the Web-based Help Desk. These are available using the commands `helpwin` and `helpdesk` or from the **Help** menu on the PC. The Function Reference is also available on the Help Desk in PDF format if you want to print out any of the function descriptions in high-quality form.
I want to find a function for a specific purpose but I don't know its name.	There are three choices. • Use `lookfor` (e.g. `lookfor inverse`) from the command line. • Use the online keyword search from the Help Desk. • Visit The MathWorks Web site and see if there is a user-contributed file to solve your problem.
I want to learn about a specific topic like sparse matrices, ordinary differential equations, or cell arrays.	See the appropriate chapter in *Using MATLAB*.
I want to know what functions are available in a general area.	Use the help window (type `helpwin` or select from **Help** menu) to see a table of contents with functions grouped by subject area, or use the Help Desk (type `helpdesk` or select from **Help** menu) to see the Function Reference grouped by subject.
I have a problem I want help with.	For tips and troubleshooting problems, use the Help Desk (type `helpdesk` or select from **Help** menu) to visit the Technical Support section of The MathWorks Web site (`www.mathworks.com`) and use the Solution Search Engine to search the Technical Support database of problem solutions.
I want to report a bug or make a suggestion.	Use the Help Desk (type `helpdesk` or select from **Help** menu) or send e-mail to `bugs@mathworks.com` or `suggest@mathworks.com`.
I want to contact The MathWorks Technical Support.	Use the Help Desk (type `helpdesk` or select from **Help** menu) to submit an e-mail help request form describing your question or problem.

About Simulink

Simulink®, a companion program to MATLAB, is an interactive system for simulating nonlinear dynamic systems. It is a graphical mouse-driven program that allows you to model a system by drawing a block diagram on the screen and manipulating it dynamically. It can work with linear, nonlinear, continuous-time, discrete-time, multivariable, and multirate systems.

Blocksets are add-ins to Simulink that provide additional libraries of blocks for specialized applications like communications, signal processing, and power systems.

Real-time Workshop® is a program that allows you to generate C code from your block diagrams and to run it on a variety of real-time systems.

About Toolboxes

MATLAB features a family of application-specific solutions called toolboxes. Very important to most users of MATLAB, toolboxes allow you to learn and apply specialized technology. Toolboxes are comprehensive collections of MATLAB functions (M-files) that extend the MATLAB environment in order to solve particular classes of problems. Many toolboxes are available from The MathWorks. Some of these are listed on the following page; contact The MathWorks or visit `www.mathworks.com` for a complete up-to-date list.

The MATLAB Product Family

How The MathWorks products fit together

MATLAB is the foundation for all The MathWorks products. MATLAB combines numeric computation, 2-D and 3-D graphics, and language capabilities in a single, easy-to-use environment.

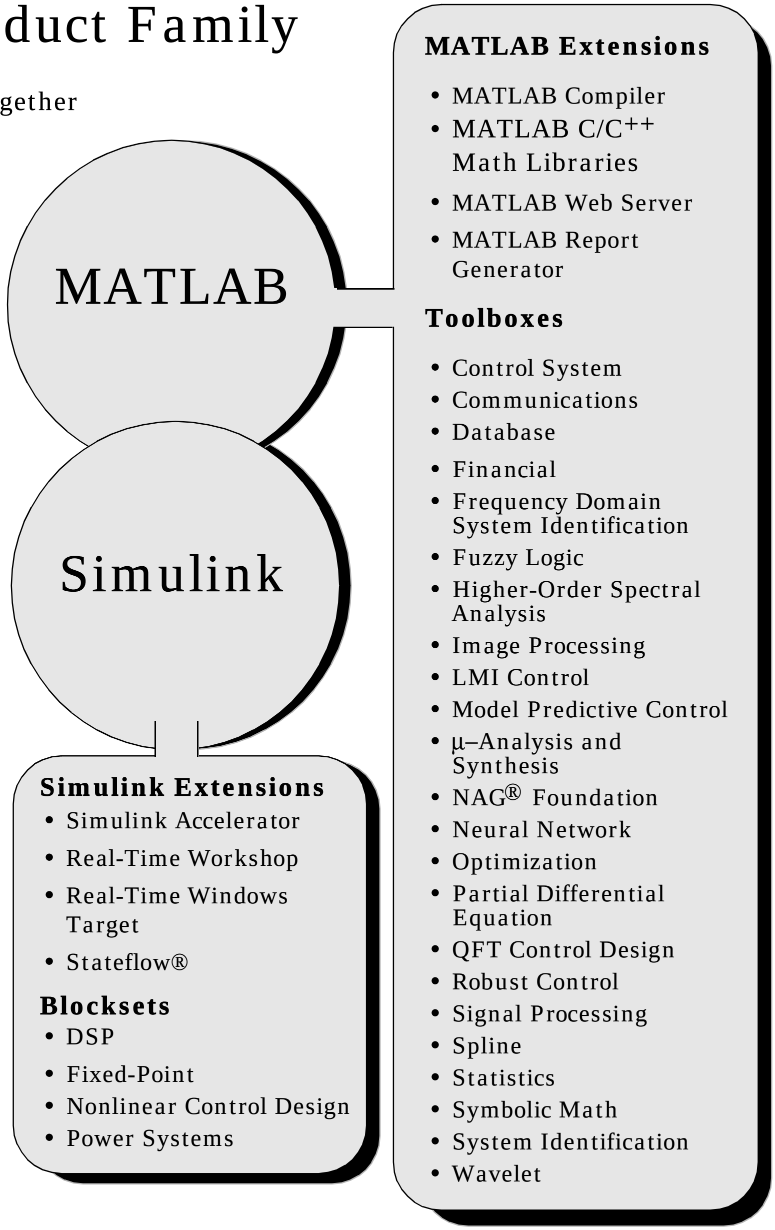
MATLAB Extensions are optional tools that support the implementation of systems developed in MATLAB.

Toolboxes are libraries of MATLAB functions that customize MATLAB for solving particular classes of problems. Toolboxes are open and extensible; you can view algorithms and add your own.

Simulink is a system for nonlinear simulation that combines a block diagram interface and “live” simulation capabilities with the core numeric, graphics, and language functionality of MATLAB.

Simulink Extensions are optional tools that support the implementation of systems developed in Simulink

Blocksets are collections of Simulink blocks designed for use in specific application areas.



MATLAB

Simulink

MATLAB Extensions

- MATLAB Compiler
- MATLAB C/C++ Math Libraries
- MATLAB Web Server
- MATLAB Report Generator

Toolboxes

- Control System
- Communications
- Database
- Financial
- Frequency Domain System Identification
- Fuzzy Logic
- Higher-Order Spectral Analysis
- Image Processing
- LMI Control
- Model Predictive Control
- μ–Analysis and Synthesis
- NAG® Foundation
- Neural Network
- Optimization
- Partial Differential Equation
- QFT Control Design
- Robust Control
- Signal Processing
- Spline
- Statistics
- Symbolic Math
- System Identification
- Wavelet

Simulink Extensions

- Simulink Accelerator
- Real-Time Workshop
- Real-Time Windows Target
- Stateflow®

Blocksets

- DSP
- Fixed-Point
- Nonlinear Control Design
- Power Systems

Contact The MathWorks or visit `www.mathworks.com` for an up-to-date product list.

MATLAB Working Environment

Using the Environment

MATLAB is both a language and a working environment. This chapter focuses on the MATLAB working environment. As a working environment, MATLAB includes facilities for managing the variables in your workspace and for importing and exporting data. MATLAB also includes tools for developing and managing M-files, MATLAB's applications.

The first part of this chapter describes general aspects of using the MATLAB working environment. In this and subsequent chapters, when it is necessary in this general material to call out features specific to a particular platform, we use icons in the text margin to highlight the information pertinent to your platform. Look for:

⊞ for Microsoft Windows information

X for UNIX information

Additional sections at the end of this chapter discuss further platform-specific MATLAB environment features:

- "Microsoft Windows Handbook"
- "UNIX Handbook"

Starting MATLAB

On Windows platforms, the installer creates a shortcut to the program file in the installation directory. You can move this shortcut to your desktop if you want. Double-click on this shortcut icon to start MATLAB.

To start MATLAB on a UNIX system, type `matlab` at the operating system prompt.

Startup Files

At startup, MATLAB automatically executes the master M-file `matlabrc.m` and, if it exists, `startup.m`.

The file `matlabrc.m`, which lives in the `local` directory, is reserved for use by The MathWorks and, on multiuser systems, by your system manager.

The file `startup.m` is for you to use. You can set default paths, define Handle Graphics defaults, or predefine variables in your workspace. For example, creating a `startup.m` with the line

```
addpath /home/me/mytools
```

adds a tools directory to your default search path.

On Windows platforms, place the `startup.m` file in the folder named `local` in the `toolbox` folder.

On UNIX workstations, place the `startup.m` file in the directory named `matlab` off of your home directory, e.g., `~/matlab`.

Startup Options

You can specify startup options for MATLAB.

Add these options to the target path for your Windows shortcut for MATLAB. If you run MATLAB from a DOS window, include these options with the startup command.

Startup Option	**Description**
`automation`	Start MATLAB as an automation server, minimized, and without the MATLAB splash screen. (For more information, see Chapter 7 of the *Application Program Interface Guide*.)
`logfile logfilename`	Automatically write output from MATLAB to the specified log file.
`minimize`	Start MATLAB minimized and without the MATLAB splash screen.
`nosplash`	Start MATLAB without displaying the MATLAB splash screen.
`r M_file`	Automatically run the specified M-file immediately after MATLAB starts.

`regserver` Modify the Windows registry with the appropriate ActiveX entries for MATLAB. (For more information, see Chapter 7 of the *Application Program Interface Guide*.)

`unregserver` Modify the Windows registry to remove the ActiveX entries for MATLAB. Use this to reset the registry. (For more information, see Chapter 7 of the *Application Program Interface Guide*.)

For example, to start MATLAB and automatically run the file `results.m`, use this target path for your Windows shortcut:

```
D:\bin\nt\matlab.exe /r results
```

For the same example, if you start MATLAB from a DOS window, automatically run the file `results.m` upon startup by typing

```
matlab /r results
```

For a list of MATLAB startup options available for UNIX, at the UNIX prompt type

```
matlab -h
```

Quitting MATLAB

To quit MATLAB at any time, type `quit` at the MATLAB prompt.

On Windows platforms, you can also quit by selecting **Exit** from the **File** menu, or by using the close box.

`quit` runs the script `finish.m`, if `finish.m` exists anywhere on the MATLAB path. `finish.m` is a file you create that contains commands you want to run when MATLAB terminates. For example, include a `save` command in your `finish.m` file to save the workspace when MATLAB quits. Or in your `finish.m` file, include code that will display a confirmation dialog box when you quit MATLAB. Two sample `finish.m` files are in `/toolbox/local`:

- `finishsav.m` – saves the workspace to a MAT-file when MATLAB quits
- `finishdlg.m` – displays a dialog allowing you to cancel quitting

The Command Window

The Command Window is the main window in which you communicate with MATLAB.

On Windows platforms, MATLAB provides a special window with Windows-only features.

On UNIX systems, the Command Window is the terminal window from which you start MATLAB.

The MATLAB interpreter displays a prompt (>>) indicating that it is ready to accept commands from you. For example, to enter a 3-by-3 matrix, you can type

```
A = [1 2 3; 4 5 6; 7 8 10]
```

When you press the **Enter** or **Return** key, MATLAB responds with

```
A =

       1     2     3
       4     5     6
       7     8    10
```

To invert this matrix, enter

```
B = inv(A)
```

MATLAB responds with the result.

Command Line Editing

Arrow and control keys on your keyboard allow you to recall, edit, and reuse commands you have typed earlier. For example, suppose you mistakenly enter

```
rho = (1+ sqt(5))/2
```

You have misspelled `sqrt`. MATLAB responds with

```
Undefined function or variable 'sqt'.
```

Instead of retyping the entire line, simply press the ↑ key. The misspelled command is redisplayed. Use the ← key to move the cursor over and insert the missing `r`. Repeated use of the ↑ key recalls earlier lines.

The commands you enter during a MATLAB session are stored in a buffer. You can use *smart recall* to recall a previous command whose first few characters you specify. For example, typing the letters `plo` and pressing the ↑ key recalls the last command that started with `plo`, as in the most recent `plot` command.

The complete list of arrow and control keys provides additional control. Many of these keys should be familiar to users of the Emacs editor.

Arrow Key	Control Key	Operation
↑	**Ctrl-p**	Recall *p*revious line.
↓	**Ctrl-n**	Recall *n*ext line.
←	**Ctrl-b**	Move *b*ack one character.
→	**Ctrl-f**	Move *f*orward one character.
ctrl- →	**Ctrl-r**	Move *r*ight one word.
ctrl- ←	**Ctrl-l**	Move *l*eft one word.
home	**Ctrl-a**	Move to beginning of line.
end	**Ctrl-e**	Move to *e*nd of line.
esc	**Ctrl-u**	Clear line.
del	**Ctrl-d**	Delete character at cursor.
backspace	**Ctrl-h**	Delete character before cursor.
	Ctrl-k	Delete (*k*ill) to end of line.

Clearing the Command Window

Use `clc` to clear the Command Window. This does not clear the workspace, but only clears the view. After using `clc`, you still can use the up arrow key to see the history of the commands, one at a time.

Paging of Output in the Command Window

To control paging of output in the Command Window, use `more`. By default, `more` is `off`. When you set `more on`, a page (a screen full) of output displays at one time. You then use

Return	To advance to the next line
Space Bar	To advance to the next page
q	To stop displaying the output

Interrupting a Running Program

You can interrupt a running program by pressing **Ctrl-c** at any time.

On Windows platforms, you may have to wait until an executing built-in function or MEX-file has finished its operation.

On UNIX systems, program execution will terminate immediately.

The format Command

The `format` command controls the numeric format of the values displayed on the screen. The command affects only how numbers are displayed, not how MATLAB computes or saves them.

On Windows platforms, you can change the default format by selecting **Preferences** from the **File** menu, and selecting the desired format from the **General** tab.

Here are various formats and the output produced from a two-element vector with components of different magnitudes.

```
x = [4/3 1.2345e–6]

format short

   1.3333    0.0000

format short e

   1.3333e+000  1.2345e–006

format short g

   1.3333  1.2345e–006

format long

   1.33333333333333   0.00000123450000

format long e

   1.333333333333333e+000    1.234500000000000e–006

format long g

   1.33333333333333               1.2345e–006

format bank

   1.33          0.00
```

```
format +

++

format rat

   4/3          1/810045

format hex

   3ff5555555555555   3eb4b6231abfd271
```

If the largest element of a matrix is larger than 10^3 or smaller than 10^{-3}, MATLAB applies a common scale factor for the short and long formats.

In addition to the `format` commands shown above

```
format compact
```

suppresses many of the blank lines that appear in the output. This lets you view more information on a screen or window. To show the blank lines, use

```
format loose
```

If you want more control over the output format, use the `sprintf` and `fprintf` functions.

Suppressing Output

If you simply type a statement and press **Return** or **Enter**, MATLAB automatically displays the results on screen. However, if you end the line with a semicolon, MATLAB performs the computation but does not display any output. This is particularly useful when you generate large matrices. For example,

```
A = magic(100);
```

Long Command Lines

If a statement does not fit on one line, use an ellipsis (three periods, ...), followed by **Return** or **Enter** to indicate that the statement continues on the next line. For example,

```
s = 1 – 1/2 + 1/3 – 1/4 + 1/5 – 1/6 + 1/7 ...
      – 1/8 + 1/9 – 1/10 + 1/11 – 1/12;
```

Blank spaces around the =, +, and – signs are optional, but they improve readability. The maximum number of characters allowed on a single line is 4096.

MATLAB Workspace

The MATLAB workspace contains a set of variables (named arrays) that you can manipulate from the MATLAB command line. You can use the who and whos commands to see what is currently in the workspace. The who command gives a short list, while the whos command also gives size and data type information.

Here is the output produced by whos on a workspace containing eight variables of different data types.

```
whos
  Name      Size         Bytes  Class

  A         4x4            128  double array
  D         3x5            120  double array
  M        10x1             40  cell array
  S         1x3            628  struct array
  h         1x11            22  char array
  n         1x1              8  double array
  s         1x5             10  char array
  v         1x14            28  char array

Grand total is 93 elements using 984 bytes
```

To delete all existing variables from the workspace, enter

```
clear
```

Loading and Saving the Workspace

MATLAB's save and load commands let you save the contents of the MATLAB workspace at any time during a session and then reload the data back into MATLAB during that session or a later one. load and save can also import and export text data files.

Saving the Workspace

The save command saves the contents of the workspace into a binary MAT-file that you can read back later with the load command. For example,

```
save june10
```

saves the entire workspace contents in the file june10.mat. If desired, you can save only certain variables by specifying the variable names after the filename.

For example,

```
save june10 x y z
```

saves only variables x, y, and z.

On Windows platforms, the save operation is also available by selecting **Save Workspace As** from the **File** menu.

Note The *MATLAB Application Program Interface Guide* provides details on reading and writing MAT-files from external C or Fortran programs.

Specifying File Format

You can control the format in which save stores data by appending flags to the filename/variable name list:

`–mat`	Use binary MAT-file form (default).
`–ascii`	Use 8-digit ASCII form.
`–ascii –double`	Use 16-digit ASCII form.
`–ascii –double –tabs`	Delimit array elements with tabs.
`–v4`	Save in format that MATLAB version 4 can open.
`–append`	Append data to existing MAT-file.

If you use the `v4` flag, you can only save data constructs that are compatible with versions of MATLAB 4; therefore, you cannot save structures, cell arrays, multidimensional arrays, or objects. In addition, you must use filenames that are supported by MATLAB version 4.

When you save workspace contents in ASCII format, save only one variable at a time. If you save more than one variable, MATLAB will create the ASCII file, but you will be unable to load it back into MATLAB later using `load`.

Loading the Workspace

The `load` command loads a MAT-file that you have previously created with `save`. For example,

```
load june10
```

loads `june10.mat` into the workspace. If the saved MAT-file `june10` contains the variables `A`, `B`, and `C`, then loading `june10` places the variables `A`, `B`, and `C` back into the workspace. If the variables already exist in the workspace, they are overwritten.

If your MAT-file has a filename extension other than `.mat`, you must use the `-mat` switch or else MATLAB expects the file to be ASCII text format.

```
load filename -mat
```

On Windows platforms, the `load` operation is also available by selecting **Load Workspace** from the **File** menu.

Loading ASCII Data Files

The load command also imports ASCII data files. It reads the contents of the file into a variable with the same name as the file (without the extension). For example,

```
load tides.dat
```

creates a variable named tides in the workspace. If the ASCII data file has m lines with n values on each line, the result is an *m*-by-*n* numeric array.

Filenames Stored in String Variables

If the filenames and variable names you are working with are stored in string variables, you can use command/function duality to call load and save as functions. In this case, the input arguments appear in the same order as they would at the command line. For example, the statements

```
save('myfile','VAR1','VAR2')
A = 'myfile';
load(A)
```

are the same as

```
save myfile VAR1 VAR2
load myfile
```

To load or save multiple files with the same prefix and successive integer suffixes, use a loop. For example, this code saves the squares of the numbers 1 through 10 in files data1 through data10:

```
file = 'data';
for i = 1:10
    j = i.^2;
    save([file int2str(i)],'j');
end
```

Wildcards

The load and save commands let you specify a wildcard character (*) to search for patterns of variable names. For example,

```
save rundate x*
```

saves all variables in the workspace that start with x in the file rundata.mat. Similarly,

```
load testdata ex1*95
```

loads from testdata.mat all the variables whose first three characters are 'ex1' and last two characters are '95', regardless of the characters between them.

Search Path

MATLAB uses a *search path* to find M-files. MATLAB's M-files are organized in directories or folders on your file system. Many of these directories of M-files are provided along with MATLAB, while others are available separately as toolboxes.

If you enter the name foo at the MATLAB prompt, the MATLAB interpreter:

1. Looks for foo as a variable.

2. Checks for foo as a built-in function.

3. Looks in the current directory for a file named foo.m.

4. Searches the directories on the search path for foo.m.

Although the actual search rules are more complicated because of the restricted scope of private functions, subfunctions, and object-oriented functions, this simplified perspective is accurate for the ordinary M-files that you usually work with.

If you have more than one function with the same name, only the first one in the search path order is found; other functions with the same name are considered to be shadowed and cannot be executed.

Changing the Search Path

You can display and change the search path for the duration of your current session using the path, addpath, and rmpath functions:

- path, by itself, returns the current search path.
- path(s), where s is a string, sets the path to s.

- `addpath /home/lib` and `path(path,'/home/lib')` both append a new directory to the path.
- `rmpath /home/lib` removes the path `/home/lib`.

The default search path remembered between sessions is defined in the file `pathdef.m` in the directory named `local` on your system. `pathdef` executes automatically each time you start MATLAB.

On Windows platforms, you can directly edit `pathdef.m` with your text editor.

On UNIX workstations you may not have file system permission to edit `pathdef.m`. In this case, put `path` and `addpath` commands in your `startup.m` file to change your path defaults.

MATLAB also provides a Path Browser with a convenient interface for viewing and changing the search path. Use `pathtool` to start the Path Browser.

Files on the Search Path

To display the search path, use `path`. Use `what` to see all of the MATLAB files in a directory. With no arguments, `what` displays the files in the current directory.

With a full or partial path, `what` lists the files in any directory on the path, for example,

```
what matlab/elfun
```

To see the code in a specific M-file, use the `type` command, for example,

```
type rank
```

To edit the M-file, use `edit`, for example,

```
edit rank
```

Note Save any M-files you create or any MATLAB-supplied M-files that you edit in a directory that is not in the `MATLAB` directory tree. If you keep your files in the `MATLAB` directory tree, they might be overwritten when you install a new version of MATLAB. Another consideration is that files in the `MATLAB/toolbox` directory tree are loaded and cached into memory at the beginning of each MATLAB session to improve performance. This cache is not updated until MATLAB is restarted. If you add any files or make changes to any files in the `toolbox` directory, you will not be able to see the changes until you restart MATLAB.

Current Directory

MATLAB maintains a *current directory* for working with M-files and MAT-files.

On Windows platforms, the initial current directory is specified in the shortcut file you use to start MATLAB. Right-click on the shortcut file, and select **Properties** to change the default.

On UNIX systems, the initial current directory is the directory you are in on your UNIX file system when you invoke MATLAB.

To display your current directory, use the `cd` command with no arguments. For example, on UNIX:

```
cd
/home/roger
```

To change your current directory, use `cd` with a path. For example, for Windows

```
cd \bigproj\phase1
```

changes the current directory to the `phase1` directory, located in `bigproj`.

Opening Files in MATLAB

You can open files in MATLAB based on their extension using the `open` function. `open` is a user-extensible function that provides an interface to file open operations. Default behavior is provided for these standard MATLAB file types. You can extend the interface to include other file types and to override the default behavior for the standard files.

Name	Action
Figure file (`*.fig`)	Open figure in a figure window
M-file (`name.m`)	Open M-file `name` in Editor
Model (`name.mdl`)	Open model `name` in Simulink
P-file (`name.p`)	Open the corresponding M-file, `name.m`, if it exists, in the Editor
Variable	Open array `name` in the Array Editor (the array must be numeric); `open` calls `openvar`
Other extensions (`name.custom`)	Open `name.custom` by calling the helper function `opencustom`, where `opencustom` is a user-defined function.

The Figure Window

MATLAB directs graphics output to a window that is separate from the Command Window. In MATLAB, this window is referred to as a figure. Graphics functions automatically create new figure windows if none currently exist. If a figure window already exists, MATLAB uses that window. If multiple figure windows exist, one is designated as the current figure and is used by MATLAB (this is generally the last figure used or the last figure you clicked the mouse in).

The `figure` function creates figure windows. For example,

```
figure
```

creates a new window and makes it the current figure.

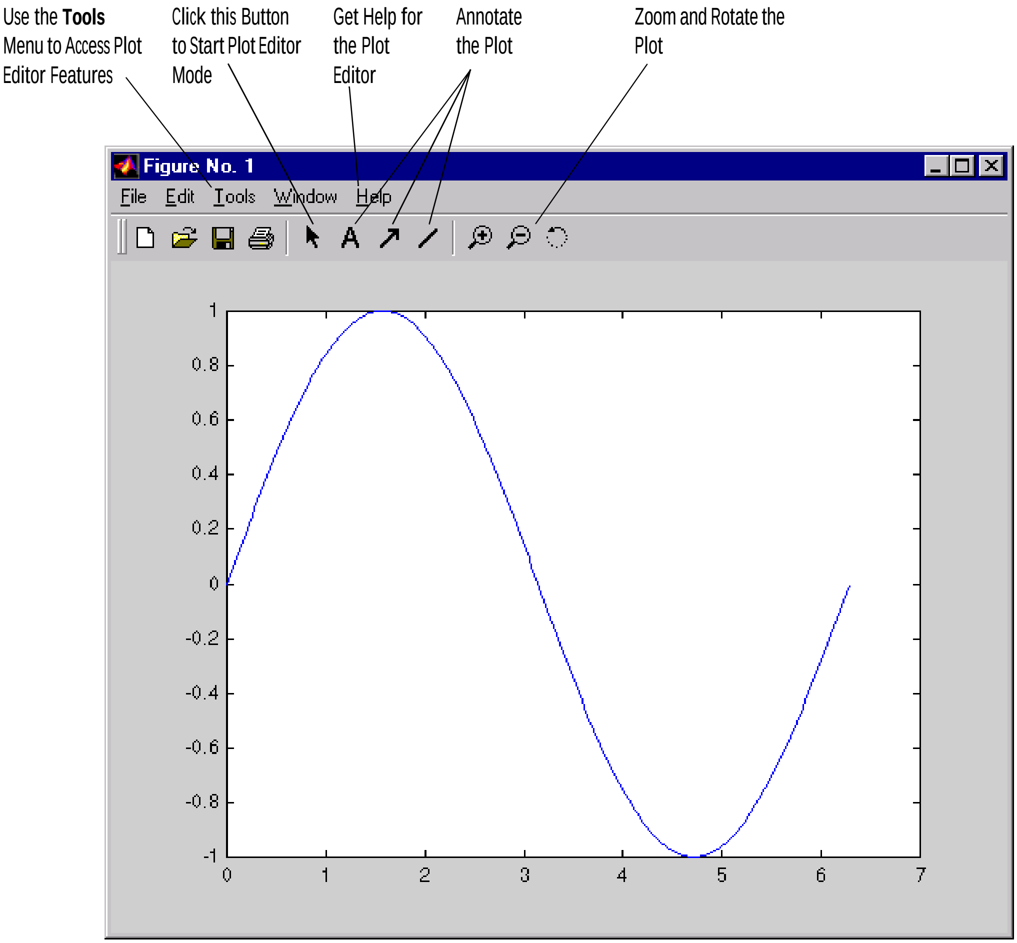
The plot function creates a plot in a figure window. For example,

```
t = 0:pi/100:2*pi;
y = sin(t);
plot(t,y)
```

draws a graph of the sine function from zero to 2π in the current figure window, if one exists, or in a new figure window if none exists.

Annotating Plots Using the Plot Editor

After creating a plot, you can make changes to it and annotate it with the Plot Editor, which is an easy-to-use graphical interface. The illustration below shows the plot in a figure window and labels the main features of the figure window and the Plot Editor.


Use the **Tools** Menu to Access Plot Editor Features
Click this Button to Start Plot Editor Mode
Get Help for the Plot Editor
Annotate the Plot
Zoom and Rotate the Plot
Figure No. 1
File
Edit
Tools
Window
Help
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7

To save a figure, select **Save** from the **File** menu. To save it using a different format, such as TIFF, for use with other applications, select **Export** from the **File** menu. You can also save from the command line – use the saveas command, including any options to save the figure in a different format.

MATLAB Graphics

For more information about visualization with MATLAB, see *Using MATLAB Graphics*.

Help and Online Documentation

Command Line Help

There are several different ways to access online information about MATLAB functions:

Command	Description
`help`	Display in the Command Window a description of the specified command
`helpwin`	Display a help window that describes the specified command and allows viewing help for other topics
`lookfor`	Display in the Command Window a brief description for all commands whose description includes the specified keyword
`helpdesk`	Display the Help Desk page in a Web browser, providing direct access to a comprehensive library of online help, PDF-formatted documentation, troubleshooting information, and The MathWorks Web site
`doc`	Display in a Web browser the reference page for the specified command, providing a description, additional remarks, and examples

The help Command

The `help` command is the most basic way to determine the syntax and behavior of a particular function. Information is displayed directly in the Command Window. For example,

```
help magic
```

displays

```
MAGIC  Magic square.
    MAGIC(N) is an N–by–N matrix constructed from
    the integers 1 through N^2 with equal row,
    column, and diagonal sums.
    Produces valid magic squares for N = 1,3,4,5....
```

Note MATLAB Command Window `help` entries use uppercase characters for the function and variable names to make them stand out from the rest of the text. When typing function names, however, always use the corresponding lowercase characters since MATLAB is case sensitive and all function names are actually in lowercase.

All the MATLAB functions are organized into logical groups, and MATLAB's directory structure is based on this grouping. For instance, all the linear algebra functions reside in the `matfun` directory. To list the names of all the functions in that directory, with a brief description of each, use

```
help matfun

Matrix functions – numerical linear algebra.

Matrix analysis.
  norm        – Matrix or vector norm.
  normest     – Estimate the matrix 2–norm
...
```

The command

```
help
```

by itself, lists all the directories, with a description of the function category each represents:

```
matlab/general
matlab/ops
...
```

The helpwin Command

The MATLAB Help Window is available by typing

```
helpwin
```

The Help Window gives you access to the same information as the `help` command, but the window interface provides convenient links to other topics.

To use the Help Window on a particular topic, type

```
helpwin topic
```

On Windows platforms, you can also access the **Help Window** by selecting the **Help Window** option under the **Help** menu, or by clicking the question mark button on the menu bar.

The lookfor Command

The `lookfor` command allows you to search for functions based on a keyword. It searches through the first line of `help` text, which is known as the H1 line, for each MATLAB function, and returns the H1 lines containing a specified keyword. For example, MATLAB does not have a function named `inverse`. So the response from

```
help inverse
```

is

```
inverse.m not found.
```

But

```
lookfor inverse
```

finds over a dozen matches. Depending on which toolboxes you have installed, you will find entries like

```
INVHILB Inverse Hilbert matrix
ACOSH   Inverse hyperbolic cosine
ERFINV  Inverse of the error function
INV     Matrix inverse
PINV    Pseudoinverse
IFFT    Inverse discrete Fourier transform
IFFT2   Two–dimensional inverse discrete Fourier transform
ICCEPS  Inverse complex cepstrum
IDCT    Inverse discrete cosine transform
```

Adding `-all` to the `lookfor` command searches the entire help entry, not just the H1 line.

The helpdesk Command

The MATLAB Help Desk provides access to a wide range of help and reference information stored on a disk or CD in your local system. Many of the underlying documents use HyperText Markup Language (HTML) and are accessed with an Internet Web browser such as Netscape Navigator or Microsoft Internet Explorer.

To access the Help Desk, type

```
helpdesk
```

On Windows platforms, you can also access the Help Desk by selecting the **Help Desk** option under the **Help** menu.

All of MATLAB's operators and functions have online reference pages in HTML format, which you can reach from the Help Desk. These pages provide more details and examples than the basic `help` entries. HTML versions of other documents are also available. A search engine can query all the online reference material.

PDF-formatted Documentation. Versions of all MATLAB documentation are available in Portable Document Format (PDF) through the Help Desk. These pages are processed by Adobe's Acrobat Reader. They reproduce the look and feel of the printed page, complete with fonts, graphics, formatting, and images. You can use links from one table of contents or index of a manual, as well as internal links, to go directly to the page of interest.

Acrobat Reader also allows you to print selected pages within a document. This is the best way to get printed copies of the online MATLAB Function Reference, which is not otherwise available in hard copy.

MathWorks Web Site. If your computer is connected to the Internet, the Help Desk provides connections to The MathWorks, the home of MATLAB. You can use electronic mail to ask questions, make suggestions, and report possible bugs. You can also use the Solution Search Engine at The MathWorks Web site to query an up-to-date data base of technical support information.

Alternatively, you can point your Web browser directly at `www.mathworks.com` to access The MathWorks Web site.

The doc Command

The `doc` command accesses the HTML reference documentation for MATLAB functions and all installed toolboxes.

For example, if you have the Control System Toolbox and Symbolic Math Toolbox installed as well as MATLAB, when you enter at the MATLAB command line

```
doc eig
```

you will see the HTML reference documentation page reflecting the MATLAB version of eig. In addition, in the Command Window you will see

```
Overloaded functions:
   doc control/eig
   doc symbolic/eig
```

To see the documentation for either of those versions of the eig function, issue the appropriate doc command with the proper path, as shown above.

The doc command starts your Web browser if it is not already running.

Help Menus and Help Buttons

In the figure window use the **Help** menu to access help for the Plot Editor, for MATLAB Graphics and to access the Help Window and Help Desk. In the **PrintFrame Editor** window, use the **Help** menu to access help for editing printframes.

In the **Page Setup** dialog box and in the Plot Editor dialog boxes, click the **Help** button to access help for those dialog boxes.

Disk File Manipulation and Shell Escape

The commands dir, type, delete, and cd implement a set of generic operating system commands for manipulating files. This table indicates how these commands map to other operating systems.

MATLAB Commands	Windows	UNIX
dir	dir	ls
type	type	cat
delete	del or erase	rm
cd	chdir	cd

For most of these commands, you can use pathnames, wildcards, and drive designators in the usual way.

Running External Programs

The exclamation point character ! is a *shell escape* and indicates that the rest of the input line is a command to the operating system. This is quite useful for invoking utilities or running other programs without quitting from MATLAB. On UNIX, for example,

```
!vi darwin.m
```

invokes the *vi* editor for a file named darwin.m. After you quit the program, the operating system returns control to MATLAB.

See the commands unix and dos in online help to run external programs that return results and status.

Data Import/Export

There are many ways to move data between MATLAB and other applications. In most cases, you can simply use MATLAB's native data exchange capabilities to read in or write out files. For more complicated data sets, you may want to create your own C or Fortran program to read or write a file.

Importing Data into MATLAB

You can introduce data from other programs into MATLAB using several methods. The best method for importing data depends on the amount and format of the data.

Method	When to Use
Enter data as an explicit list of elements	If you have a small amount of data, it is easy to type the data explicitly using brackets ([ ]). This method is awkward for larger amounts of data because you can't edit your input if you make a mistake, but must correct it using assignment statements. See *Getting Started with MATLAB* for more information on this technique.
Create data in an M-file	Use a text editor to create an M-file that enters the data as an explicit list of elements. This method is useful when the data is not already in digital form and must be entered anyway. Although similar to the first method, this method has the advantage of allowing you to use your text editor to change the data and to fix mistakes. You can then just rerun the M-file to re-enter the data.

Load data from an ASCII data file	An ASCII data file stores the data in ASCII form, with each row having the same number of values, and terminating with new lines (carriage returns), with spaces separating the numbers. You can edit ASCII data files using a normal text editor. You can read ASCII data files directly into MATLAB using the `load` function. This creates a variable whose name is the same as the filename. See “Loading and Saving the Workspace” for details on `load`. You can also use `dlmread` if you need to specify alternate value delimiters. `dlmread` is discussed in “Delimiter-Separated Text Files”.	
Read data using `fopen`, `fread`, and MATLAB’s file I/O functions	This method is useful for loading data files from other applications that have their own established file formats. These functions are discussed in detail in Chapter 15.	
Use a specialized file reader function for application-specific formats	`dlmread`	Read ASCII data file.
	`textread`	Read string and numeric data from a file into MATLAB variables using conversion specifiers.
	`wk1read`	Read spreadsheet (`WK1`) file.
	`imread`	Read image from graphics file.
	`auread`	Read Sun (`.au`) sound file.
	`wavread`	Read Microsoft WAVE (`.wav`) sound file.

Develop a MEX-file to read the data	This is the best method if C or Fortran routines are already available for reading data files from other applications. See the *MATLAB Application Program Interface Guide* for more information.
Develop a Fortran or C translation program	Develop a program to translate your data into MAT-file format and then read the MAT-file into MATLAB with the load command. See the *MATLAB Application Program Interface Guide* for more information.

Exporting Data from MATLAB

There are several methods for getting MATLAB data to other applications:

Method	**When to Use**
Use the `diary` command	For small arrays, use the `diary` command to create a diary file and display the variables, echoing them into this file. The output of `diary` includes the MATLAB commands used during the session, which is useful for inclusion in documents and reports. You can use your text editor to edit the diary file, removing unwanted text.
Save the data in ASCII form	Use the `save` command with the `–ascii` option. See “Loading and Saving the Workspace” for details on `save`. You can also use `dlmwrite` if you need to specify alternate value delimiters. `dlmwrite` is discussed in “Delimiter-Separated Text Files”.
Write the data in a special format	Use `fwrite` and the other low-level I/O functions. This method is useful for writing data files in the file formats required by other applications. These functions are discussed in detail in Chapter 15.

Use a specialized file write function for application-specific formats	`dlmwrite`	Write ASCII data file.
	`wk1write`	Write spreadsheet (`WK1`) file.
	`imwrite`	Write image to graphics file.
	`auwrite`	Write Sun (`.au`) sound file.
	`wavwrite`	Write Microsoft WAVE (`.wav`) sound file.
Develop a MEX-file to write the data	This is the best method if C or Fortran routines are already available for writing data files in the form needed by other applications. See the *MATLAB Application Program Interface Guide* for more information.	
Write out the data as a MAT-file	Use the `save` command, and then write a program in Fortran or C to translate the MAT-file into the desired format. See the *MATLAB Application Program Interface Guide* for more information.	

Delimiter-Separated Text Files

The functions `dlmread` and `dlmwrite` let you read and write delimiter-separated values from an ASCII data file. A *delimiter* is any character that separates the file's values. These functions are also useful for reading or writing into a specific MATLAB variable name.

For example, consider a file named `ph.dat` whose contents are separated by semicolons.

```
7.2;8.5;6.2;6.6
5.4;9.2;8.1;7.2
```

To read the entire contents of this file into an array named `A`, use

```
A = dlmread('ph.dat', ';');
```

The second argument to dlmread specifies the delimiter, which in the previous example is a semicolon. In addition to the delimiter you specify, dlmread also interprets all whitespace characters as delimiters. So, the preceding dlmread command works even if the contents of ph.dat are

```
7.2;   8.5;       6.2;6.6
5.4;   9.2   ;8.1;7.2
```

Note The first argument to dlmread is a filename, not a file identifier. Do not open the file with fopen before using dlmread or dlmwrite.

Similarly, dlmwrite writes delimiter-separated text to an external file.

```
A =

     1     2     3
     4     5     6

dlmwrite('myfile',A,';')
```

myfile now contains

```
1;2;3
4;5;6
```

Reading Files that Have a Uniform Format

The function textread reads string and numeric data from a file into MATLAB variables using the conversion specifiers you indicate. Conversion specifiers denote, for example, the length of the field and the data format. textread is most useful for files with a known and uniform format, such as comma- or tab-delimited files, but you can also use it for reading free format files.

For example, the file mydata.dat is

```
Sally    Type1 12.34 45 Yes
```

To read mydata.dat as a free format file, use the % conversion format.

```
[names,types,x,y,answer] = textread('mydata.dat','%s %s %f ...
   %d %s',1)
```

where %s reads a whitespace-separated string, %f reads a floating point value, and %d reads a signed integer.

MATLAB returns

```
names =
    'Sally'
types =
    'Type1'
x =
  12.34000000000000
y =
    45
answer =
    'Yes'
```

See all of the allowable delimiters on the reference page for textread.

Exchanging Data Files Between Platforms

It's sometimes necessary to work with MATLAB implementations on different computer systems, or to transmit MATLAB applications to users on other systems. MATLAB applications consist of *M-files*, containing functions and scripts, and *MAT-files*, containing binary data. Both types of files can be transported directly between different computers:

- M-files consist of ordinary text. They are machine independent. While different platforms terminate lines with various combinations of CR (carriage return) and LF (line feed) characters, the MATLAB interpreter tolerates all possible combinations. (However, text editors and other tools may not work correctly with M-files from other platforms.)
- MAT-files are binary and machine dependent, but they can be transported between machines because they contain a machine signature in the file header. MATLAB checks the signature when it loads a file and, if a signature indicates that a file is foreign, performs the necessary conversion.

To use MATLAB across different platforms, you need a program for exchanging both binary and text data between the machines. When using these programs, be sure to transmit MAT-files in *binary file mode* and M-files in *ASCII file mode*. Failure to set these modes correctly usually corrupts the data.

The diary Command

The `diary` command creates a verbatim copy of your MATLAB session in a disk file (excluding graphics). You can view and edit the resulting text file using any word processor. To create a file on your disk called `sept23.out` that contains all the commands you enter, as well as MATLAB's output, enter

```
diary sept23.out
```

To stop recording the session, use

```
diary off
```

Memory Utilization

MATLAB requires a contiguous area of memory to store each matrix. In particular, images and movies can consume large amounts of memory. In addition to the storage required for the matrix, the pixmap used to draw the image requires memory proportional to the area of the image on the screen. A color image of 500-by-500 pixels uses one megabyte of memory. To limit the amount of memory required for these operations, limit the size of the images you display.

Resolving Memory Errors

If you do not have a “chunk” of memory large enough to allocate a matrix, an out of memory error may occur even though you seem to have enough available memory. To consolidate the fragmented memory, you can use the MATLAB `pack` command, or you can allocate larger matrices earlier in the MATLAB session.

If you run out of memory often, use these tips:

For Windows, increase virtual memory by using **System Properties** for **Performance**, which you can access from the **Control Panel**.

For UNIX, ask your system manager to increase your swap space. For VAX/VMS, ask your system manager to increase your working set and/or pagefile quota.

MATLAB’s Memory Management

MATLAB uses the standard C functions `malloc` and `free` to allocate dynamic memory. These routines maintain a pool of memory that is allocated from the operating system relatively slowly. `malloc` and `free` allocate memory from this pool for MATLAB much more quickly. If the pool runs low, `malloc` asks the operating system for another large chunk of memory to replenish the pool.

As MATLAB releases memory, the pool can grow very large. To maintain speed, `malloc` and `free` do not return the additional memory to the operating system. These routines make the assumption that if you need a large amount of memory once, you will need it again. A side effect of this algorithm is that, once MATLAB has used a certain amount of memory, it is no longer available to other programs even if MATLAB is no longer using it. The memory in the pool only returns to the operating system when MATLAB terminates.

X

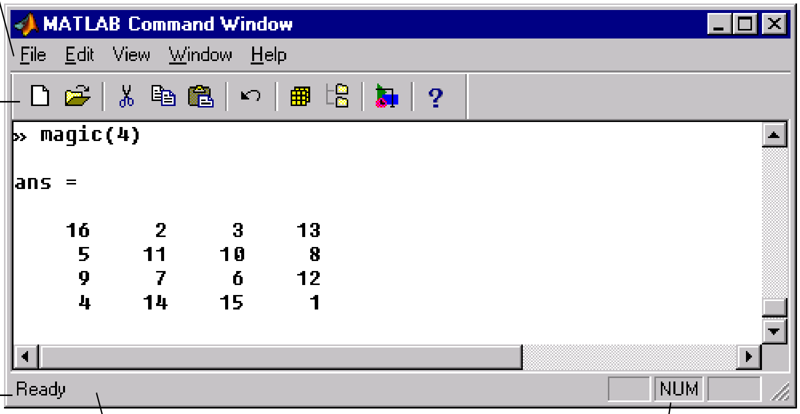
If you use an operating system tool such as ps on UNIX, the display indicates the total sum of the memory allocated by MATLAB plus the contents of the pool. This number can be deceiving because it indicates the highest level of memory use, which may or may not be the current usage.

Microsoft Windows Handbook

This section describes several MATLAB environment tools and their use in the Windows environment.

Command Window

The Command Window appears when you first start MATLAB. You use it to run MATLAB commands, to launch MATLAB tools such as the Editor/ Debugger, and to start toolboxes.



```
>> magic(4)

ans =

    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1
```

Use Menus as an Alternative to Typing Commands

Use Toolbar for Easy Access to Popular Operations

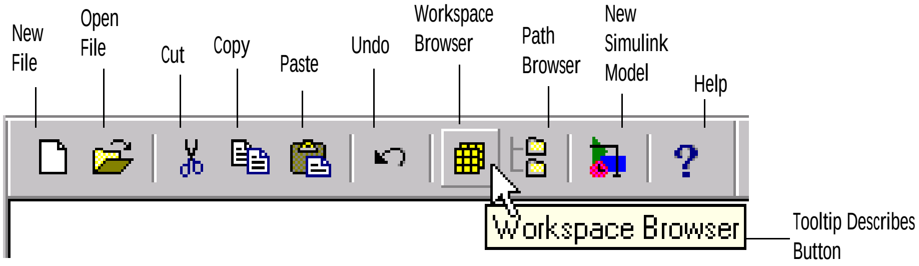
Status Bar

Displays Description of

Displays Status of Cap, Num, and Scroll Locks

Toolbar

The toolbar in the Command Window provides easy access to popular operations. Hold the cursor over a button; a tooltip appears describing the button and additional information appears in the status bar.

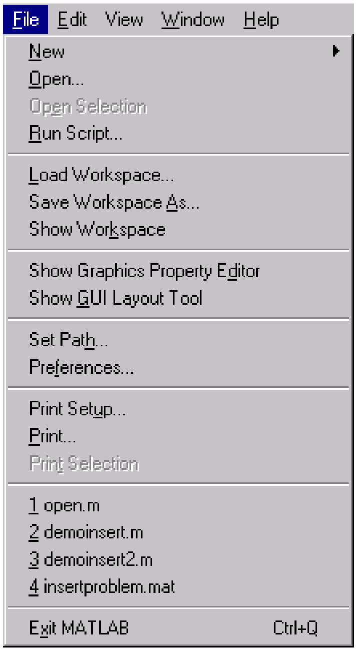


Show or Hide the Toolbar. To remove the toolbar from the Command Window, select **Toolbar** from the **View** menu; the menu item becomes unchecked. To display the toolbar, select **Toolbar** from the **View** menu; the menu item becomes checked and the toolbar appears. This does not affect the setting for **Show Toolbar** in the **Preferences** dialog box. The preferences setting pertains to the toolbar status when you first start MATLAB.

Move the Toolbar. You can move the toolbar to another position. Click and hold on any separator bar (the line between groups of buttons) and then drag the toolbar to a new location. If you move the toolbar to an inside edge of the Command Window, it will become docked, meaning it will not overlap the contents of the Command Window. The new toolbar position is maintained when you restart MATLAB.

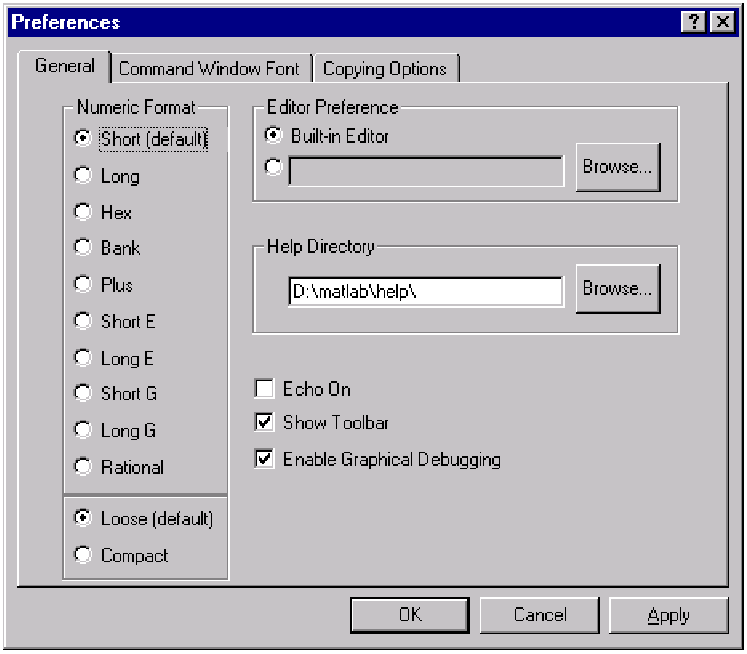
Menus

The Command Window menus provide access to some operations not available from the toolbar. Hold the cursor on a menu item and a description of the item appears in the status bar.



Preferences

Set preferences to control the appearance and operation of the Command Window. Select **Preferences** from the **File** menu. The **Preferences** dialog box appears, from which you set **General**, **Command Window Font**, and **Copying Options** preferences.



Preferences

General | Command Window Font | Copying Options

Numeric Format

● Short (default)
○ Long
○ Hex
○ Bank
○ Plus
○ Short E
○ Long E
○ Short G
○ Long G
○ Rational

● Loose (default)
○ Compact

Editor Preference

● Built-in Editor
○

Help Directory

D:\matlab\help\

□ Echo On
☑ Show Toolbar
☑ Enable Graphical Debugging

General Preferences.

- **Numeric Format** – Specify the default numeric format. For more information see "The format Command", or the reference page for `format`.
- **Editor Preference** – Use MATLAB's Editor, or specify another. For more information about MATLAB's Editor, see "Editor/Debugger".
- **Help Directory** – Specify the directory in which MATLAB help files reside.
- **Echo On** – Turn M-file echoing on; see the `echo` reference page for more information.
- **Show Toolbar** – Show or hide the toolbar in the Command Window.
- **Enable Graphical Debugging** – At each breakpoint, automatically show the Debugger.

Command Window Font. Specify the font characteristics for the text displayed in the Command Window.

Copying Options. Specify options used when copying items from MATLAB to the Windows clipboard for pasting into other applications.

Editor/Debugger

The Editor/Debugger provides basic text editing operations as well as access to M-file debugging tools. The Editor/Debugger offers a graphical user interface. It supports automatic indenting and syntax highlighting; for details see the “General Options” section under “View Menu”. You can also use debugging commands in the Command Window. See Chapter 3 for more information about MATLAB’s debugging capabilities.

To specify the default editor for MATLAB, select **Preferences** from the **File** menu in the Command Window. On the **General** page, select MATLAB’s Editor/Debugger or specify another.

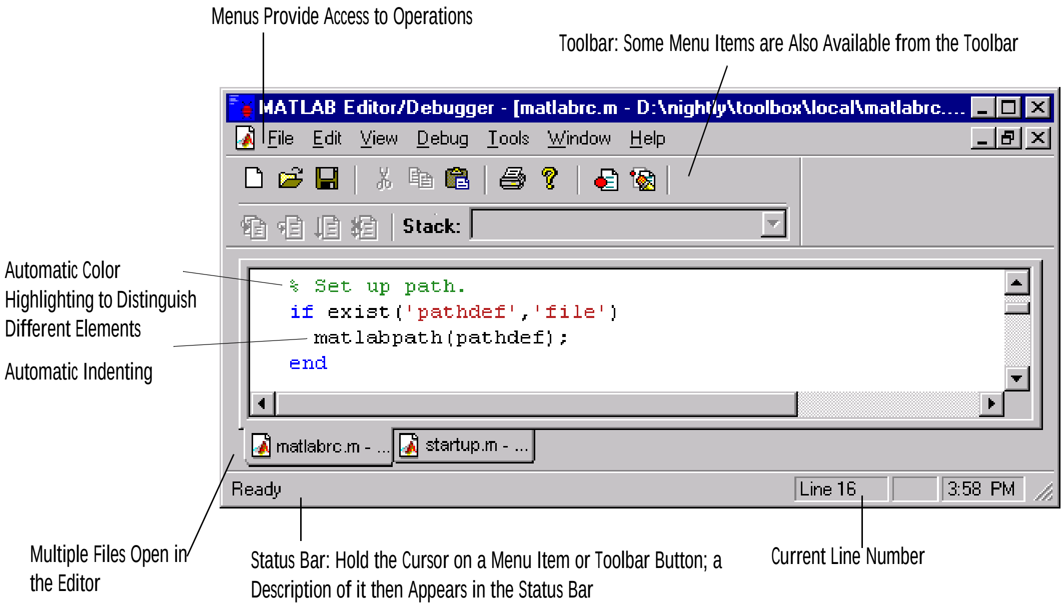
Starting the Editor/Debugger

To start the Editor/Debugger, select **New** from the **File** menu, or click the new file (page icon) button on the toolbar, or type `edit` at the command line.

To start the Editor/Debugger, opening it to a particular file, select **Open** from the **File** menu, or click the open file (folder icon) button on the toolbar, or type `edit filename` at the command line.

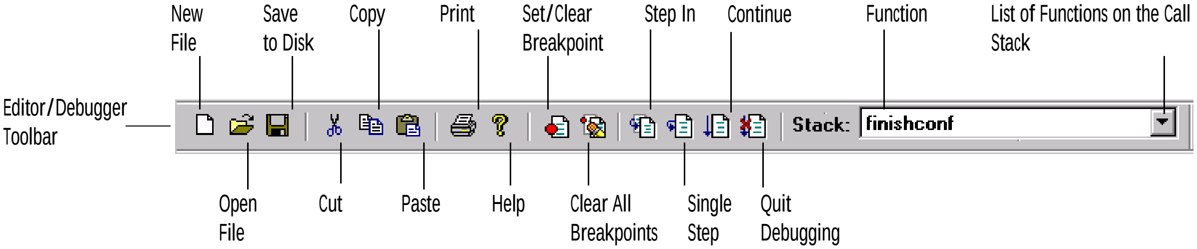
Do not use the Editor/Debugger while you are running an M-file in the Command Window or you will get an error.

When you run the Editor/Debugger without MATLAB open, it becomes a pure text editor; you cannot use it as a debugger. To use the debugger, launch the Editor/Debugger from within MATLAB or with MATLAB open.



```
% Set up path.
if exist('pathdef','file')
  matlabpath(pathdef);
end
```

Menus Provide Access to Operations
Toolbar: Some Menu Items are Also Available from the Toolbar
MATLAB Editor/Debugger - [matlabrc.m - D:\nightly\toolbox\local\matlabrc....
File Edit View Debug Tools Window Help
Stack:
Automatic Color Highlighting to Distinguish Different Elements
Automatic Indenting
matlabrc.m - ...
startup.m - ...
Ready
Line 16
3:58 PM
Multiple Files Open in the Editor
Status Bar: Hold the Cursor on a Menu Item or Toolbar Button; a Description of it then Appears in the Status Bar
Current Line Number



Title Bar

The name of the file you are editing appears in the title bar. If there is an asterisk (*) appearing after the name in the title bar, it indicates that you have made changes to the file but have not saved those changes.

File Menu

Use the **File** menu items to open, save, and print files, and to exit from the Editor/Debugger. If you have a color printer and want to print in color, before printing, select **Options** from the **Tools** menu and check the **Print in color** checkbox.

Edit Menu

Use the Edit menu items to find and replace text, and to go to the line you specify.

View Menu

Use **Evaluate Selection** to evaluate an expression and display the answer in the Command Window. Use **Auto Indent Selection** to indent the selected text according to MATLAB syntax.

Debug Menu

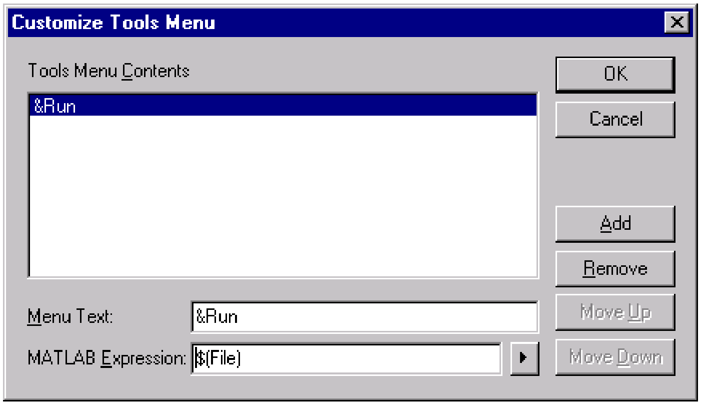
The **Debug** menu provides an interface to the graphical M-file Debugger. See Chapter 3 for information about debugging within MATLAB.

Tools Menu

The **Tools** menu provides access to several dialog boxes that allow you to control existing features and to define new features for use with the Editor/ Debugger.

Run. Initially present on the **Tools** menu, **Run** saves all files and runs the current file. It is an example of the type of command you can create and add to the **Tools** menu using the **Customize** menu item.

Customize. Use **Customize** to create your own custom tool that can run any MATLAB command.



Menu Text: &Run

MATLAB Expression: $(File)

There are two edit fields in the **Customize Tools Menu**:

- **Menu Text**: Supply the name for the menu item that you want to add to the **Tools** menu. To create a mnemonic for the menu command, precede a character with &. Use this mnemonic by typing **Alt** followed by **t** (the **t** indicates the **Tools** menu) followed by the mnemonic character. (Note: The **Alt** key need not be held down while pressing the keys that follow.) For instance, in the example above, you can execute the **Run** command by choosing **Run** on the **Tools** menu or by typing **Alt**, **t**, and **r** in sequence.
- **MATLAB Expression**: In this box, type the expression you want MATLAB to evaluate when you select the custom tool from the **Tools** menu. Any valid MATLAB expression is allowed. You can also include any of six special substitution variables. The variables may be typed directly or inserted via the submenu, as shown above.

  When your custom tool is chosen, information about the current file is substituted into the expression, replacing the substitution variables, and the expression is evaluated in MATLAB. Different information is substituted for the substitution variables depending upon whether the expression is executed by the Editor, Path Browser, or Array Editor.

The table summarizes the variables and the substitutions made for them within the various environment tools.

Submenu Item	Variable Inserted	Invoked from Editor	Invoked from Path Browser	Invoked from Array Editor
Pathname	`$(Pathname)`	Inserts complete pathname of this file.	Inserts complete pathname of file selected in right-hand panel.	Inserts name of variable.
File	`$(File)`	Inserts name of this function without its extension.	Inserts name of file selected in right-hand panel without extension.	Inserts name of variable.
Selection	`$(Sel)`	Inserts the text that is highlighted in this view.	Inserts complete pathname of file selected in right-hand panel.	Inserts indexing expression corresponding to selection (numbers in bottom right panel).

Submenu Item	Variable Inserted	Invoked from Editor	Invoked from Path Browser	Invoked from Array Editor
Quoted Selection	`$(QuotedSel)`	Adds additional single quotes to strings to preserve quotes (e.g., `who’s` becomes '`who’’s’`).	Adds single quotes to selected pathname and transmits to MATLAB.	--
Beginning of Selection	`$(BeginSel)`	Inserts character offset of first character in the selection.	--	--
End of Selection	`$(EndSel)`	Inserts character offset of last character in the selection.	--	--

When you invoke your custom tool:

1. All open files are saved.
2. Appropriate values are substituted for all substitution variables in the tool’s MATLAB expression.
3. The expression is evaluated in MATLAB.
4. The current file is reloaded if it has changed.

Here are some examples of expressions you can create using the **Customize** command and several substitution variables. Observe the differing results when you invoke the expression in the Editor, Path Browser, or Array Editor.

Expression	Result

`(DOS)  !copy /Y $(Pathname) $(Pathname)~`
`(UNIX) !/bin/cp $(Pathname) $(Pathname)~`
	Backup file.

`mex $(Pathname)`	Runs Mex script on a C file.
`which $(File)`	Display pathname of function.
`doc $(Sel)`	Call Help Desk for selected function.
`edit $(Sel)`	Edit selected function.
`disp('Selection: $(BeginSel) through $(EndSel)')`	Indicates in Command Window beginning and end of selected Editor text.
`$(File)($(Sel)) = $(File)($(Sel))*2;`	When invoked from Array Editor, doubles the values of selected array elements.

Here is a longer example demonstrating how to create a custom tool using MATLAB to script the Editor. First, create an M-file called `censor.m`:

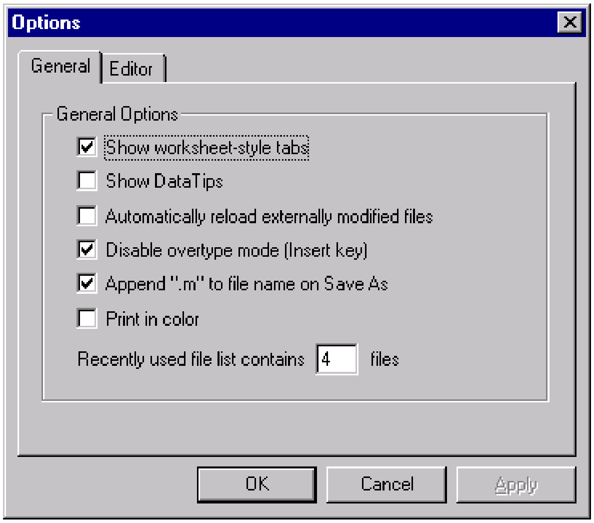
```
function censor(pathname,selstart,selend)
% Read the specified file into an array
fp=fopen(pathname,'r+');
txt=fread(fp);
fseek(fp,0,-1);
txt = char(txt');

% Insert your code here.
% For example, the next line replaces the selection with X's;
txt(selstart:selend)='X';

% Write the modified array back to the file
fwrite(fp,txt');
fclose(fp);
```

Now, use **Customize** from the **Tools** menu to create the MATLAB expression:

```
censor('$(Pathname)',$(BeginSel),$(EndSel));
```

Also, create the menu text for this expression by typing `&Censor` in the **Menu Text** box. In the example we have provided, when you run **Censor** from the **Tools** menu, text you have highlighted in the Editor is replaced with `X`'s.

General Options. Choose **Options** from the **Tools** menu and select the **General** tab.



General Options

☑ Show worksheet-style tabs

□ Show DataTips

□ Automatically reload externally modified files

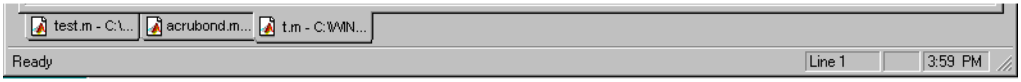
☑ Disable overtype mode (Insert key)

☑ Append ".m" to file name on Save As

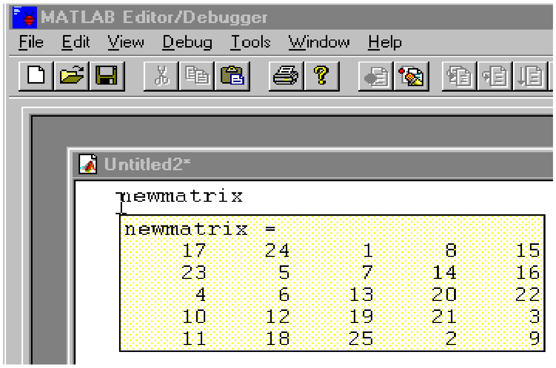
□ Print in color

Recently used file list contains 4 files

- **Show worksheet-style tabs**: controls if the **Editor** window displays tabs for navigating among open files.



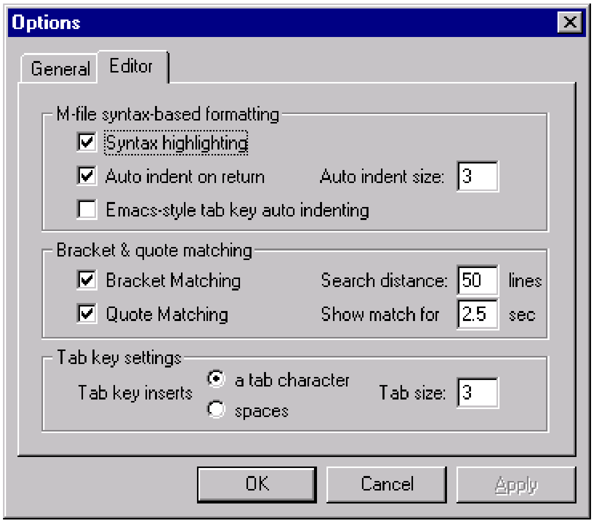
- **Show DataTips**: When the cursor remains positioned over a variable, the item is expanded and the results are displayed in the **Editor** window.



```
newmatrix
newmatrix =
    17    24     1     8    15
    23     5     7    14    16
     4     6    13    20    22
    10    12    19    21     3
    11    18    25     2     9
```

- **Automatically reload externally modified files**: Files changed outside of the Editor are automatically reloaded. Otherwise, the Editor asks if you want to reload the file.
- **Disable overtype mode (Insert key)**: Disables the **Insert** key, which switches overtype mode on and off.
- **Append "m" to filename on Save As**: The `.m` extension is appended to the filename provided in the **Save As** dialog box if that extension is not already present. If this option is not checked, the file is saved with the filename exactly as typed.
- **Print in color**: Prints the M-file in color when a color printer is used.
- **Recently used file list**: Controls number of files listed on the recently used file list under the **File** menu. The Editor must be restarted for the new number to take effect.

Editor Options. Choose **Options** from the **Tools** menu and select the **Editor** tab.



M-file syntax-based formatting

☑ Syntax highlighting

☑ Auto indent on return Auto indent size: 3

□ Emacs-style tab key auto indenting

Bracket & quote matching

☑ Bracket Matching Search distance: 50 lines

☑ Quote Matching Show match for 2.5 sec

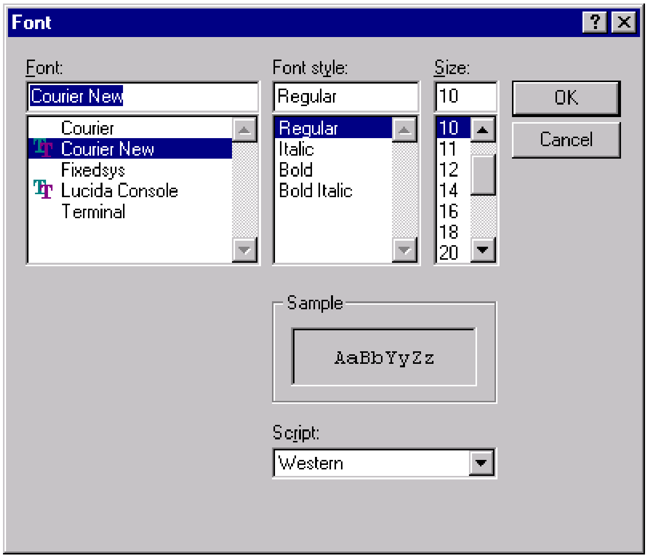
Tab key settings

Tab key inserts ● a tab character ○ spaces Tab size: 3

- **M-file syntax-based formatting**:
  - **Syntax highlighting**: As you type into the Editor, the text is colored according to the kind of text entered.
  - **Auto indent on return**: Pressing the **Return** or **Enter** key indents the current and next lines.
  - **Auto indent size**: Type the number of columns to indent for each level of nested code. The default accelerator for indenting in the MATLAB Editor is **Ctrl-I**.
  - **Emacs-style tab key auto indenting**: The Emacs editor uses the **Tab** key to indent the current line. If you want the **Tab** key to indent the current line, check this option.
- **Bracket and quote matching**: The Editor can indicate which bracket, parenthesis, brace, or quote balances another in your file. Four settings control this matching:
  - **Bracket Matching**: Enables matching for brackets, braces, and parentheses.
  - **Quote Matching**: Enables matching for quotes.

  - **Search distance:** Controls the number of lines forward or backward the Editor searches for a match.
  - **Show match for**: Controls duration of the match indicator.
- **Tab key settings**: These settings control what happens when you type a tab character (with **Emacs-style tab key auto indenting** off) and when the Editor encounters a pre-existing tab in your file.
  - **Tab key inserts**: Chooses whether the **Tab** key inserts **a tab character** or a number of **spaces** equivalent to a tab character.
  - **Tab size:** Controls number of space equivalents between tab stops. Note that the **Auto indent size** parameter is separately tunable from this setting. Automatic indenting always inserts spaces.

Font. Use the **Font** submenu to change the font family, style, and size for the text in the Editor/Debugger and the Path Browser.



Font

Font: Courier New

Font style: Regular

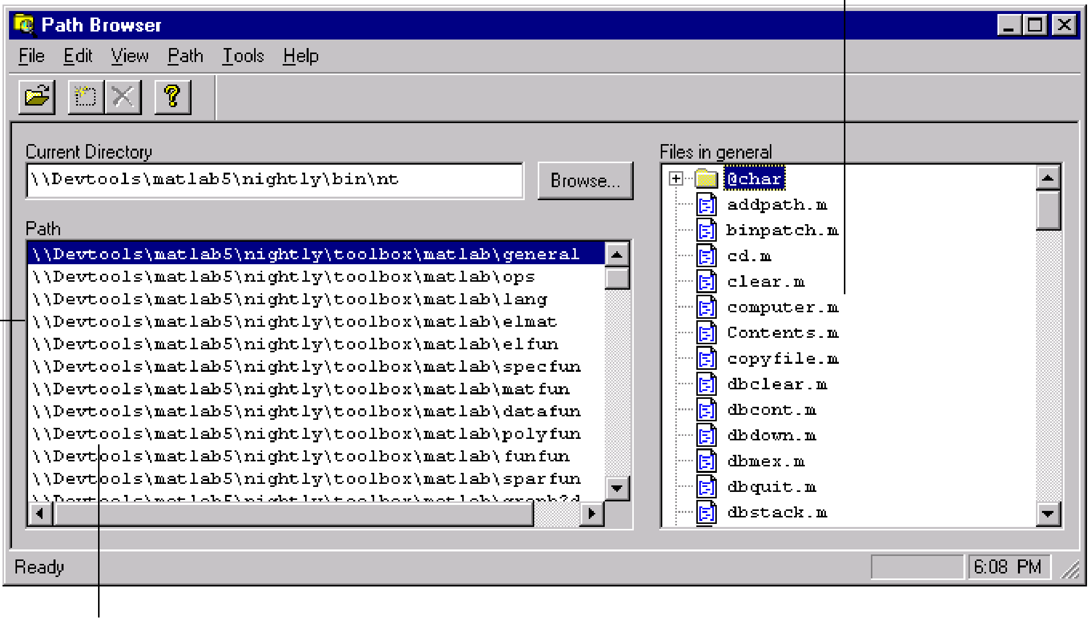
Size: 10

Script: Western

Path Browser

The Path Browser lets you view and modify MATLAB's search path and see all of its files. To open the Path Browser, select **Set Path** from the **File** menu, or click the **Path Browser** toolbar button. You can also access the Path Browser using the `pathtool` command or, from the Editor/Debugger, select **Path Browser** from the **View** menu.

Double-click on a File to Open It



Current Directory

```
\\Devtools\matlab5\nightly\bin\nt
```

Path

```
\\Devtools\matlab5\nightly\toolbox\matlab\general
\\Devtools\matlab5\nightly\toolbox\matlab\ops
\\Devtools\matlab5\nightly\toolbox\matlab\lang
\\Devtools\matlab5\nightly\toolbox\matlab\elmat
\\Devtools\matlab5\nightly\toolbox\matlab\elfun
\\Devtools\matlab5\nightly\toolbox\matlab\specfun
\\Devtools\matlab5\nightly\toolbox\matlab\matfun
\\Devtools\matlab5\nightly\toolbox\matlab\datafun
\\Devtools\matlab5\nightly\toolbox\matlab\polyfun
\\Devtools\matlab5\nightly\toolbox\matlab\funfun
\\Devtools\matlab5\nightly\toolbox\matlab\sparfun
\\Devtools\matlab5\nightly\toolbox\matlab\graph2d
```

Files in general

```
@char
addpath.m
binpatch.m
cd.m
clear.m
computer.m
Contents.m
copyfile.m
dbclear.m
dbcont.m
dbdown.m
dbmex.m
dbquit.m
dbstack.m
```

Directories on Search Path

To Move a Directory, Drag It to the Desired Position

Use the menus in the Path Browser to:

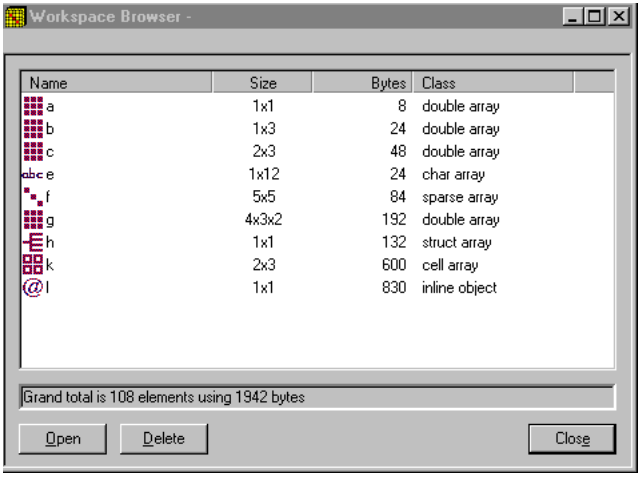
- Add a directory to the front of the path.
- Remove a selected directory from the path.
- Save settings to the `pathdef.m` file.
- Restore default settings.

Click **Browse** to find a directory to add to the path. The **Browse for Folder** dialog box opens. Note that the current directory displayed in this dialog box is the directory you had selected in the **Path Browser** window. If you want the **Browse for Folder** dialog box to open to the current directory, click in the **Current Directory** field in the **Path Browser** window and then click **Browse**.

Workspace Browser

The Workspace Browser lets you view the contents of the current MATLAB workspace. It provides a graphical representation of the `whos` display.

To open the Workspace Browser, select **Show Workspace** from the **File** menu, click the **Workspace Browser** toolbar button, or type `workspace` at the command line.



Name	Size	Bytes	Class
a	1x1	8	double array
b	1x3	24	double array
c	2x3	48	double array
e	1x12	24	char array
f	5x5	84	sparse array
g	4x3x2	192	double array
h	1x1	132	struct array
k	2x3	600	cell array
l	1x1	830	inline object

Grand total is 108 elements using 1942 bytes

You can resize the columns of information by dragging the column header borders. The workspace is sorted by variable name. Sorting by other fields is not supported.

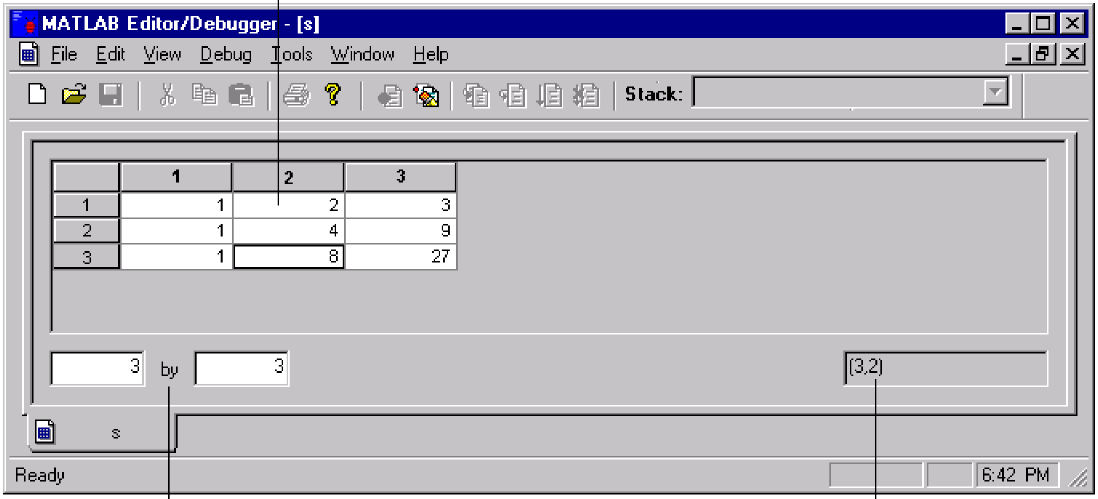
To clear a variable, select the variable and click **Delete**. Shift-click to select multiple variables.

To rename a variable, first select it, then click its name. After a short delay, type a new name and press **Enter** to complete the name change.

Editing Arrays

The MATLAB Editor/Debugger provides a visual representation of two-dimensional numeric arrays, which allows for easy editing. To see and edit a graphical representation of a variable, select a variable's icon in the Workspace Browser and click **Open**, or double-click the icon. The variable is displayed in the **Editor/Debugger** window, where you can edit it. You can only use this feature with numeric arrays.

Current Values: Change Any Value By Editing It in the Cell


MATLAB Editor/Debugger - [s]
File Edit View Debug Tools Window Help
Stack:
s

	1	2	3
1	1	2	3
2	1	4	9
3	1	8	27

3 by 3

[3,2]

Ready

6:42 PM

Current Dimensions: Add or Remove Rows and Columns By Editing these Dimensions

Current Cell

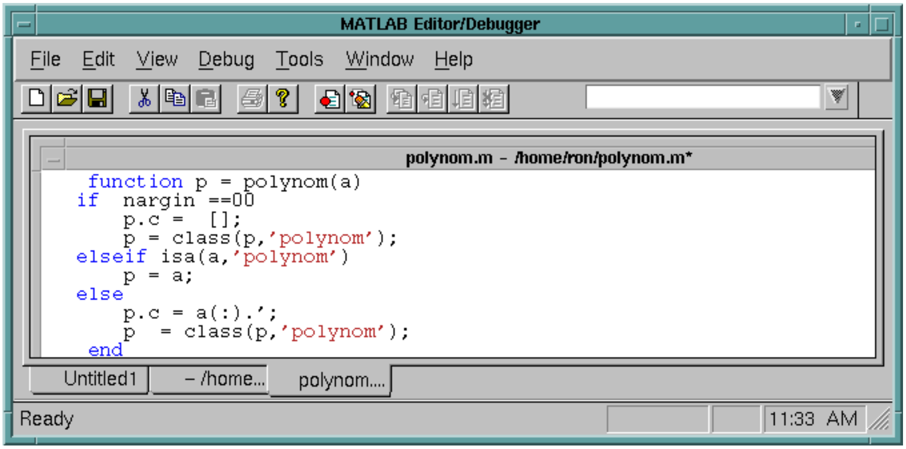
UNIX Handbook

This section describes several MATLAB environment tools and their use with the UNIX operating system.

Editor/Debugger

MATLAB provides a built-in combined Editor/Debugger that offers basic text editing operations as well as access to M-file debugging tools. This is essentially the same Editor/Debugger as the one provided for Windows platforms. See the section, “Microsoft Windows Handbook,” for a discussion of file editing with this tool.

The Editor/Debugger offers a graphical user interface. To use the Editor/Debugger, type `edit` at the MATLAB prompt. You can also use debugging commands in the Command Window. Refer to Chapter 3 for a discussion of MATLAB’s debugging capabilities.



```
 function p = polynom(a)
if  nargin ==00
    p.c =  [];
    p = class(p,'polynom');
elseif isa(a,'polynom')
    p = a;
else
    p.c = a(:).';
    p  = class(p,'polynom');
 end
```

Note You cannot use the Editor on UNIX platforms to cut and paste to and from X Window Systems applications. The Editor on UNIX platforms does not accept Japanese characters as input.

Setting the Default Editor

The editing and debugging features are both set On by default when MATLAB is installed. If you want to substitute a different editor (such as Emacs) or not use graphical debugging, you can turn these tools Off by setting the appropriate variable in your ~home/.Xdefaults file:

```
matlab*builtInEditor: Off
matlab*graphicalDebugger: Off
```

Run

```
xrdb –merge ~home/.Xdefaults
```

before starting MATLAB.

If you set the Editor Off, the option

```
matlab*externalEditorCommand: $EDITOR $FILE &
```

controls what the edit command does. MATLAB substitutes $EDITOR with the name of your default editor and $FILE with the filename. This option can be modified to any sort of command line you want.

Changing the Editor During a Session

To turn off the builtinEditor during a MATLAB session, use

```
system_dependent('builtinEditor','off')
```

Then edit uses the editor defined for your UNIX EDITOR environment variable. To turn the MATLAB Editor/Debugger back on during the session, use

```
system_dependent('builtinEditor','on')
```

You can include the system_dependent command in your startup.m file. See the matlabrc reference page for more information.

Saving Editor Options

The MATLAB Editor stores your options in a registry file located under the directory $HOME/.windu. Communication with this file is handled through a daemon (called windu_registryd41 or windu_registryd40), which is started automatically when you start MATLAB, and is shut down shortly after you quit MATLAB.

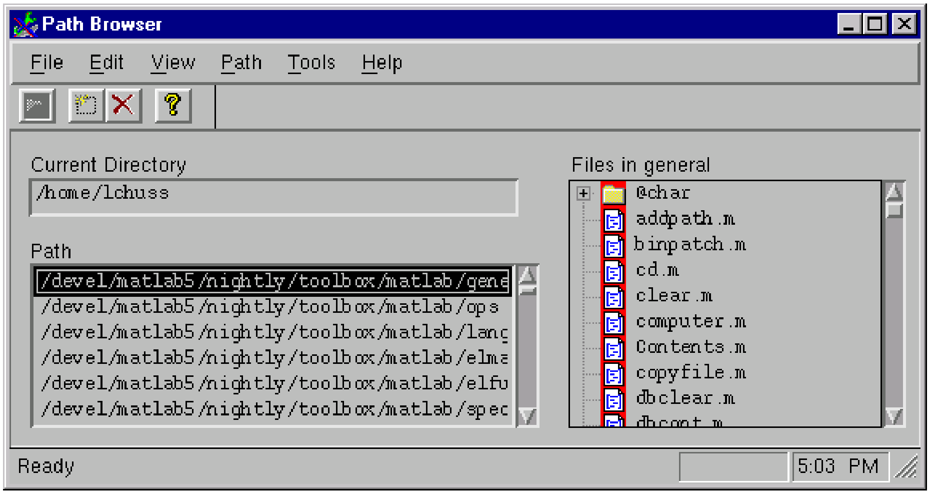
To view the registry, type the command `regedit`. The `regedit` command starts the registry editor. Your settings are located under `HKEY_CURRENT_USER/ Software/MathWorks`.

This registry is not portable across different UNIX platforms. The registry can only be read and written to on the platform on which it was created. If you start the MATLAB Editor on a different platform, it will start a registry daemon with the default options. These options can be saved and retrieved from the registry daemon, but once the daemon is shut down, the options will be lost.

When you start the Editor, if you get messages about missing options, it is possible that your registry has somehow been corrupted. To fix this problem, kill your registry daemon and delete the registry database directory. Then restart MATLAB and it will create a new registry with the default options.

Path Browser

The Path Browser lets you view and modify MATLAB's search path and see all of its files. To open the Path Browser, type `pathtool` at the command line. This is essentially the same Path Browser as the one provided for Windows platforms. See the section, "Microsoft Windows Handbook," for a discussion of this tool.



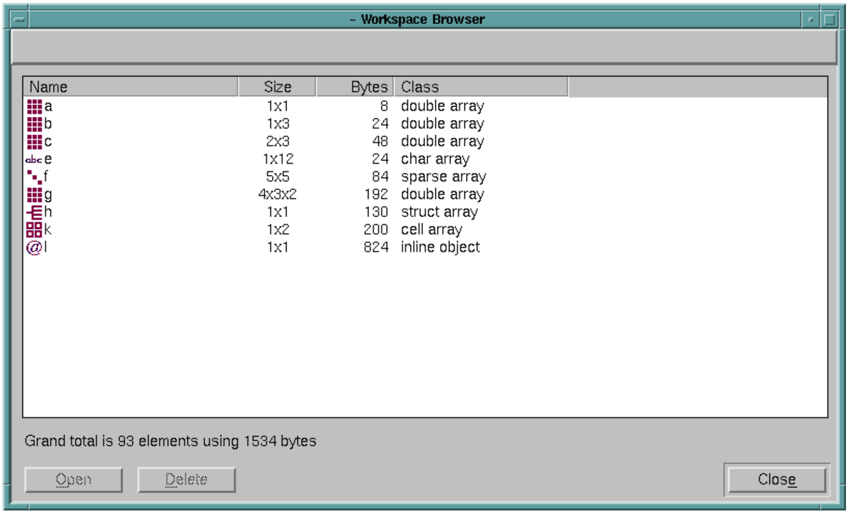
Current Directory: /home/lchuss

```
/devel/matlab5/nightly/toolbox/matlab/gene
/devel/matlab5/nightly/toolbox/matlab/ops
/devel/matlab5/nightly/toolbox/matlab/lang
/devel/matlab5/nightly/toolbox/matlab/elma
/devel/matlab5/nightly/toolbox/matlab/elfu
/devel/matlab5/nightly/toolbox/matlab/spec
```

```
@char
addpath.m
binpatch.m
cd.m
clear.m
computer.m
Contents.m
copyfile.m
dbclear.m
dbcont.m
```

Workspace Browser

MATLAB provides a Workspace Browser that lets you view the contents of the current MATLAB workspace. It provides a graphical representation of the whos display. To open the Workspace Browser, type workspace at the command line This is essentially the same Workspace Browser as the one provided for Windows platforms. See the section, “Microsoft Windows Handbook,” for a discussion of this tool.

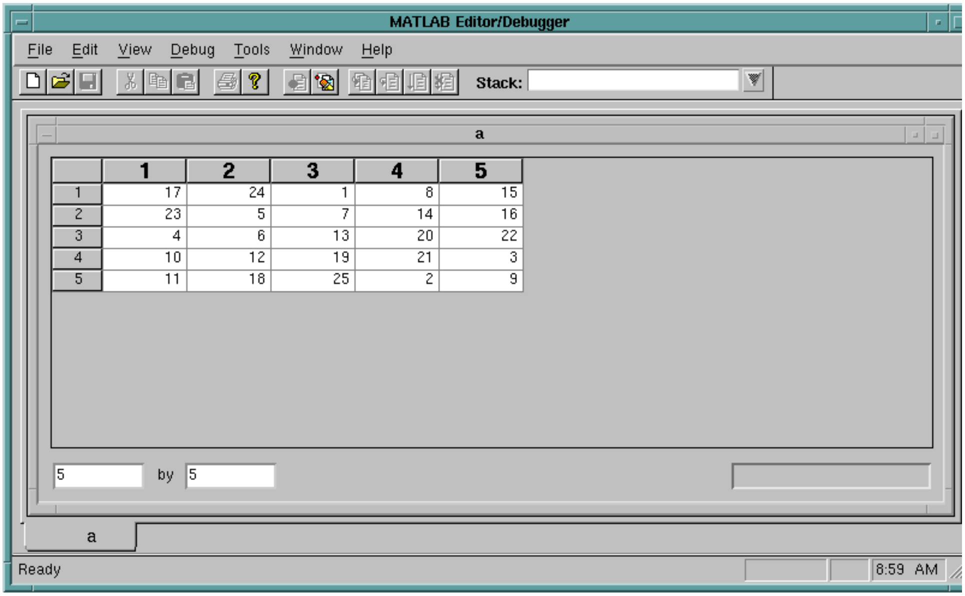


Name	Size	Bytes	Class
a	1x1	8	double array
b	1x3	24	double array
c	2x3	48	double array
e	1x12	24	char array
f	5x5	84	sparse array
g	4x3x2	192	double array
h	1x1	130	struct array
k	1x2	200	cell array
l	1x1	824	inline object

Grand total is 93 elements using 1534 bytes

Editing Arrays

The MATLAB Editor/Debugger provides a visual representation of two-dimensional numeric arrays, which allows for easy editing. This is essentially the same array editing facility as the one provided for Windows platforms. See the section, “Microsoft Windows Handbook,” for a discussion of this tool.



	1	2	3	4	5
1	17	24	1	8	15
2	23	5	7	14	16
3	4	6	13	20	22
4	10	12	19	21	3
5	11	18	25	2	9

5 by 5

Preferences

An optimization option in the .Xdefaults file starts the environment tools but does not display them until you explicitly ask for them. If you want to turn this feature off, set the matlab*preloadIDE variable to Off. (The default is on.)

The environment tools will not display well on 16-bit X displays. If you are using such a display, turn the environment tools off in your .Xdefaults file:

```
matlab*builtInEditor: Off
matlab*graphicalDebugger: Off
matlab*preloadIDE: Off
```

License Management Tools

Although your system administrator has probably taken care of the details of installing and configuring MATLAB’s license manager, some background information is helpful for you to know. This section contains some of the same information provided for the system administrator. For complete details, see the section by the same name in the *MATLAB Installation Guide for UNIX*.

On UNIX platforms, MATLAB uses a license manager called *FLEXlm* to manage the per-computer or per-user licensing. FLEXlm manages per-user licenses with a *key* system. Each time a user invokes MATLAB, the license manager considers that one key in use. When the total number of licensed keys are in use, no more users can invoke MATLAB.

FLEXlm consists of a *license daemon* and a *product daemon* that run on a server node. On UNIX computers, the server node is usually the file server on which MATLAB is installed. Throughout this section, references to the `matlab` directory refer to the directory where the contents of the MATLAB CD is installed.

The license and product daemons run in the background on the server node. They are responsible for checking in and out licenses as users invoke and quit MATLAB.

License Administration

A number of license administration tools are available in `matlab/etc`, including

`lmdown`	Shut down all license daemons.
`lmhostid`	Display hostid of the machine on which you are running.
`lmstat`	Show the current status of all network licensing activities. The command `lmstat -a` displays all information. Use the switch `-f` instead of `-a` to display a list of who is using what features. `lmstat -h` displays usage help.
`lmstart`	Start license daemons. (If license daemons are already running, you must first use `lmdown` to shut them down.)

Understanding the License File

The License File `license.dat` contains the details of your license, such as the number of keys you have for MATLAB, the toolboxes that you purchased, and the hostids of the licensed CPUs. If you upgrade your license or need to move the license server to a different machine, The MathWorks can give you new information by e-mail or telephone.

Your system administrator should edit the License File to reflect your licensing information. If you edit the file yourself, follow the instructions in the *Installation Guide*, or run lmdown followed by lmstart. To run lmdown, you must be a member of the UNIX group lmadmin, or a member of group 0 if lmadmin does not exist.

The matlab script in matlab/bin sets the environment variable LM_LICENSE_FILE to contain the pathname where license.dat is stored. This pathname is normally matlab/etc/license.dat wherever MATLAB is stored. If necessary, you can change this environment variable to point to some other location.

The file /usr/tmp/lm_TMW5.log, where the license daemon's output usually is redirected, contains a log of all license check-outs, check-ins, and denials. A new entry is recorded in the log each time a transaction occurs. To save file space, you can delete it occasionally.

Creating a Local Options File

You can instruct the license manager to:

- Reserve one or more keys for a particular user, group of users, or host
- Specify the users, groups of users, or hosts that have permission to access one or more products

To use these options, you can create a local options file and list its pathname as the fourth field on the DAEMON line in the license.dat file. Depending on the length of your path, this line may become fairly long. In the following example, this line is shown on two lines; however, you should keep it all on one line:

```
DAEMON MLM /usr/local/matlab/etc/lm_matlab
/usr/local/matlab/etc/local.options
```

A local options file is not required. If it does exist, it can have one line or many lines, reflecting your special needs. The license manager allocates keys according to these options until all keys are in use. If you try to reserve more than the authorized number of keys in the options file, a warning message appears in the license.log file.

A local options file might look similar to this one:

```
RESERVE 1 MATLAB USER patricia
RESERVE 3 MATLAB HOST pegasus
RESERVE 1 CONTROL GROUP devels

INCLUDE SIGNAL HOST labrea
INCLUDE SIGNAL USER tom
EXCLUDE SIMULINK GROUP devels
GROUP devels andrea tom fred
```

The lines starting with RESERVE contain the number of keys for a particular product set aside for a specific user, group, or host. This does not limit the number of keys for that group or host; it simply ensures that a key will be available when you want it (unless the specified number of reserved keys has already been reached).

The lines starting with INCLUDE contain the products to be restricted to a particular user, group, or host; only that user, group, or host is allowed to use this product. You can have multiple INCLUDE lines for the same feature, including different users, groups, or hosts.

The lines starting with EXCLUDE contain the features to be restricted from a particular user, host, or group; that user, group, or host is not allowed to use that product. You may have multiple EXCLUDE lines for the same feature as well.

Any line starting with GROUP defines the members of a group name used in the previous lines of this file. (License manager groups are distinct from UNIX protection groups and any other groups defined outside of MATLAB.) If a group name is used in a RESERVE, INCLUDE, or EXCLUDE line, the group membership must be defined in a GROUP line.

3

Debugger and Profiler

MATLAB Debugger

The MATLAB Debugger helps you identify programming errors in your MATLAB code. Using the Debugger, you can view the contents of the workspace at any time during function execution, view the function call stack, and execute M-file code line by line.

MATLAB's Debugger has a graphical user interface and provides a command line interface as well.

These sections step you through an example debugging session. Although the example M-files might be simpler than your own MATLAB code, the debugging concepts demonstrated here remain the same.

Debugging: An Overview

Debugging is the process by which you isolate and fix problems with your code. Debugging helps to correct two kinds of errors:

- Syntax errors, such as misspelling a function name or omitting a parenthesis. MATLAB detects most syntax errors and displays a message describing the error and showing its line number in the M-file.
- Runtime errors. These errors are usually algorithmic in nature; for example, you might modify the wrong variable or perform a calculation incorrectly. Runtime errors are apparent when an M-file produces unexpected results.

You can usually correct syntax errors easily based on MATLAB's error messages. Runtime errors are more difficult to track down because the function's local workspace is lost when the error forces a return to the MATLAB base workspace. Use any of the following techniques to isolate the cause of runtime errors:

- Remove selected semicolons from the statements in your M-file. Semicolons suppress the display of intermediate calculations in the M-file. By removing the semicolons, you instruct MATLAB to display these results on your screen as the M-file executes.
- Add `keyboard` statements to the M-file. Keyboard statements stop M-file execution at the point where they appear and allow you to examine and change the function's local workspace. This mode is indicated by a special prompt, "`K>>`." Resume function execution by typing `return` and pressing the **Return** key.

- Comment out the leading function declaration and run the M-file as a script. This makes the intermediate results accessible in the base workspace.
- Use the MATLAB Debugger.

  The Debugger is useful for correcting runtime errors precisely because it enables you to access function workspaces and examine or change the values they contain. The Debugger allows you to set and clear *breakpoints*, specific lines in an M-file at which execution halts. It also lets you change workspace contexts, view the function call stack, and execute the lines in an M-file one by one. This section describes these tasks in detail.

Note The M-file breakpoint information is closely associated with the copy of the M-file that MATLAB holds in memory. If you clear the M-file by editing or by issuing `clear Mfile`, all of `Mfile`'s breakpoints are also cleared.

M-Files For An Example Session

To try the Debugger, first create an M-file called `variance.m` that accepts an input vector and returns an unbiased variance estimate. This file calls another M-file, `sqsum`, that computes the mean-removed squared sum for the input vector.

```
function y = variance(x)
mu = sum(x)/length(x);
tot = sqsum(x,mu);
y = tot/(length(x)–1);
```

Create the `sqsum.m` file exactly as it is shown below, complete with a planted bug.

```
function tot = sqsum(x,mu)
tot = 0;
for i = 1:length(mu)
    tot = tot + ((x(i)–mu).^2);
end
```

Note The example above is coded for illustrative purposes only. Whenever possible, avoid `for` loops and use vectorization for the most efficient execution.

Trial Run

Try out the M-files to see if they work correctly. Use MATLAB's `std` function to compute results.

Create a test vector at the command line.

```
v = [1 2 3 4 5];
```

Compute the variance using `std`.

```
var1 = std(v).^2

var1 =

    2.5000
```

Now try the `variance` function from above.

```
myvar1 = variance(v)

myvar1 =

     1
```

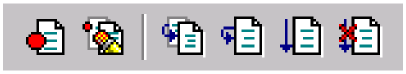
The answer is wrong. Let's use the Debugger to isolate the error in the M-files. The following section shows a sample session using the Editor/Debugger graphical user interface, as well as one from the command line.

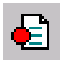
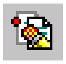
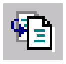
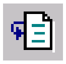
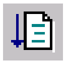
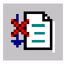
Debugging Using the Graphical User Interface

To start debugging:

- If you have just created the M-files using the **Editor/Debugger** window, you can continue from this point.
- If you've created the M-files using an external text editor, start the Editor/Debugger and then click the **Open M-file** button on the toolbar.

The Editor/Debugger toolbar contains a series of debugging icons.



Toolbar Button	Purpose	Description	Equivalent Command
	Set/Clear Breakpoint	Set or clear a breakpoint at the line containing the cursor.	`dbstop/` `dbclear`
	Clear All Breakpoints	Clear all breakpoints that are currently set.	`dbclear all`
	Step In	Execute the current line of the M-file and, if the line is a call to another function, step into that function.	`dbstep in`
	Single Step	Execute the current line of the M-file.	`dbstep`
	Continue	Continue execution of M-file until completion or until another breakpoint is encountered.	`dbcont`
	Quit Debugging	Exit the debugging state.	`dbquit`

A right-button mouse click in the **Editor** window produces a pop-up menu of some of the options.

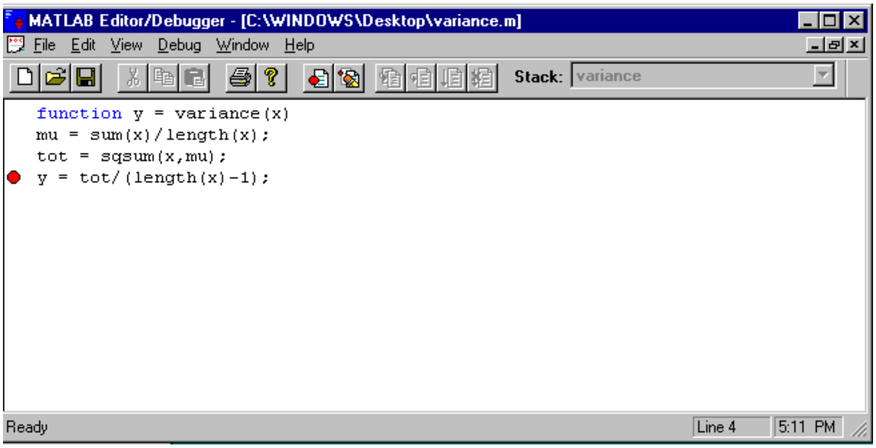
Setting Breakpoints

Most debugging sessions start by setting a breakpoint. Breakpoints stop M-file execution at specified lines and allow you to view or change values in the function's workspace before resuming execution. A breakpoint is set or cleared at the line containing the cursor. A red stop sign (●) next to a line indicates that a breakpoint is set at that line. If the line selected for a breakpoint is not a valid executable line, then the breakpoint is set at the next executable line.

Note The Debugger's **Breakpoints** menu also lets you halt M-file execution if your code generates a warning, error, or `NaN` or `Inf` value.

At the beginning of the debugging session, you're not sure where the error in the `variance` function is, or even if it's in the `variance.m` or `sqsum.m` file. A logical place to insert a breakpoint is after the computation of the mean and the mean-removed squared sum. Open `variance.m` and set a breakpoint at line 4.

```
y = tot/(length(x)-1);
```


MATLAB Editor/Debugger - [C:\WINDOWS\Desktop\variance.m]
File Edit View Debug Window Help
Stack: variance
Ready
Line 4
5:11 PM

```
function y = variance(x)
mu = sum(x)/length(x);
tot = sqsum(x,mu);
y = tot/(length(x)-1);
```

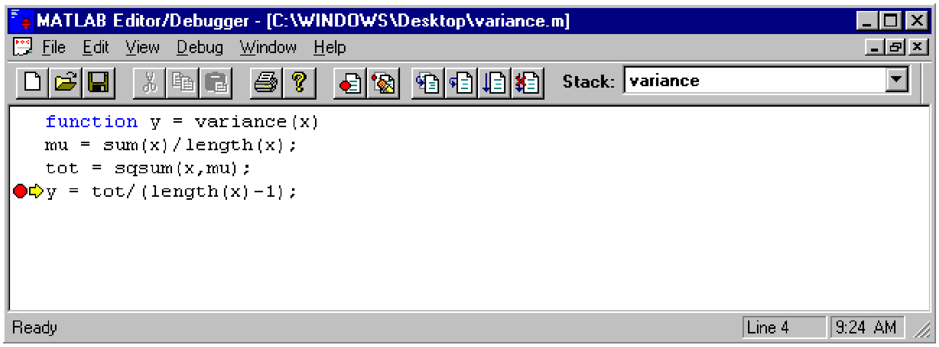
The line number is indicated at the bottom right of the status bar. Set the breakpoint by positioning the cursor in the line of text and click on the breakpoint icon in the toolbar. Alternatively, you can choose **Set Breakpoint** from the **Debug** menu, or right-click to bring up the context menu and choose **Set/Clear Breakpoint.**

Examining Variables

To get to the breakpoint and check the values of interest, first execute the function from the Command Window.

```
variance(v)
```

When execution of an M-file pauses at a breakpoint, the yellow arrow to the left of the text (⇨) shows the next line to execute. A downward yellow arrow (⇩) appearing to the left of the text indicates a pause at the end of the script or function. This allows you to examine variables before returning to the calling function.



MATLAB Editor/Debugger - [C:\WINDOWS\Desktop\variance.m]
File Edit View Debug Window Help
Stack: variance
Ready
Line 4
9:24 AM

```
  function y = variance(x)
  mu = sum(x)/length(x);
  tot = sqsum(x,mu);
●⇨y = tot/(length(x)-1);
```

Check the values of mu and tot from the Debugger. Highlight the text of each variable and right-click to bring up the context menu and choose **Evaluate Selection**. Or, alternatively, choose **Evaluate Selection** from the **View** menu.

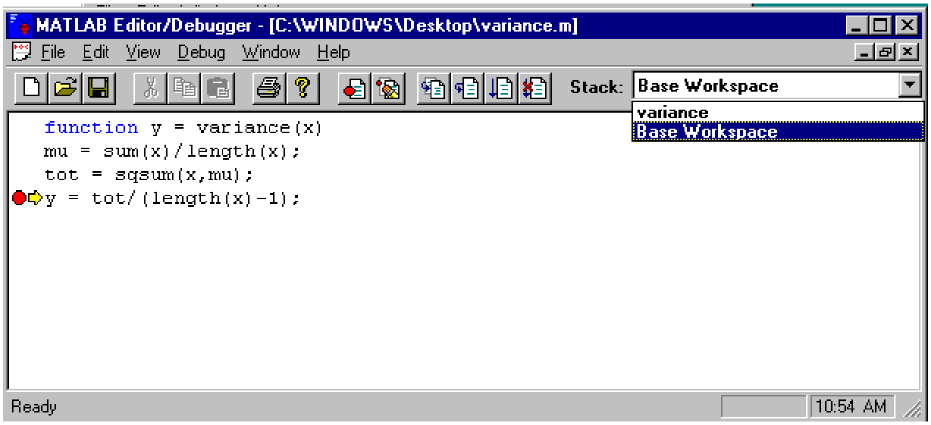
Both the selection and the result are displayed in the Command Window.

```
K>> mu

mu =
      3

K>> tot

tot =
      4
```

The problem is in the sqsum function.

Changing Workspace Context

Use the **Stack** pull-down menu in the upper-right corner of the **Debugger** window to change workspace contexts. To step out from the variance function and see the base workspace contents, select **Base Workspace** from the menu.


MATLAB Editor/Debugger - [C:\WINDOWS\Desktop\variance.m]
File Edit View Debug Window Help
Stack: Base Workspace
variance
Base Workspace
Ready
10:54 AM

```
function y = variance(x)
mu = sum(x)/length(x);
tot = sqsum(x,mu);
y = tot/(length(x)-1);
```

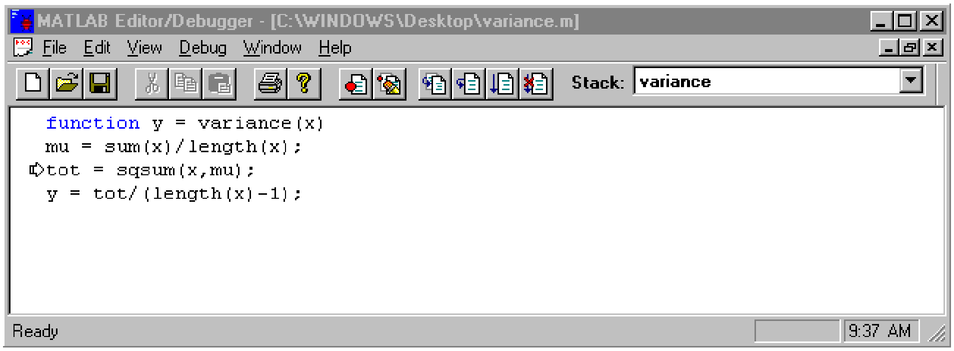
Check the workspace context using whos or the graphical Workspace Browser.

The variables v and myvar1, as well as any other variables you may have created, show up in the listing. To return to the variance workspace context, select **Variance** from the menu.

Stepping Through Code and Continuing Execution

Clear the breakpoint at line 4 in variance.m by placing the cursor on the line and selecting **Clear Breakpoint** from the **Debug** menu. (Or alternatively right-click to bring up the context menu and choose **Clear Breakpoint**). Continue executing the M-file by selecting **Continue** from the **Debug** menu.

Open the sqsum.m file and set a breakpoint at line 4 to check both the loop indices and the computations that take place inside the loop. Run variance again, still using the vector v as input. Execution pauses at line 4 of sqsum. If we look at the variance function in the Editor/Debugger, we see the call to sqsum indicated by an arrow with a vertical line through it.


MATLAB Editor/Debugger - [C:\WINDOWS\Desktop\variance.m]
File Edit View Debug Window Help
Stack: variance
Ready
9:37 AM

```
 function y = variance(x)
 mu = sum(x)/length(x);
➪tot = sqsum(x,mu);
 y = tot/(length(x)-1);
```

Evaluate the loop index i.

```
K>> i

i =
      1
```

Then select **Single Step** from the **Debug** menu to execute the next line.

Evaluate the variable tot.

```
K>> tot

tot =
       4
```

Select **Single Step** again. sqsum only goes through the for loop once.

```
for i  = 1:length(mu)
```

The loop only iterates until the length of mu, a scalar, rather than the length of x, the input vector.

Select **Quit Debugging** from the **Debug** menu to end the M-file execution.

To see if changing tot to its expected value produces the correct answer, clear the breakpoint from sqsum and set a breakpoint on line 4 of variance.m. Run variance once again.

```
variance(v)
```

Execution pauses after control returns from sqsum, but before variance uses the returned value of tot. From the Command Window, set tot to its correct value of 10.

```
K>> tot = 10


tot =
       10
```

Select **Continue Execution** from the **Debug** menu, and the result is correct.

End the Debug Session

Select the **Exit Editor/Debugger** from the **File** menu to end the debugging session.

Debugging from the Command Line

The MATLAB debugging commands are a set of tools that allow you to debug your M-files from the command line. The most general form for each debugging command is shown below.

Description	Syntax
Set breakpoint.	`dbstop at line_num in file_name`
Remove breakpoint.	`dbclear at line_num in file_name`

Stop on warning, error, or `NaN/Inf` generation.	`dstop if warning`
	`error`
	`naninf`
	`infnan`

Resume execution.	`dbcont`

Description	Syntax
List function call stack.	`dbstack`
List all breakpoints.	`dbstatus file_name`
Execute one or more lines.	`dbstep nlines`
List M-file with line numbers.	`dbtype file_name`
Change local workspace context (down).	`dbdown`
Change local workspace context (up).	`dbup`
Quit debug mode.	`dbquit`

For more information about any of these debugging commands, type `help` or `doc` followed by the command name.

Example Command Line Debugging Session

You can perform all debugging tasks from the command line. To follow this example, use the M-files from the beginning of this chapter.

```
function y = variance(x)function tot = sqsum(x,mu)
mu = sum(x)/length(x);tot = 0;
tot= sqsum(x,mu);for i = 1:length(mu)
y = tot/(length(x)–1); tot = tot + ((x(i)–mu).^2);
           end
```

Setting Breakpoints

`dbstop` inserts a breakpoint at a specified line. The M-file halts before the line actually executes. Set breakpoints in `variance` after the computation of the mean (line 2), and after the computation of the mean-removed squared sum (line 3) using

```
dbstop variance 3
dbstop variance 4
```

Stepping Through Code and Using Keyboard Mode

Take the vector v = [1 2 3 4 5] as example input. The expected values for the mean and the mean-removed squared sum are 3 and10, respectively. See if you get the expected results at the breakpoints. Execute the function from the command line.

```
variance(v)
```

MATLAB displays the next line to be executed, and its line number.

```
3   tot = sqsum(x,mu);
K>>
```

When execution stops at a breakpoint, you're automatically in keyboard mode, as indicated by the K>> prompt. At this prompt, you can type standard MATLAB commands. When you're finished, type dbcont to resume execution.

Note On some platforms, a debugging window may automatically appear when a function stops at a breakpoint.

When execution halts at the first breakpoint, use whos to see what variables are now in the workspace.

```
whos
```

To check the value of mu

```
K>> mu

mu =

     3
```

Use dbstep to step one line in the function. When execution stops again before line 4, check the value of tot to see if the mean-removed squared sum calculation matches the expected value.

```
K>> dbstep
4   y = tot/(length(x)-1);
K>> tot

tot =

     4
```

It appears the problem may be in the sqsum function.

Changing Workspace Context

Use dbup and dbdown to move between function workspaces and the base workspace. To step up from the variance function and see the base workspace contents, use

```
dbup
whos
```

The test variable v, as well as any other variables you may have created, shows up in the listing. To step back down to the variance workspace, use

```
dbdown
```

Displaying an M-File with Line Numbers

Without leaving keyboard mode, use dbtype to view sqsum. Set breakpoints to check both the loop indices and the computations that take place inside the loop.

```
K>> dbtype sqsum

1    function tot = sqsum(x,mu)
2    tot = 0;
3    for i = 1:length(mu)
4        tot = tot + ((x(i)-mu).^2);
5    end

K>> dbstop sqsum 4
K>> dbstop sqsum 5
```

Viewing the Function Call Stack and Continuing Execution

Return from keyboard mode using dbquit. At the MATLAB prompt, type

```
dbclear variance
```

to clear the breakpoints from the variance function, while retaining the new sqsum breakpoints.

Run variance again, still using the vector v as input:

```
variance(v)
4       tot = tot + ((x(i)–mu).^2);
```

Use dbstack to view the function call stack, verifying that variance did call sqsum.

```
K>> dbstack
  In Pat:Applications:V5:sqsum.m at line 4
  In Pat:Applications:V5:variance.m at line 3
```

Check the value of the loop index i, then the value of tot. After checking tot, dbstep again to check the next value of i.

```
K>> i

i =

     1

K>> dbstep
5   end
K>> tot

tot =

     4

K>> dbstep
End of M-file function Pat:Applications:V5:sqsum.m.
```

The function only goes through the loop once, ending after the first iteration. From the for statement

```
for i = 1:length(mu)
```

you can see that there is a mistake in the line. The loop only iterates until the length of mu, a scalar, rather than the length of x, the input vector.

To see if changing tot to its expected value produces the correct answer, type

```
K>> tot = 10

tot =

    10
```

Use the dbup command to move up one workspace context, into the variance function workspace. It's clear that the variance function was taking the returned tot value of 4, dividing by length(x)–1, also 4 in this case, and coming up with the incorrect answer of 1. Verify that it comes up with the correct answer now that tot has the correct value, using dbcont to continue execution.

```
K>> dbup
In workspace belonging to Pat:Applications:V5:variance.m.
K>> dbcont

ans =

    2.5000
```

Ending A Debugging Session

Use dbquit to end the debugging session and return to the base workspace. Edit sqsum.m so that its for statement runs from 1 to length(x) rather than 1 to length(mu).

```
for i = 1:length(x)
```

Repeating the original trial run, we now get the expected results:

```
variance(v)

ans =

    2.5000

variance(w)

ans =

  468.3876
```

M-File Profiler

One way to improve the performance of your M-files is to profile them. MATLAB provides an M-file profiler that lets you see how much computation time each line of an M-file uses.

Profiling: An Overview

Profiling is a way to measure where a program spends its time. Measuring is a much better method than guessing where the most execution time is spent. You probably deal with obvious speed issues at design time and can then discover unanticipated effects through measurement. One key to effective coding is to create an original implementation that is as simple as possible and then use a profiler to identify bottlenecks if speed is an issue. Premature optimization often increases code complexity unnecessarily without providing a real gain in performance.

Use a profiler to identify functions that are consuming the most time, then determine why you are calling them and look for ways to minimize their use. It is often helpful to decide whether the number of times a particular function is called is reasonable. Because programs often have several layers, your code may not explicitly call the most expensive functions. Rather, functions within your code may be calling other, time-consuming functions that can be several layers down in the code. In this case it's important to determine which of your functions are responsible for such calls.

The profiler often helps to uncover problems that you can solve by:

- Avoiding unnecessary computation, which can arise from oversight.
- Changing your algorithm to avoid costly functions.
- Avoiding recomputation by storing results for future use.

When you reach the point where most of the time is spent on calls to a small number of built-in functions, you have probably optimized the code as much as you can expect.

How the Profiler Works

Use the `profile` command to generate and view statistics. Start the profiler using `profile on`, and specify any options you want to use. Then execute your M-file. The profiler counts how many seconds each line in the M-files use. The profiler works cumulatively, that is, adding to the count for each M-file you execute until you clear the statistics. Use `profile report` to display the statistics gathered in an HTML-formatted report in a Web browser.

The profile Command

Here is a summary of the main forms of `profile`. For details about these and other options, type `help profile` or `doc profile`.

Syntax	Options	Description
`profile on`		Starts the profiler, clearing previously recorded statistics.
	`-detail level`	Specifies the level of function to be profiled.
	`-history`	Specifies that the exact sequence of function calls is to be recorded.
`profile report`		Suspends the profiler, generates a profile report in HTML format, and displays the report in your Web browser.
	`basename`	Saves the report in the file `basename` in the current directory.
`profile plot`		Suspends the profiler and displays in a figure window a bar graph of the functions using the most execution time.

`profile resume`		Restarts the profiler without clearing previously recorded statistics.
`profile clear`		Clears the statistics recorded by the profiler.
`profile off`		Suspends the profiler.
`profile status`		Displays a structure containing the current profiler status.
`stats = profile('info')`		Suspends the profiler and displays a structure containing profiler results.

An Example Using the Profiler

This example demonstrates how to run the profiler.

1 Start the profiler.

```
profile on -detail builtin -history
```

The `-detail builtin` option instructs the profiler to gather statistics for built-in functions, in addition to the default M-functions, M-subfunctions, and MEX-functions.

The `-history` option instructs the profiler to track the exact sequence of entry and exit calls.

2 Execute an M-file. This example runs the Lotka-Volterra predator-prey population model. For more information about this model, type `lotkademo` to run a demonstration.

```
[t,y] = ode23('lotka',[0 2],[20;20]);
```

3 Generate the profile report and save the results to the file `lotkaprof`.

```
profile report lotkaprof
```

This suspends the profiler, displays the profile report in a Web browser window, and saves the results. See “Viewing Profiler Results” for more information.

4 Restart the profiler, without clearing the existing statistics.

```
profile resume
```

The profiler is now ready to continue gathering statistics for any more M-files you run. It will add these new statistics to those generated in the previous steps.

5 Stop the profiler when you are finished gathering statistics.

```
profile off
```

Viewing Profiler Results

Profile Reports

To display profiler results, type

```
profile report
```

`profile report` suspends the profiler. The results appear in a report in a Web browser window. The report opens with a summary report, and from that page, you can access the detail and history reports.

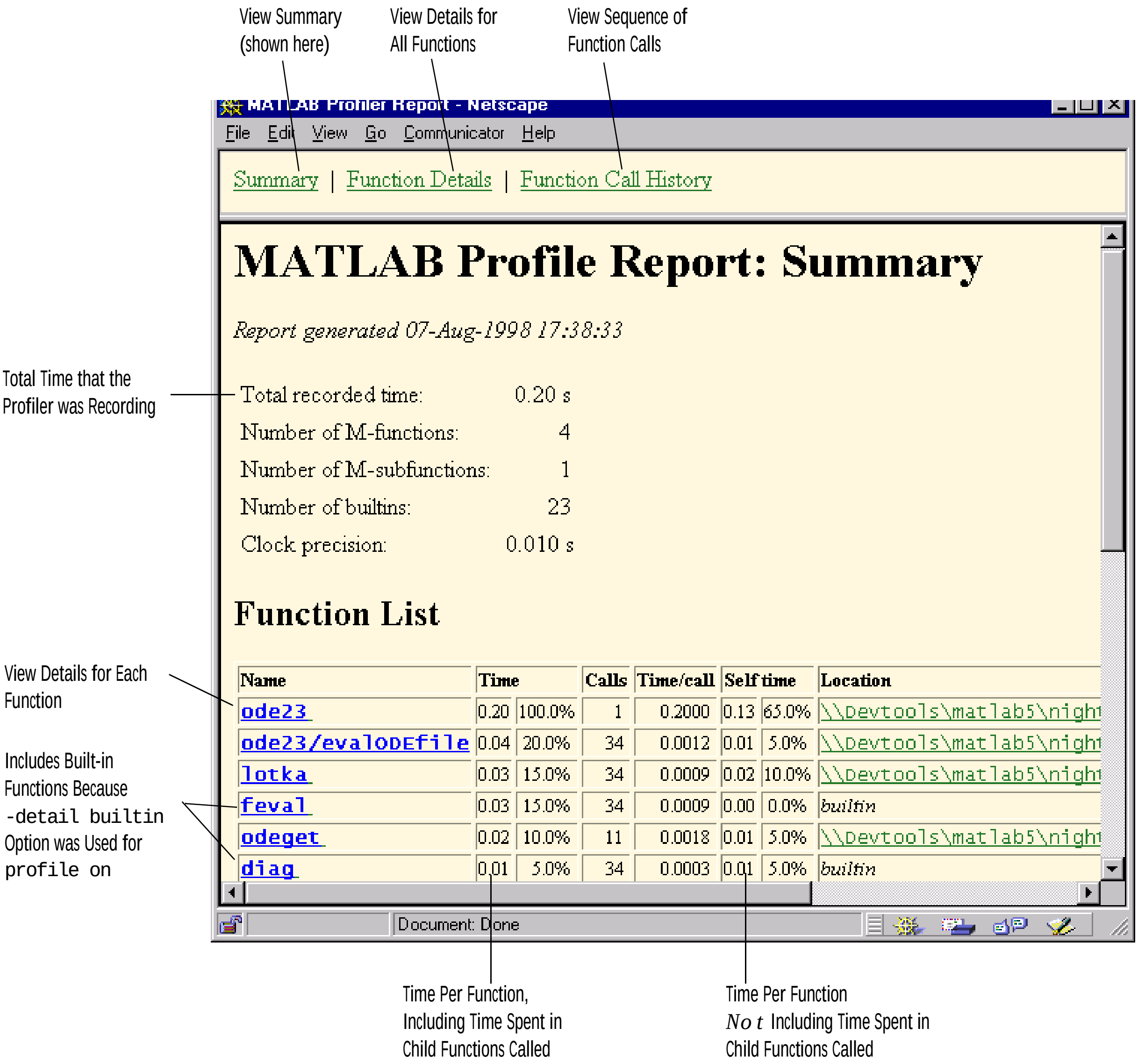
Summary Profile Report. The summary report presents statistics about the overall execution and provides summary statistics for each function called. Values reported include:

- **Clock precision** – the precision of the profiler’s time measurement. When Time for a function is 0, it is actually a positive value, but smaller than the profiler can detect given the clock precision.
- **Time** columns – the total time spent in a function, including all child functions called. Because the time for a function includes time spend on child

functions, the times do not add up to the **Total recorded time** and the percentages add up to more than 100%.

- **Self time** columns – total time spent in a function, *not* including time for any child functions called. Adding the **Self time** values for all functions listed equals the **Total recorded time**. The **Self time** percentages for all functions add up to approximately 100%.

Below is the summary report for the Lotka-Volterra model described in “An Example Using the Profiler”.



Summary | Function Details | Function Call History

MATLAB Profile Report: Summary

Report generated 07-Aug-1998 17:38:33

Total recorded time:	0.20 s
Number of M-functions:	4
Number of M-subfunctions:	1
Number of builtins:	23
Clock precision:	0.010 s

Function List

Name	Time		Calls	Time/call	Self time		Location
ode23	0.20	100.0%	1	0.2000	0.13	65.0%	\\Devtools\matlab5\night
ode23/evalODEfile	0.04	20.0%	34	0.0012	0.01	5.0%	\\Devtools\matlab5\night
lotka	0.03	15.0%	34	0.0009	0.02	10.0%	\\Devtools\matlab5\night
feval	0.03	15.0%	34	0.0009	0.00	0.0%	*builtin*
odeget	0.02	10.0%	11	0.0018	0.01	5.0%	\\Devtools\matlab5\night
diag	0.01	5.0%	34	0.0003	0.01	5.0%	*builtin*

View Summary (shown here)

View Details for All Functions

View Sequence of Function Calls

Total Time that the Profiler was Recording

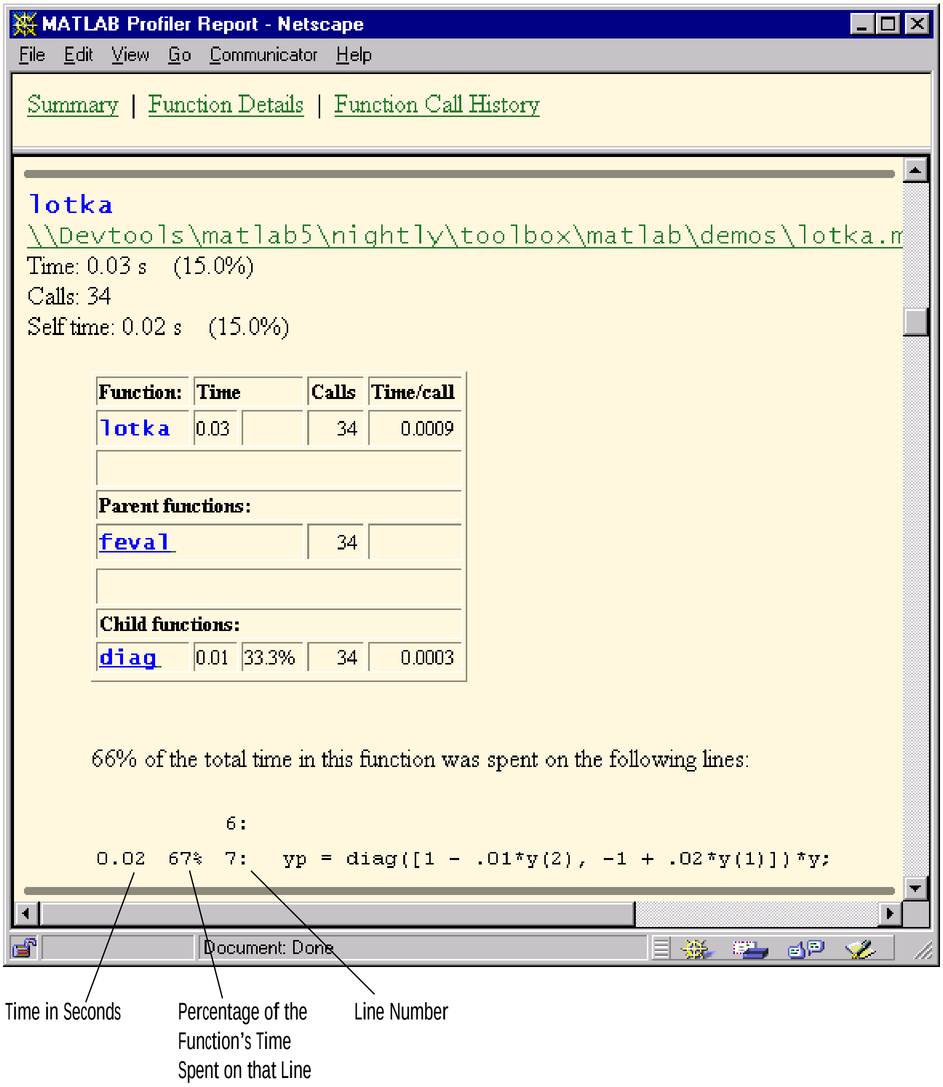
View Details for Each Function

Includes Built-in Functions Because `-detail builtin` Option was Used for `profile on`

Time Per Function, Including Time Spent in Child Functions Called

Time Per Function *No t* Including Time Spent in Child Functions Called

Function Details Profile Report. The function details report provides statistics for the parent and child functions of a function, and reports the line numbers on which the most time was spent. Below is the detail report for the `lotka` function, which is one of the functions called in “An Example Using the Profiler”.


MATLAB Profiler Report - Netscape
File
Edit
View
Go
Communicator
Help
Summary | Function Details | Function Call History
Document: Done
Time in Seconds
Percentage of the Function’s Time Spent on that Line
Line Number

lotka
\\Devtools\matlab5\nightly\toolbox\matlab\demos\lotka.m
Time: 0.03 s (15.0%)
Calls: 34
Self time: 0.02 s (15.0%)

Function:	Time		Calls	Time/call
lotka	0.03		34	0.0009
Parent functions:				
feval			34	
Child functions:				
diag	0.01	33.3%	34	0.0003

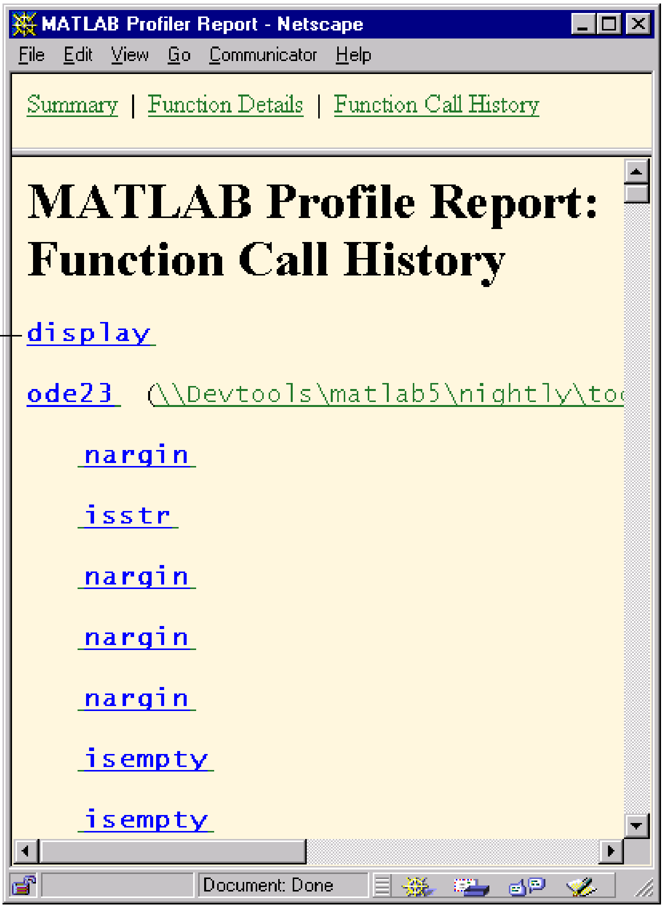
66% of the total time in this function was spent on the following lines:

```
                  6:
0.02    67%    7:    yp = diag([1 - .01*y(2), -1 + .02*y(1)])*y;
```

Function Call History Profile Report. The function call history displays the exact sequence of functions called. To view this report, you must have started the profiler using the `-history` option.

```
profile on -history
```

The profiler records up to 10,000 function entry and exit events. For more than 10,000 events, the profiler continues to record other profile statistics, but not the sequence of calls. Below is the history report generated from “An Example Using the Profiler”.


MATLAB Profiler Report - Netscape
File Edit View Go Communicator Help
Summary | Function Details | Function Call History
Document: Done

MATLAB Profile Report: Function Call History

display

ode23 (\\Devtools\matlab5\nightly\to

nargin

isstr

nargin

nargin

nargin

isempty

isempty

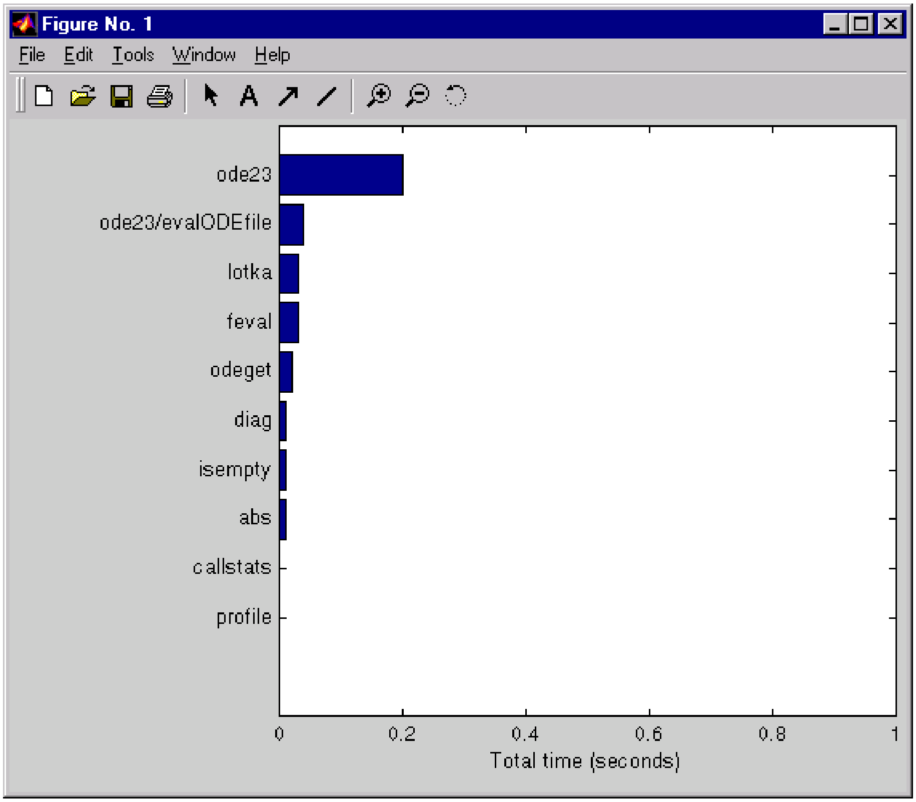
Exact Sequence of Calls

Profile Plot

To view a bar graph for the functions using the most execution time, type

```
profile plot
```

`profile plot` suspends the profiler. The bar graph appears in a figure window. This is the bar graph generated from "An Example Using the Profiler".



Saving Profile Reports

Type

```
profile report basename
```

The profiler saves the report to the file basename in the current directory. Later you can view the saved results using your Web browser.

Another way to save results is with the info = profile command, which displays a structure containing the profiler results. Save this structure so that later you can generate and view the profile report using profreport(info).

Example Using Structure of Profiler Results

The profiler results are stored in a structure that you can view or access. This example illustrates how you can view the results.

1 Run the profiler for code that computes the Lotka-Volterra predator-prey population model.

```
profile on -detail builtin -history
[t,y] = ode23('lotka',[0 2],[20;20]);
```

2 To view the structure containing profiler results, type

```
stats = profile('info')
```

MATLAB returns

```
stats =
   FunctionTable: [28x1 struct]
   FunctionHistory: [2x774 double]
   ClockPrecision: 0.00999999999840
```

3 You can view and access the contents of the structure. For example, type

```
stats.FunctionTable
```

MATLAB displays the FunctionTable structure.

```
ans =
28x1 struct array with fields:
    FunctionName
    MfileName
    Type
    NumCalls
    TotalTime
    TotalRecursiveTime
    Children
    Parents
    ExecutedLines
```

4 To view the contents of an element in the `FunctionTable` structure, type, for example,

```
stats.FunctionTable(2)
```

MATLAB returns the second element in the structure.

```
ans =
          FunctionName: ’ode23’
             MfileName: [1x56 char]
                  Type: ’M-function’
              NumCalls: 1
             TotalTime: 0.65100000000166
    TotalRecursiveTime: 0.65100000000166
              Children: [21x1 struct]
               Parents: [0x1 struct]
         ExecutedLines: [159x3 double]
```

5 Save the results.

```
save profstats
```

6 In a later session, generate the profiler report using the saved results. Type

```
load profstats
profreport(stats)
```

MATLAB displays the profile report.

4

Matrices and Linear Algebra

Matrices and Linear Algebra

A *matrix* is a two-dimensional array of real or complex numbers. *Linear algebra* defines many matrix operations that are directly supported by MATLAB. Matrix arithmetic, linear equations, eigenvalues, singular values, and matrix factorizations are included.

The linear algebra functions are located in the `matfun` directory in the MATLAB Toolbox.

Category	**Function**	**Description**
Matrix analysis	`norm`	Matrix or vector norm.
	`normest`	Estimate the matrix 2-norm.
	`rank`	Matrix rank.
	`det`	Determinant.
	`trace`	Sum of diagonal elements.
	`null`	Null space.
	`orth`	Orthogonalization.
	`rref`	Reduced row echelon form.
	`subspace`	Angle between two subspaces.
Linear equations	`\` and `/`	Linear equation solution.
	`inv`	Matrix inverse.
	`cond`	Condition number for inversion.
	`condest`	1-norm condition number estimate.
	`chol`	Cholesky factorization.
	`cholinc`	Incomplete Cholesky factorization.
	`lu`	LU factorization.
	`luinc`	Incomplete LU factorization.

Category	Function	Description
	`qr`	Orthogonal-triangular decomposition.
	`nnls`	Nonnegative least-squares.
	`pinv`	Pseudoinverse.
	`lscov`	Least squares with known covariance.
Eigenvalues and singular values	`eig`	Eigenvalues and eigenvectors.
	`svd`	Singular value decomposition.
	`eigs`	A few eigenvalues.
	`svds`	A few singular values.
	`poly`	Characteristic polynomial.
	`polyeig`	Polynomial eigenvalue problem.
	`condeig`	Condition number for eigenvalues.
	`hess`	Hessenberg form.
	`qz`	QZ factorization.
	`schur`	Schur decomposition.
Matrix functions	`expm`	Matrix exponential.
	`logm`	Matrix logarithm.
	`sqrtm`	Matrix square root.
	`funm`	Evaluate general matrix function.

Matrices in MATLAB

Informally, the terms matrix and array are often used interchangeably. More precisely, a matrix is a two-dimensional rectangular array of real or complex numbers that represents a linear transformation. The linear algebraic operations defined on matrices have found applications in a wide variety of technical fields. (The Symbolic Math Toolboxes extend MATLAB's capabilities to operations on various types of nonnumeric matrices.)

MATLAB has dozens of functions that create different kinds of matrices. Two of them can be used to create a pair of 3-by-3 example matrices for use throughout this chapter. The first example is symmetric.

```
A = pascal(3)

A =

     1     1     1
     1     2     3
     1     3     6
```

The second example is not symmetric.

```
B = magic(3)

B =

     8     1     6
     3     5     7
     4     9     2
```

Another example is a 3-by-2 rectangular matrix of random integers.

```
C = fix(10*rand(3,2))

C =

     9     4
     2     8
     6     7
```

A c*olumn vector* is an *m*-by-1 matrix, a *row vector* is a 1-by-*n* matrix and a *scalar* is a 1-by-1 matrix. The statements

```
u = [3; 1; 4]

v = [2 0 –1]

s = 7
```

produce a column vector, a row vector, and a scalar.

```
u =

     3
     1
     4

v =

     2     0    –1

s =

     7
```

Addition and Subtraction

Addition and subtraction of matrices is defined just as it is for arrays, element-by-element. Adding `A` to `B` and then subtracting `A` from the result recovers `B`.

```
X = A + B

X =

     9     2     7
     4     7    10
     5    12     8

Y = X –A

Y =

     8     1     6
     3     5     7
     4     9     2
```

Addition and subtraction require both matrices to have the same dimension, or one of them be a scalar. If the dimensions are incompatible, an error results.

```
X = A + C

Error using ==> +
Matrix dimensions must agree.

w = v + s

w =

     9     7     6
```

Vector Products and Transpose

A row vector and a column vector of the same length can be multiplied in either order. The result is either a scalar, the *inner* product, or a matrix, the *outer* product.

```
x = v*u

x =

     2

X = u*v

X =

     6     0    –3
     2     0    –1
     8     0    –4
```

For real matrices, the *transpose* operation interchanges a_{ij} and a_{ji}. MATLAB uses the apostrophe (or single quote) to denote transpose. Our example matrix A is *symmetric*, so A' is equal to A. But B is not symmetric.

```
X = B'

X =

     8     3     4
     1     5     9
     6     7     2
```

Transposition turns a row vector into a column vector.

```
x = v'

x =

     2
     0
    –1
```

If x and y are both real column vectors, the product x*y is not defined, but the two products

```
x′*y
```

and

```
y′*x
```

are the same scalar. This quantity is used so frequently, it has three different names: *inner* product, *scalar* product, or *dot* product.

For a complex vector or matrix, z, the quantity z′ denotes the *complex conjugate transpose*. The unconjugated complex transpose is denoted by z.′, in analogy with the other array operations. So if

```
z = [1+2i 3+4i]
```

then z′ is

```
    1–2i
    3–4i
```

while z.' is

```
    1+2i
    3+4i
```

For complex vectors, the two scalar products x'*y and y'*x are complex conjugates of each other and the scalar product x'*x of a complex vector with itself is real.

Matrix Multiplication

Multiplication of matrices is defined in a way that reflects composition of the underlying linear transformations and allows compact representation of systems of simultaneous linear equations. The matrix product $C = AB$ is defined when the column dimension of A is equal to the row dimension of B, or when one of them is a scalar. If A is m-by-p and B is p-by-n, their product C is m-by-n. The product can actually be defined using MATLAB's for loops, colon notation, and vector dot products.

```
for i = 1:m
     for j = 1:n
        C(i,j) = A(i,:)*B(:,j);
     end
  end
```

MATLAB uses a single asterisk to denote matrix multiplication. The next two examples illustrate the fact that matrix multiplication is not commutative; *AB* is usually not equal to *BA*.

```
X = A*B

X =

    15    15    15
    26    38    26
    41    70    39

Y = B*A

Y =

    15    28    47
    15    34    60
    15    28    43
```

A matrix can be multiplied on the right by a column vector and on the left by a row vector.

```
x = A*u

  x =

       8
      17
      30

  y = v*B

  y =

      12    –7    10
```

Rectangular matrix multiplications must satisfy the dimension compatibility conditions.

```
X = A*C

X =

    17    19
    31    41
    51    70

Y = C*A

Error using ==> *
Inner matrix dimensions must agree.
```

Anything can be multiplied by a scalar.

```
w = s*v

w =

    14     0    –7
```

The Identity Matrix

Generally accepted mathematical notation uses the capital letter I to denote *identity* matrices, matrices of various sizes with ones on the main diagonal and zeros elsewhere. These matrices have the property that $AI = A$ and $IA = A$ whenever the dimensions are compatible. The original version of MATLAB could not use I for this purpose because it did not distinguish between upper and lowercase letters and `i` already served double duty as a subscript and as the complex unit. So an English language pun was introduced. The function

```
eye(m,n)
```

returns an m-by-n rectangular identity matrix and `eye(n)` returns an n-by-n square identity matrix.

The Kronecker Tensor Product

The Kronecker product, kron(X,Y), of two matrices is the larger matrix formed from all possible products of the elements of X with those of Y. If X is *m*-by-*n* and Y is *p*-by-*q*, then kron(X,Y) is *mp*-by-*nq*. The elements are arranged in the order

```
[X(1,1)*Y  X(1,2)*Y  . . .  X(1,n)*Y
                     . . .
 X(m,1)*Y  X(m,2)*Y  . . .  X(m,n)*Y]
```

The Kronecker product is often used with matrices of zeros and ones to build up repeated copies of small matrices. For example, if X is the 2-by-2 matrix

```
X =

     1     2
     3     4
```

and I = eye(2,2) is the 2-by-2 identity matrix, then the two matrices

```
kron(X,I)
```

and

```
kron(I,X)
```

are

```
     1     0     2     0
     0     1     0     2
     3     0     4     0
     0     3     0     4
```

and

```
     1     2     0     0
     3     4     0     0
     0     0     1     2
     0     0     3     4
```

Vector and Matrix Norms

The p-norm of a vector x

$$,\|x\|_p = \left(\sum|x_i|^p\right)^{1/p}$$

is computed by `norm(x,p)`. This is defined by any value of $p > 1$, but the most common values of p are 1, 2, and ∞. The default value is $p = 2$, which corresponds to *Euclidean length*.

```
[norm(v,1) norm(v) norm(v,inf)]

ans =

    3.0000    2.2361    2.0000
```

The p-norm of a matrix A,

$$\|A\|_p = \frac{\max}{x}\|Ax\|_p/\|x\|_p$$

can be computed for $p = 1$, 2, and ∞ by `norm(A,p)`. Again, the default value is $p = 2$.

```
[norm(C,1) norm(C) norm(C,inf)]

ans =

   19.0000   14.8015   13.0000
```

Solving Linear Equations

One of the most important problems in technical computing is the solution of simultaneous linear equations. In matrix notation, this problem can be stated as follows:

Given two matrices A and B, does there exist a unique matrix X so that $AX = B$ or $XA = B$?

It is instructive to consider a 1-by-1 example.

Does the equation

$$7x = 21$$

have a unique solution ?

The answer, of course, is yes. The equation has the unique solution $x = 3$. The solution is easily obtained by *division*:

$$x = 21/7 = 3$$

The solution is *not* ordinarily obtained by computing the inverse of 7, that is $7^{-1} = 0.142857\ldots$, and then multiplying 7^{-1} by 21. This would be more work and, if 7^{-1} is represented to a finite number of digits, less accurate. Similar considerations apply to sets of linear equations with more than one unknown; MATLAB solves such equations without computing the inverse of the matrix.

Although it is not standard mathematical notation, MATLAB uses the division terminology familiar in the scalar case to describe the solution of a general system of simultaneous equations. The two division symbols, *slash*, /, and *backslash*, \, are used for the two situations where the unknown matrix appears on the left or right of the coefficient matrix.

`X = A\B` denotes the solution to the matrix equation $AX = B$.

`X = B/A` denotes the solution to the matrix equation $XA = B$.

You can think of “dividing” both sides of the equation $AX = B$ or $XA = B$ by A. The coefficient matrix `A` is always in the “denominator”.

The dimension compatibility conditions for `X = A\B` require the two matrices `A` and `B` to have the same number of rows. The solution `X` then has the same number of columns as `B` and its row dimension is equal to the column dimension of `A`. For `X = B/A`, the roles of rows and columns are interchanged.

In practice, linear equations of the form $AX = B$ occur more frequently than those of the form $XA = B$. Consequently, backslash is used far more frequently than slash. The remainder of this section concentrates on the backslash operator; the corresponding properties of the slash operator can be inferred from the identity

```
(B/A)′ = (A′\B′)
```

The coefficient matrix `A` need not be square. If `A` is m-by-n, there are three cases.

$m = n$.	Square system. Seek an exact solution.
$m > n$.	Overdetermined system. Find a least squares solution.
$m < n$.	Underdetermined system. Find a basic solution with at most m nonzero components.

The backslash operator employs different algorithms to handle different kinds of coefficient matrices. The various cases, which are diagnosed automatically by examining the coefficient matrix, include:

- Permutations of triangular matrices
- Symmetric, positive definite matrices
- Square, nonsingular matrices
- Rectangular, overdetermined systems
- Rectangular, underdetermined systems

Square Systems

The most common situation involves a square coefficient matrix `A` and a single right-hand side column vector `b`. The solution, `x = A\b`, is then the same size as `b`. For example

```
x = A\u

x =

    10
   –12
     5
```

It can be confirmed that `A*x` is exactly equal to `u`.

If `A` and `B` are square and the same size, then `X = A\B` is also that size.

```
X = A\B

X =

    19    -3    –1
   –17     4    13
     6     0    –6
```

It can be confirmed that `A*X` is exactly equal to `B`.

Both of these examples have exact, integer solutions. This is because the coefficient matrix was chosen to be `pascal(3)`, which has a determinant equal to one. A later section considers the effects of roundoff error inherent in more realistic computation.

A square matrix *A* is *singular* if it does not have linearly independent columns. If *A* is singular, the solution to *AX* = *B* either does not exist, or is not unique. The backslash operator, `A\B`, issues a warning if A is nearly singular and raises an error condition if exact singularity is detected.

Overdetermined Systems

Overdetermined systems of simultaneous linear equations are often encountered in various kinds of curve fitting to experimental data. Here is a hypothetical example. A quantity *y* is measured at several different values of time, *t*, to produce the following observations:

t	*y*
0.0	0.82
0.3	0.72
0.8	0.63
1.1	0.60
1.6	0.55
2.3	0.50

This data can be entered into MATLAB with the statements

```
t = [0 .3 .8 1.1 1.6 2.3]';
y = [.82 .72 .63 .60 .55 .50]';
```

It is believed that the data can be modeled with a decaying exponential function.

$$y(t) \approx c_1 + c_2 e^{-t}$$

This equation says that the vector y should be approximated by a linear combination of two other vectors, one the constant vector containing all ones and the other the vector with components e^{-t}. The unknown coefficients, c_1 and c_2, can be computed by doing a *least squares fit*, which minimizes the sum of the squares of the deviations of the data from the model. There are six equations in two unknowns, represented by the 6-by-2 matrix.

```
   E = [ones(size(t)) exp(-t)]

    E =

        1.0000    1.0000
        1.0000    0.7408
        1.0000    0.4493
        1.0000    0.3329
        1.0000    0.2019
        1.0000    0.1003
```

The least squares solution is found with the backslash operator.

```
    c = E\y

    c =

        0.4760
        0.3413
```

In other words, the least squares fit to the data is

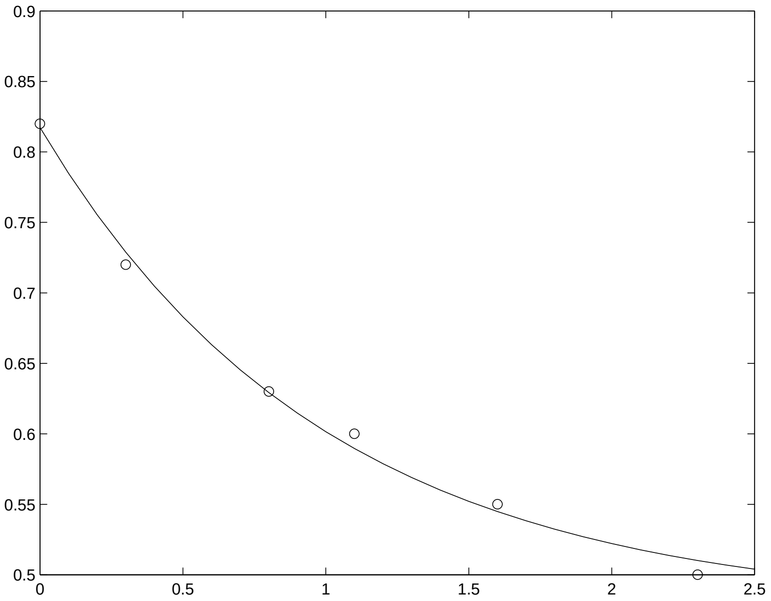
$$y(t) \approx 0.4760 + 0.3413 \;\; e^{-t}$$

The following statements evaluate the model at regularly spaced increments in t, and then plot the result, together with the original data.

```
    T = (0:0.1:2.5)';
    Y = [ones(size(T)) exp(-T)]*c;
    plot(T,Y,'-',t,y,'o')
```

You can see that `E*c` is not exactly equal to `y`, but that the difference might well be less than measurement errors in the original data.

A rectangular matrix A is *rank deficient* if it does not have linearly independent columns. If A is rank deficient, the least squares solution to $AX = B$ is not unique. The backslash operator, `A\B`, issues a warning if A is rank deficient and produces a *basic* solution that has as few nonzero elements as possible.



Undetermined Systems

Underdetermined linear systems involve more unknowns than equations. When they are accompanied by additional constraints, they are the purview of *linear programming*. By itself, the backslash operator deals only with the unconstrained system. The solution is never unique. MATLAB finds a *basic* solution, which has at most m nonzero components, but even this may not be unique. The particular solution actually computed is determined by the QR factorization with column pivoting (see a later section on the QR factorization).

Here is a small, random example.

```
R = fix(10*rand(2,4))

R =

     6     8     7     3
     3     5     4     1

b = fix(10*rand(2,1))

b =

     1
     2
```

The linear system $Rx = b$ involves two equations in four unknowns. Since the coefficient matrix contains small integers, it is appropriate to display the solution in *rational* format. The particular solution is obtained with

```
format rat

p = R\b

p =

    0
   5/7
    0
 –11/7
```

One of the nonzero components is `p(2)` because `R(:,2)` is the column of `R` with largest norm. The other nonzero component is `p(4)` because `R(:,4)` dominates after `R(:,2)` is eliminated.

The complete solution to the overdetermined system can be characterized by adding an arbitrary vector from the null space, which can be found using the `null` function with an option requesting a “rational” basis.

```
Z = null(R,’r’)

Z =

    –1/2         –7/6
    –1/2          1/2
      1            0
      0            1
```

It can be confirmed that `A*Z` is zero and that any vector of the form

```
x = p + Z*q
```

for an arbitrary vector `q` satisfies `R*x = b`.

Inverses and Determinants

If A is square and nonsingular, the equations $AX = I$ and $XA = I$ have the same solution, X. This solution is called the *inverse* of A, is denoted by A^{-1}, and is computed by the function `inv`. The determinant of a matrix is useful in theoretical considerations and some types of symbolic computation, but its scaling and roundoff error properties make it far less satisfactory for numeric computation. Nevertheless, the function `det` computes the determinant of a square matrix.

```
d = det(A)
X = inv(A)

d =

     1

X =

     3    –3     1
    –3     5    –2
     1    –2     1
```

Again, because `A` is symmetric, has integer elements, and has determinant equal to one, so does its inverse. On the other hand,

```
d = det(B)
X = inv(B)

d =

–360

X =

    0.1472   –0.1444    0.0639
   –0.0611    0.0222    0.1056
   –0.0194    0.1889   –0.1028
```

Closer examination of the elements of `X`, or use of `format rat`, would reveal that they are integers divided by 360.

If A is square and nonsingular, then without roundoff error, X = inv(A)*B would theoretically be the same as X = A\B and Y = B*inv(A) would theoretically be the same as Y = B/A. But the computations involving the backslash and slash operators are preferable because they require less computer time, less memory, and have better error detection properties.

Pseudoinverses

Rectangular matrices do not have inverses or determinants. At least one of the equations $AX = I$ and $XA = I$ does not have a solution. A partial replacement for the inverse is provided by the *Moore-Penrose pseudoinverse*, which is computed by the pinv function.

```
X = pinv(C)

X =

    0.1159   –0.0729    0.0171
   –0.0534    0.1152    0.0418
```

The matrix

```
Q = X*C

Q =

    1.0000    0.0000
    0.0000    1.0000
```

is the 2-by-2 identity, but the matrix

```
P = C*X

P =

    0.8293   –0.1958    0.3213
   –0.1958    0.7754    0.3685
    0.3213    0.3685    0.3952
```

is not the 3-by-3 identity. However, P acts like an identity on a portion of the space in the sense that P is symmetric, P*C is equal to C and X*P is equal to X.

If `A` is *m*-by-*n* with *m* > *n* and full rank *n*, then each of the three statements

```
x = A\b
x = pinv(A)*b
x = inv(A’*A)*A’*b
```

theoretically computes the same least squares solution `x`, although the backslash operator does it faster.

However, if `A` does not have full rank, the solution to the least squares problem is not unique. There are many vectors `x` that minimize

```
norm(A*x –b)
```

The solution computed by `x = A\b` is a *basic* solution; it has at most *r* nonzero components, where *r* is the rank of `A`. The solution computed by `x = pinv(A)*b` is the *minimal norm* solution; it also minimizes `norm(x)`. An attempt to compute a solution with `x = inv(A'*A)*A'*b` fails because `A'*A` is singular.

Here is an example to illustrates the various solutions.

```
A = [ 1  2  3
      4  5  6
      7  8  9
      10 11 12]
```

does not have full rank. Its second column is the average of the first and third columns. If

```
b = A(:,2)
```

is the second column, then an obvious solution to `A*x = b` is `x = [0 1 0]'`. But none of the approaches computes that `x`. The backslash operator gives

```
x = A\b

Warning: Rank deficient, rank = 2.

x =

      0.5000
           0
      0.5000
```

This solution has two nonzero components. The pseudoinverse approach gives

```
y = pinv(A)*b

y =

    0.3333
    0.3333
    0.3333
```

There is no warning about rank deficiency. But `norm(y) = 0.5774` is less than `norm(x) = 0.7071`. Finally

```
z = inv(A'*A)*A'*b
```

fails completely.

```
Warning: Matrix is singular to working precision.

z =

   Inf
   Inf
   Inf
```

LU, QR, and Cholesky Factorizations

MATLAB's linear equation capabilities are based on three basic matrix factorizations.

- Cholesky factorization for symmetric, positive definite matrices
- Gaussian elimination for general square matrices
- Orthogonalization for rectangular matrices

These three factorizations are available through the `chol`, `lu`, and `qr` functions.

All three of these factorizations make use of *triangular* matrices where all the elements either above or below the diagonal are zero. Systems of linear equations involving triangular matrices are easily and quickly solved using either *forward* or *back substitution*.

Cholesky Factorization

The Cholesky factorization expresses a symmetric matrix as the product of a triangular matrix and its transpose.

$$A = R'R$$

where R is an upper triangular matrix.

Not all symmetric matrices can be factored in this way; the matrices that have such a factorization are said to be *positive definite*. This implies that all the diagonal elements of A are positive and that the offdiagonal elements are "not too big." The Pascal matrices provide an interesting example. Throughout this chapter, our example matrix `A` has been the 3-by-3 Pascal matrix. Let's temporarily switch to the 6-by-6.

```
A = pascal(6)

A =

     1     1     1     1     1     1
     1     2     3     4     5     6
     1     3     6    10    15    21
     1     4    10    20    35    56
     1     5    15    35    70   126
     1     6    21    56   126   252
```

The elements of A are binomial coefficients. Each element is the sum of its north and west neighbors. The Cholesky factorization is

```
R = chol(A)

R =

     1     1     1     1     1     1
     0     1     2     3     4     5
     0     0     1     3     6    10
     0     0     0     1     4    10
     0     0     0     0     1     5
     0     0     0     0     0     1
```

The elements are again binomial coefficients. The fact that R'*R is equal to A demonstrates an identity involving sums of products of binomial coefficients.

The Cholesky factorization also applies to complex matrices. Any complex matrix which has a Cholesky factorization satisfies $A' = A$ and is said to be *Hermitian positive definite*.

The Cholesky factorization allows the linear system

```
A*x = b
```

to be replaced by

```
R'*R*x = b
```

Because the backslash operator recognizes triangular systems, this can be solved quickly with

```
x = R\(R'\b)
```

If A is n-by-n, the computational complexity of chol(A) is $O(n^3)$, but the complexity of the subsequent backslash solutions is only $O(n^2)$.

LU Factorization

Gaussian elimination, or LU factorization, expresses any square matrix as the product of a permutation of a lower triangular matrix and an upper triangular matrix

$$A = L\,U$$

where L is a permutation of a lower triangular matrix with ones on its diagonal and U is an upper triangular matrix.

The permutations are necessary for both theoretical and computational reasons. The matrix

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

cannot be expressed as the product of triangular matrices without interchanging its two rows. Although the matrix

$$\begin{bmatrix} \varepsilon & 1 \\ 1 & 0 \end{bmatrix}$$

can be expressed as the product of triangular matrices, when ε is small the elements in the factors are large and magnify errors, so even though the permutations are not strictly necessary, they are desirable. *Partial pivoting* ensures that the elements of L are bounded by one in magnitude and that the elements of U are not much larger than those of A.

For example

```
[L,U] = lu(B)

L =

    1.0000         0         0
    0.3750    0.5441    1.0000
    0.5000    1.0000         0

U =

    8.0000    1.0000    6.0000
         0    8.5000   –1.0000
         0         0    5.2941
```

The LU factorization of A allows the linear system

```
A*x = b
```

to be solved quickly with

```
x = U\(L\b)
```

Determinants and inverses are computed from the LU factorization using

```
det(A) = det(L)*det(U) =±±prod(diag(U))
```

and

```
inv(A) = inv(U)*inv(L)
```

QR Factorization

An *orthogonal* matrix, or a matrix with *orthonormal columns*, is a real matrix whose columns all have unit length and are perpendicular to each other. If *Q* is orthogonal, then

$$Q' Q = I$$

The simplest orthogonal matrices are two-dimensional coordinate rotations.

$$\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

For complex matrices, the corresponding term is *unitary*. Orthogonal and unitary matrices are desirable for numerical computation because they preserve length, preserve angles, and do not magnify errors.

The orthogonal, or QR, factorization expresses any rectangular matrix as the product of an orthogonal or unitary matrix and an upper triangular matrix. A column permutation may also be involved.

```
A = Q R
```

or

```
A P = Q R
```

where *Q* is orthogonal or unitary, *R* is upper triangular, and *P* is a permutation.

There are four variants of the QR factorization– full or economy size and with or without column permutation.

Overdetermined linear systems involve a rectangular matrix with more rows than columns, that is m-by-n with $m > n$. The *full* size QR factorization produces a square, m-by-m orthogonal Q and a rectangular m-by-n upper triangular R.

```
[Q,R] = qr(C)

Q =

   –0.8182    0.3999   –0.4131
   –0.1818   –0.8616   –0.4739
   –0.5455   –0.3126    0.7777

R =

  –11.0000   –8.5455
         0   –7.4817
         0         0
```

In many cases, the last m - n columns of Q are not needed because they are multiplied by the zeros in the bottom portion of R. So the *economy* size QR factorization produces a rectangular, m-by-n Q with orthonormal columns and a square n-by-n upper triangular R. For our 3-by-2 example, this is not much of a saving, but for larger, highly rectangular matrices, the savings in both time and memory can be quite important.

```
[Q,R] = qr(C,0)

Q =

   –0.8182    0.3999
   –0.1818   –0.8616
   –0.5455   –0.3126

R =

  –11.0000   –8.5455
         0   –7.4817
```

In contrast to the LU factorization, the QR factorization does not require any pivoting or permutations. But an optional column permutation, triggered by the presence of a third output argument, is useful for detecting singularity or rank deficiency. At each step of the factorization, the column of the remaining unfactored matrix with largest norm is used as the basis for that step. This ensures that the diagonal elements of *R* occur in decreasing order and that any linear dependence among the columns will almost certainly be revealed by examining these elements. For our small example, the second column of `C` has a larger norm than the first, so the two columns are exchanged.

```
[Q,R,P] = qr(C)

Q =

   –0.3522    0.8398   –0.4131
   –0.7044   –0.5285   –0.4739
   –0.6163    0.1241    0.7777

R =

  –11.3578   –8.2762
         0    7.2460
         0         0

P =

     0     1
     1     0
```

When the economy size and column permutations are combined, the third output argument is a permutation vector, rather than a permutation matrix.

```
[Q,R,p] = qr(C,0)

Q =

   –0.3522    0.8398
   –0.7044   –0.5285
   –0.6163    0.1241

R =

  –11.3578   –8.2762
         0    7.2460

p =

     2     1
```

The QR factorization transforms an overdetermined linear system into an equivalent triangular system. The expression

```
norm(A*x – b)
```

is equal to

```
norm(Q*R*x – b)
```

Multiplication by orthogonal matrices preserves the Euclidean norm, so this expression is also equal to

```
norm(R*x – y)
```

where `y = Q'*b`. Since the last $m - n$ rows of R are zero, this expression breaks into two pieces

```
norm(R(1:n,1:n)*x – y(1:n))
```

and

```
norm(y(n+1:m))
```

When `A` has full rank, it is possible to solve for `x` so that the first of these expressions is zero. Then the second expression gives the norm of the residual. When `A` does not have full rank, the triangular structure of `R` makes it possible to find a basic solution to the least squares problem.

Matrix Powers and Exponentials

If A is a square matrix and p is a positive integer, then A^p multiplies A by itself p times.

```
X = A^2

X =

     3     6    10
     6    14    25
    10    25    46
```

If A is square and nonsingular, then A^(–p) multiplies inv(A) by itself p times.

```
Y = B^(–3)

Y =

     0.0053   –0.0068    0.0018
    –0.0034    0.0001    0.0036
    –0.0016    0.0070   –0.0051
```

Fractional powers, like A^(2/3), are also permitted; the results depend upon the distribution of the eigenvalues of the matrix.

Element-by-element powers are obtained with .^. For example

```
X = A.^2

A =

     1     1     1
     1     4     9
     1     9    36
```

The function

```
sqrtm(A)
```

computes A^(1/2) by a more accurate algorithm. The m in sqrtm distinguishes this function from sqrt(A) which, like A.^(1/2), does its job element-by-element.

A system of linear, constant coefficient, ordinary differential equations can be written

$$dx/dt = Ax$$

where $x = x(t)$ is a vector of functions of t and A is a matrix independent of t. The solution can be expressed in terms of the *matrix exponential*,

$$x(t) = e^{tA}x(0)$$

The function

```
expm(A)
```

computes the matrix exponential. An example is provided by the 3-by-3 coefficient matrix

```
A =

     0    –6    –1
     6     2   –16
    –5    20   –10
```

and the initial condition, $x(0)$

```
x0 =

     1
     1
     1
```

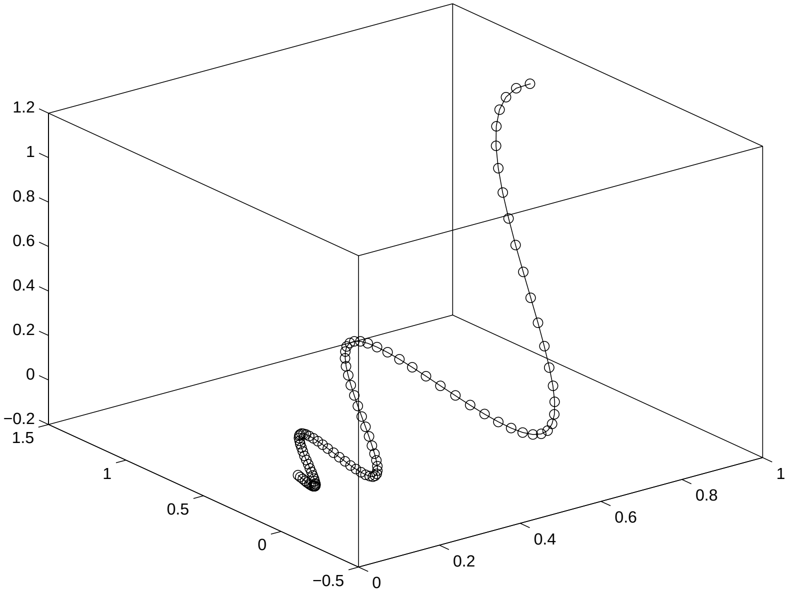
The matrix exponential is used to compute the solution, $x(t)$, to the differential equation at 101 points on the interval $0 \le t \le 1$ with

```
X = [];
for t = 0:.01:1
   X = [X expm(t*A)*x0];
end
```

A three-dimensional phase plane plot obtained with

```
plot3(X(1,:),X(2,:),X(3,:),'-o')
```

shows the solution spiraling in towards the origin. This behavior is related to the eigenvalues of the coefficient matrix, which are discussed in the next section.



Eigenvalues

An *eigenvalue* and *eigenvector* of a square matrix A are a scalar λ and a vector v that satisfy

$$Av = \lambda v$$

With the eigenvalues on the diagonal of a diagonal matrix Λ and the corresponding eigenvectors forming the columns of a matrix V, we have

$$AV = V\Lambda$$

If V is nonsingular, this becomes the *eigenvalue decomposition*

$$A = V\Lambda V^{-1}$$

A good example is provided by the coefficient matrix of the ordinary differential equation in the previous section.

```
A =

     0    –6    –1
     6     2   –16
    –5    20   –10
```

The statement

```
lambda = eig(A)
```

produces a column vector containing the eigenvalues. For this matrix, the eigenvalues are complex.

```
lambda =

  –3.0710
  –2.4645+17.6008i
  –2.4645-17.6008i
```

The real part of each of the eigenvalues is negative, so $e^{\lambda t}$ approaches zero as t increases. The nonzero imaginary part of two of the eigenvalues, $\pm\omega$, contributes the oscillatory component, $\sin(\omega t)$, to the solution of the differential equation.

With two output arguments, `eig` computes the eigenvectors and stores the eigenvalues in a diagonal matrix.

```
[V,D] = eig(A)

V =

  –0.8326           –0.1203+ 0.2123i  –0.1203– 0.2123i
  –0.3553            0.4691+ 0.4901i   0.4691– 0.4901i
  –0.4248            0.6249– 0.2997i   0.6249+ 0.2997i

D =

  –3.0710                  0                 0
        0           –2.4645+17.6008i        0
        0                  0          –2.4645–17.6008i
```

The first eigenvector is real and the other two vectors are complex conjugates of each other. All three vectors are normalized to have Euclidean length, `norm(v,2)`, equal to one.

The matrix `V*D*inv(V)`, which can be written more succinctly as `V*D/V`, is within roundoff error of `A`. And, `inv(V)*A*V`, or `V\A*V`, is within roundoff error of `D`.

Some matrices do not have an eigenvector decomposition. These matrices are *defective*, or *not diagonalizable*. For example,

```
A =

     6    12    19
    –9   –20   –33
     4     9    15
```

For this matrix

```
[V,D] = eig(A)
```

produces

```
V =

    0.4741    0.4082   –0.4082
   –0.8127   –0.8165    0.8165
    0.3386    0.4082   –0.4082

D =

   –1.0000         0         0
         0    1.0000         0
         0         0    1.0000
```

There is a double eigenvalue at $\lambda = 1$. The second and third columns of `V` are negatives of each other; they are merely different normalizations of the single eigenvector corresponding to $\lambda = 1$. For this matrix, a full set of linearly independent eigenvectors does not exist.

The optional Symbolic Math Toolbox extends MATLAB's capabilities by connecting to Maple, a powerful computer algebra system. One of the functions provided by the toolbox computes the Jordan Canonical Form. This is appropriate for matrices like our example, which is 3-by-3 and has exactly known, integer elements.

```
[X,J] = jordan(A)

X =

   –1.7500    1.5000    2.7500
    3.0000   –3.0000   –3.0000
   –1.2500    1.5000    1.2500

J =

    –1     0     0
     0     1     1
     0     0     1
```

The Jordan Canonical Form is an important theoretical concept, but it is not a reliable computational tool for larger matrices, or for matrices whose elements are subject to roundoff errors and other uncertainties.

MATLAB's advanced matrix computations do not require eigenvalue decompositions. They are based, instead, on the *Schur decomposition*,

$$A = U\ S\ U^{\mathrm{T}}$$

where U is an orthogonal matrix and S is a block upper triangular matrix with 1-by-1 and 2-by-2 blocks on the diagonal. The eigenvalues are revealed by the diagonal elements and blocks of S, while the columns of U provide a basis with much better numerical properties than a set of eigenvectors. The Schur decomposition of our defective example is

```
[U,S] = schur(A)

U =

    0.4741   –0.6571    0.5861
   –0.8127   –0.0706    0.5783
    0.3386    0.7505    0.5675

S =

   –1.0000   21.3737   44.4161
         0    1.0081    0.6095
         0   –0.0001    0.9919
```

The double eigenvalue is contained in the lower 2-by-2 block of `S`.

Singular Value Decomposition

A *singular value* and corresponding *singular vectors* of a rectangular matrix A are a scalar σ and a pair of vectors u and v that satisfy

$$\begin{aligned} Av &= \sigma u \\ A^{T}u &= \sigma v \end{aligned}$$

With the singular values on the diagonal of a diagonal matrix Σ and the corresponding singular vectors forming the columns of two orthogonal matrices U and V, we have

$$\begin{aligned} AV &= U\Sigma \\ A^{T}U &= V\Sigma \end{aligned}$$

Since U and V are orthogonal, this becomes the *singular value decomposition*

$$A = U\Sigma V^{T}$$

The full singular value decomposition of an m-by-n matrix involves an m-by-m U, an m-by-n Σ, and an n-by-n V. In other words, U and V are both square and Σ is the same size as A. If A has many more rows than columns, the resulting U can be quite large, but most of its columns are multiplied by zeros in Σ. In this situation, the *economy* sized decomposition saves both time and storage by producing an m-by-n U, an n-by-n Σ and the same V.

The eigenvalue decomposition is the appropriate tool for analyzing a matrix when it represents a mapping from a vector space into itself, as it does for an ordinary differential equation. On the other hand, the singular value decomposition is the appropriate tool for analyzing a mapping from one vector space into another vector space, possibly with a different dimension. Most systems of simultaneous linear equations fall into this second category.

If A is square, symmetric, and positive definite, then its eigenvalue and singular value decompositions are the same. But, as A departs from symmetry and positive definiteness, the difference between the two decompositions increases. In particular, the singular value decomposition of a real matrix is always real, but the eigenvalue decomposition of a real, nonsymmetric matrix might be complex.

For the example matrix

```
A =

     9     4
     6     8
     2     7
```

the full singular value decomposition is

```
[U,S,V] = svd(A)

U =

    0.6105   –0.7174    0.3355
    0.6646    0.2336   –0.7098
    0.4308    0.6563    0.6194

S =

   14.9359         0
         0    5.1883
         0         0

V =

    0.6925   –0.7214
    0.7214    0.6925
```

You can verify that U*S*V′ is equal to A to within roundoff error. For this small problem, the economy size decomposition is only slightly smaller.

```
[U,S,V] = svd(A,0)

U =

    0.6105   –0.7174
    0.6646    0.2336
    0.4308    0.6563

S =

   14.9359         0
         0    5.1883

V =

    0.6925   –0.7214
    0.7214    0.6925
```

Again, U*S*V' is equal to A to within roundoff error.

Polynomials and Interpolation

Polynomials

MATLAB provides functions for standard polynomial operations, such as polynomial roots, evaluation, and differentiation. In addition, there are functions for more advanced applications, such as curve fitting and partial fraction expansion.

The polynomial functions live in a directory called `polyfun` in the MATLAB Toolbox.

Function	Description
`conv`	Multiply polynomials.
`deconv`	Divide polynomials.
`poly`	Polynomial with specified roots.
`polyder`	Polynomial derivative.
`polyfit`	Polynomial curve fitting.
`polyval`	Polynomial evaluation.
`polyvalm`	Matrix polynomial evaluation.
`residue`	Partial-fraction expansion (residues).
`roots`	Find polynomial roots.

The Symbolic Math Toolbox contains additional specialized support for polynomial operations.

Representing Polynomials

MATLAB represents polynomials as row vectors containing coefficients ordered by descending powers. For example, consider the equation

$$p(x) = x^3 - 2x - 5$$

This is the celebrated example Wallis used when he first represented Newton's method to the French Academy. To enter this polynomial into MATLAB, use

```
p = [1 0 –2 –5];
```

Polynomial Roots

The roots function calculates the roots of a polynomial.

```
r = roots(p)

r =
      2.0946
     –1.0473 +      1.1359i
     –1.0473 –      1.1359i
```

By convention, MATLAB stores roots in column vectors. The function poly returns to the polynomial coefficients.

```
p2 = poly(r)

p2 =
     1   8.8818e-16   –2   –5
```

poly and roots are inverse functions, up to ordering, scaling, and roundoff error.

Characteristic Polynomials

The poly function also computes the coefficients of the characteristic polynomial of a matrix.

```
A = [1.2 3 –0.9; 5 1.75 6; 9 0 1];
poly(A)

ans =
    1.0000   –3.9500   –1.8500  –163.2750
```

The roots of this polynomial, computed with roots, are the *characteristic roots*, or eigenvalues, of the matrix A. (Use eig to compute the eigenvalues of a matrix directly.)

Polynomial Evaluation

The polyval function evaluates a polynomial at a specified value. To evaluate p at $s = 5$, use

```
polyval(p,5)

ans =
   110
```

It is also possible to evaluate a polynomial in a matrix sense. In this case $p(s) = x^3 - 2x - 5$ becomes $p(X) = X^3 - 2X - 5I$, where X is a square matrix and I is the identity matrix. For example, create a square matrix X and evaluate the polynomial p at X.

```
X = [2 4 5; -1 0 3; 7 1 5];
Y = polyvalm(p,X)

Y =
   377   179   439
   111    81   136
   490   253   639
```

Convolution and Deconvolution

Polynomial multiplication and division correspond to the operations convolution and deconvolution. The functions conv and deconv implement these operations.

Consider the polynomials $a(s) = s^2 + 2s +3$ and $b(s) = 4s^2 + 5s + 6$. To compute their product,

```
a = [1 2 3]; b = [4 5 6];
c = conv(a,b)

c =
     4    13    28    27    18
```

Use deconvolution to divide a(*s*) back out of the product:

```
[q,r] = deconv(c,a)

q =
     4     5     6


r =
     0     0     0     0     0
```

Polynomial Derivatives

The polyder function computes the derivative of any polynomial. To obtain the derivative of the polynomial p = [1 0 –2 –5],

```
q = polyder(p)

q =
     3     0    –2
```

polyder also computes the derivative of the product or quotient of two polynomials. For example, create two polynomials a and b:

```
a = [1 3 5];
b = [2 4 6];
```

Calculate the derivative of the product a*b by calling polyder with a single output argument:

```
c = polyder(a,b)

c =
     8    30    56    38
```

Calculate the derivative of the quotient a/b by calling polyder with two output arguments:

```
[q,d] = polyder(a,b)

q =
    –2    –8    –2


d =
     4    16    40    48    36
```

q/d is the result of the operation.

Polynomial Curve Fitting

polyfit finds the coefficients of a polynomial that fits a set of data in a least-squares sense.

```
p = polyfit(x,y,n)
```

x and y are vectors containing the *x* and *y* data to be fitted, and n is the order of the polynomial to return. For example, consider the *x*-*y* test data:

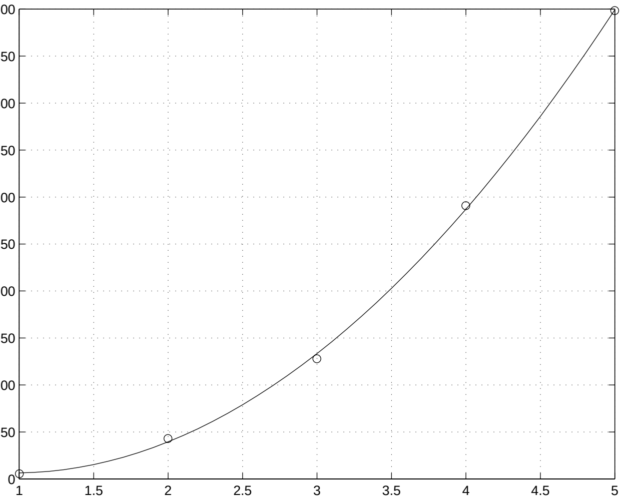
```
x = [1 2 3 4 5]; y = [5.5 43.1 128 290.7 498.4];
```

A third order polynomial that approximately fits the data is

```
p = polyfit(x,y,3)

p =
   –0.1917   31.5821  –60.3262   35.3400
```

Compute the values of the polyfit estimate over a finer range, and plot the estimate over the real data values for comparison.

```
x2 = 1:.1:5;
y2 = polyval(p,x2);
plot(x,y,'o',x2,y2)
grid on
```



To use these functions in an application example, see Chapter 6.

Partial Fraction Expansion

residue finds the partial fraction expansion of the ratio of two polynomials. This is particularly useful for applications that represent systems in transfer function form. For polynomials b and a, if there are no multiple roots,

$$\frac{b(s)}{a(s)} = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2} + \ldots + \frac{r_n}{s - p_n} + k_s$$

where r is a column vector of residues, p is a column vector of pole locations, and k is a row vector of direct terms. Consider the transfer function

$$\frac{-4+8s^{-1}}{1+6s^{-1}+8s^{-2}}$$

```
b = [-4 8];
a = [1 6 8];
[r,p,k] = residue(b,a)

r =
   -12
     8


p =
    -4
    -2


k =
     []
```

Given three input arguments (`r`, `p`, and `k`), `residue` converts back to polynomial form:

```
[b2,a2] = residue(r,p,k)

b2 =
    -4     8


a2 =
     1     6     8
```

Interpolation

Interpolation is a process for estimating values that lie between known data points. It has important applications in areas such as signal and image processing. MATLAB provides a number of interpolation techniques that let you balance the smoothness of the data fit with speed of execution and memory usage.

The interpolation functions live in a directory called `polyfun` in the MATLAB Toolbox.

Function	Description
`griddata`	Data gridding and surface fitting.
`interp1`	One-dimensional interpolation (table lookup).
`interp2`	Two-dimensional interpolation (table lookup).
`interp3`	Three-dimensional interpolation (table lookup).
`interpft`	One-dimensional interpolation using FFT method.
`interpn`	N-D interpolation (table lookup).
`spline`	Cubic spline data interpolation.

One-Dimensional Interpolation

There are two kinds of one-dimensional interpolation in MATLAB:

- Polynomial interpolation
- FFT-based interpolation

Polynomial Interpolation

The function `interp1` performs one-dimensional interpolation, an important operation for data analysis and curve fitting. This function uses polynomial techniques, fitting the supplied data with polynomial functions between data points and evaluating the appropriate function at the desired interpolation points. Its most general form is

```
yi = interp1(x,y,xi,method)
```

y is a vector containing the values of a function, and x is a vector of the same length containing the points for which the values in y are given. xi is a vector containing the points at which to interpolate. *method* is an optional string specifying an interpolation method.

There are four different interpolation methods for one-dimensional data:

- *Nearest neighbor interpolation* (method = 'nearest'). This method sets the value of an interpolated point to the value of the nearest existing data point. It uses the same algorithm as the round function to determine which value to choose: values of xi with decimal portion less than 0.5 receive the preceding value; values of xi with decimal portion greater than or equal to 0.5 receive the succeeding value. Out-of range points receive a value of NaN (Not a Number).
- *Linear interpolation* (method = 'linear'). This method fits separate functions between each pair of existing data points, and returns the value of the relevant function at the points specified by xi. This is the default method for the interp1 function. Out-of-range points receive a value of NaN.
- *Cubic spline interpolation* (method = 'spline'). This method uses a series of functions to obtain interpolated data points, determining separate functions between each pair of existing data points. At its endpoint (an existing data point), each function has at least the same first and second derivatives as the function following it.
- *Cubic interpolation* (method = 'cubic'). This method fits a cubic function through y, and returns the value of this function at the points specified by xi. Out-of-range points receive a value of NaN.

All of these methods require that x be *monotonic*, that is, either always increasing or always decreasing from point to point. Each method works with non-uniformed spaced data. If x is already equally spaced, you can speed execution time by prepending an asterisk to the method string, for example, '*cubic'.

Speed, Memory, and Smoothness Considerations

When choosing an interpolation method, keep in mind that some require more memory or longer computation time than others. However, you may need to trade off these resources to achieve the desired smoothness in the result.

- Nearest neighbor interpolation is the fastest method. However, it provides the worst results in terms of smoothness.
- Linear interpolation uses more memory than the nearest neighbor method, and requires slightly more execution time. Unlike nearest neighbor interpolation its results are continuous, but the slope changes at the vertex points.
- Cubic interpolation requires more memory and execution time than either the nearest neighbor or linear methods. However, both the interpolated data and its derivative are continuous.
- Cubic spline interpolation has the longest relative execution time, although it requires less memory than cubic interpolation. It produces the smoothest results of all the interpolation methods. You may obtain unexpected results, however, if your input data is non-uniform and some points are much closer together than others.

The relative performance of each method holds true even for interpolation of two-dimensional or multidimensional data. For a graphical comparison of interpolation methods, see the section “Comparing Interpolation Methods” on page 5-13.

FFT-Based Interpolation

The function `interpft` performs one-dimensional interpolation using an FFT-based method. This method calculates the Fourier transform of a vector that contains the values of a periodic function. It then calculates the inverse Fourier transform using more points. Its form is

```
y = interpft(x,n)
```

`x` is a vector containing the values of a periodic function, sampled at equally spaced points. `n` is the number of equally spaced points to return.

Two-Dimensional Interpolation

The function interp2 performs two-dimensional interpolation, an important operation for image processing and data visualization. Its most general form is

```
ZI = interp2(X,Y,Z,XI,YI,method)
```

Z is a rectangular array containing the values of a two-dimensional function, and X and Y are arrays of the same size containing the points for which the values in Z are given. XI and YI are matrices containing the points at which to interpolate the data. method is an optional string specifying an interpolation method.

There are three different interpolation methods for two-dimensional data:

- *Nearest neighbor interpolation* (method = 'nearest'). This method fits a piecewise constant surface through the data values. The value of an interpolated point is the value of the nearest point.
- *Bilinear interpolation* (method = 'linear'). This method fits a bilinear surface through existing data points. The value of an interpolated point is a combination of the values of the four closest points. This method is piecewise bilinear, and is faster and less memory-intensive than bicubic interpolation.
- *Bicubic interpolation* (method = 'cubic'). This method fits a bicubic surface through existing data points. The value of an interpolated point is a combination of the values of the sixteen closest points. This method is piecewise bicubic, and produces a much smoother surface than bilinear interpolation. This can be a key advantage for applications like image processing. Use bicubic interpolation when the interpolated data and its derivative must be continuous.

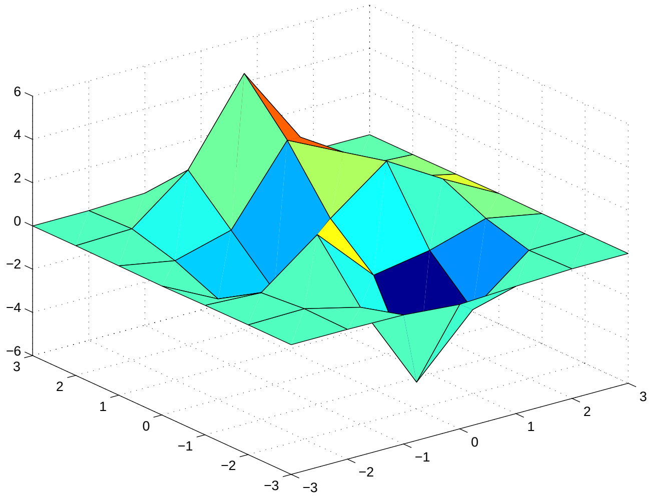
All of these methods require that X and Y be monotonic, that is, either always increasing or always decreasing from point to point. You should prepare these matrices using the meshgrid function, or else be sure that the “pattern” of the points emulates the output of meshgrid. In addition, each method automatically maps the input to an equally spaced domain before interpolating. If X and Y are already equally spaced, you can speed execution time by prepending an asterisk to the method string, for example, '*cubic'.

Comparing Interpolation Methods

This example compares two-dimensional interpolation methods on a 7-by-7 matrix of data.

1 Generate the peaks function at low resolution:

```
[x,y] = meshgrid(–3:1:3);
z = peaks(x,y);
surf(x,y,z)
```



2 Generate a finer mesh for interpolation:

```
[xi,yi] = meshgrid(–3:0.25:3);
```

3 Interpolate using nearest neighbor interpolation:

```
zi1 = interp2(x,y,z,xi,yi,'nearest');
```

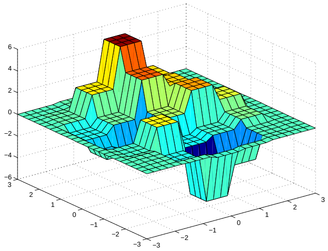
4 Interpolate using bilinear interpolation:

```
zi2 = interp2(x,y,z,xi,yi,'bilinear');
```

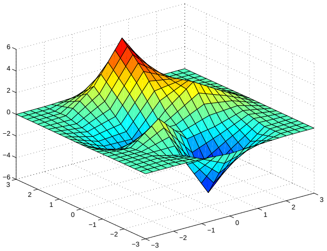
5 Interpolate using bicubic interpolation:

```
zi3 = interp2(x,y,z,xi,yi,'bicubic');
```

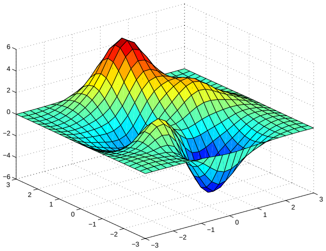
6 Compare the surface plots for the different interpolation methods



`surf(xi,yi,zi1) % nearest`

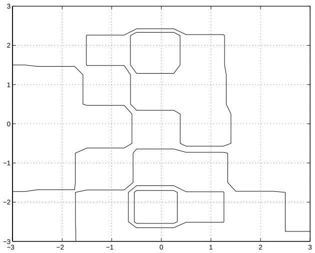


`surf(xi,yi,zi2) % bilinear`

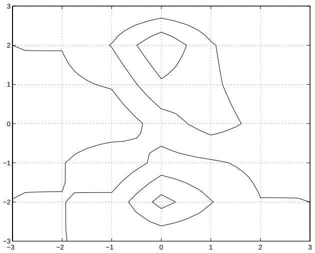


`surf(xi,yi,zi3) % bicubic`

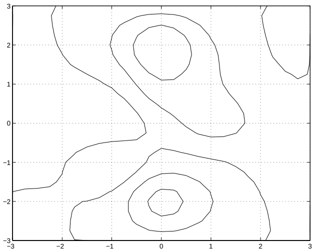
7 Compare the contour plots for the different interpolation methods:



```
contour(xi,yi,zi1)
% nearest
```



```
contour(xi,yi,zi2)
% bilinear
```



```
contour(xi,yi,zi3)
% bicubic
```

Notice that the bicubic method, in particular, produces smoother contours. This is not always the primary concern, however. For some applications, such as medical image processing, a method like nearest neighbor may be preferred because it doesn’t generate any “new” data values.

Interpolation and Multidimensional Arrays

Several interpolation functions operate specifically on multidimensional data:

Function	Description
`interp3`	Three-dimensional data interpolation.
`interpn`	Multidimensional data interpolation.
`ndgrid`	Multidimensional data gridding (`ndfun` directory).

Interpolation of Three-Dimensional Data

The function `interp3` performs three-dimensional interpolation, finding interpolated values between points of a three-dimensional set of samples `V`. You must specify a set of known data points:

- `X`, `Y`, and `Z` matrices specify the points for which values of `V` are given.
- A matrix `V` contains values corresponding to the points in `X`, `Y`, and `Z`.

The most general form for `interp3` is

```
   VI = interp3(X,Y,Z,V,XI,YI,ZI,method)
```

`XI`, `YI`, and `ZI` are the points at which `interp3` interpolates values of `V`. For out-of-range values, `interp3` returns `NaN`.

There are three different interpolation methods for three-dimensional data:

- *Nearest neighbor interpolation* (`method = 'nearest'`). This method chooses the value of the nearest point.
- *Trilinear interpolation* (`method = 'linear'`). This method uses piecewise linear interpolation based on the values of the nearest eight points.
- *Tricubic interpolation* (`method = 'cubic'`). This method uses piecewise cubic interpolation based on the values of the nearest sixty-four points.

All of these methods require that `X`, `Y`, and `Z` be *monotonic*, that is, either always increasing or always decreasing in a particular direction. In addition, you should prepare these matrices using the `meshgrid` function, or else be sure that the “pattern” of the points emulates the output of `meshgrid`.

Each method automatically maps the input to an equally spaced domain before interpolating. If x is already equally spaced, you can speed execution time by prepending an asterisk to the method string, for example, '*cubic'.

Interpolation of Higher-Dimensional Data

The function interpn performs multidimensional interpolation, finding interpolated values between points of a multidimensional set of samples V. The most general form for interpn is

```
VI = interpn(X1,X2,X3...,V,Y1,Y2,Y3,...,method)
```

1, 2, 3, ... are matrices that specify the points for which values of V are given. V is a matrix that contains the values corresponding to these points. 1, 2, 3, ... are the points for which interpn returns interpolated values of V. For out-of-range values, interpn returns NaN.

Y1, Y2, Y3, ... must be either arrays of the same size, or vectors. If they are vectors of different sizes, interpn passes them to ndgrid and then uses the resulting arrays.

There are three different interpolation methods for multidimensional data:

- *Nearest neighbor interpolation* (method = 'nearest'). This method chooses the value of the nearest point.
- *Linear interpolation* (method = 'linear'). This method uses piecewise linear interpolation based on the values of the nearest two points in each dimension.
- *Cubic interpolation* (method = 'cubic'). This method uses piecewise cubic interpolation based on the values of the nearest four points in each dimension.

All of these methods require that X1, X2,X3 be monotonic. In addition, you should prepare these matrices using the ndgrid function, or else be sure that the “pattern” of the points emulates the output of ndgrid.

Each method automatically maps the input to an equally spaced domain before interpolating. If X is already equally spaced, you can speed execution time by prepending an asterisk to the method string; for example, '*cubic'.

Multidimensional Data Gridding

The ndgrid function generates arrays of data for multidimensional function evaluation and interpolation. ndgrid transforms the domain specified by a series of input vectors into a series of output arrays. The i'th dimension of these output arrays are copies of the elements of input vector x_i.

The syntax for ndgrid is

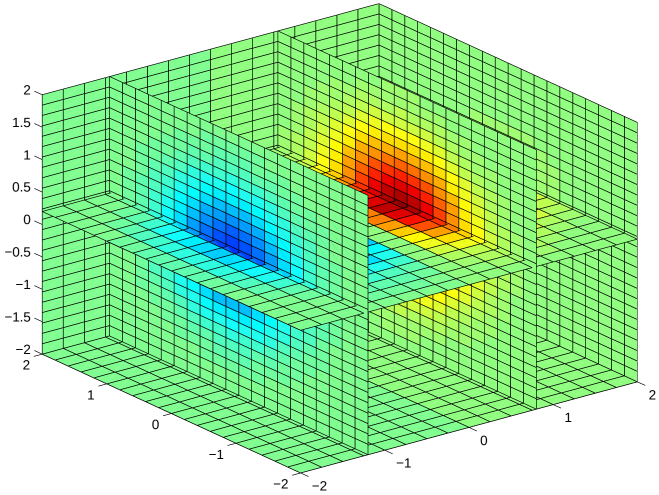
```
[X1,X2,X3,...] = ndgrid(x1,x2,x3,...)
```

For example, assume that you want to evaluate a function of three variables over a given range. Consider the function

$$z = x_2 e^{(-x_1^2 - x_2^2 - x_3^2)}$$

for $-2\pi \le x_1 \le 0$, $2\pi \le x_2 \le 4\pi$, and $0 \le x_3 \le 2\pi$. To evaluate and plot this function:

```
x1 = -2:0.2:2;
x2 = -2:0.25:2;
x3 = -2:0.16:2;
[X1,X2,X3] = ndgrid(x1,x2,x3);
z = X2.*exp(-X1.^2 -X2.^2 -X3.^2);
slice(X2,X1,X3,z,[-1.2.8 2],2,[-2 0.2])
```



Triangulation and Interpolation of Scattered Data

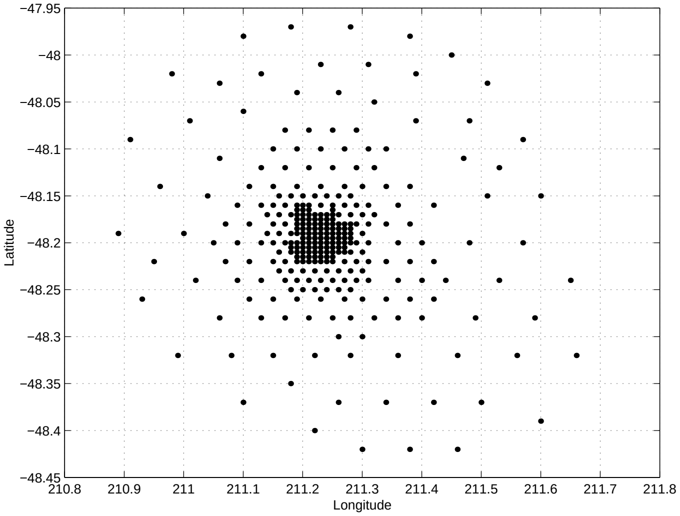
MATLAB provides routines that aid in the analysis of closest-point problems and geometric analysis:

Function	Description
`convhull`	Convex hull.
`delaunay`	Delaunay triangulation.
`dsearch`	Search Delaunay triangulation for nearest point.
`inpolygon`	True for points inside polygonal region.
`polyarea`	Area of polygon.
`rectint`	Area of intersection for two or more rectangles.
`tsearch`	Closest triangle search.
`voronoi`	Voronoi diagram.

Delaunay Triangulation

The delaunay function returns a set of triangles such that no data points are contained in any triangle's circumcircle. To try delaunay, load the seamount data set supplied with MATLAB and view the data as a simple scatter plot.

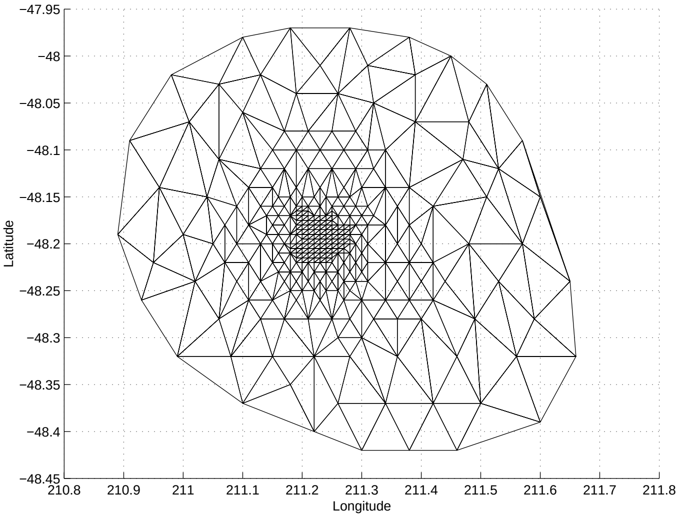
```
load seamount
plot(x,y,'.','markersize',12)
xlabel('Longitude'), ylabel('Latitude')
grid on
```



Note For information on seamount, see Parker, R. L., L. Shure, & J. Hildebrand, "The Application of Inverse Theory to Seamount Magnetism." *Reviews of Geophysics*. Vol 25, 1987: pp 17-40.

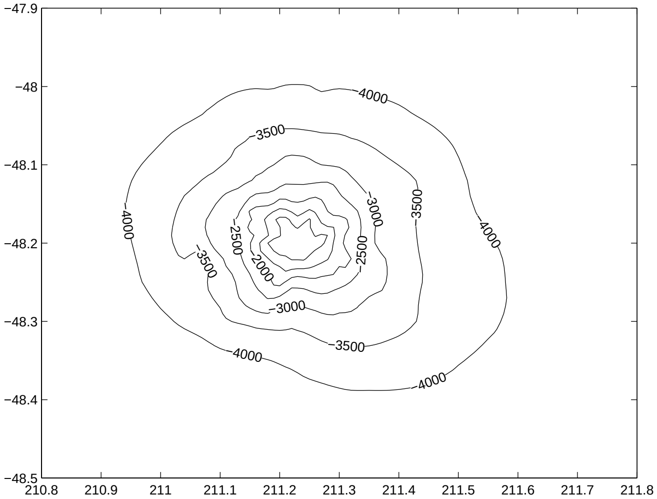
Apply Delaunay triangulation and overplot the resulting triangles on the scatter plot:

```
tri = delaunay(x,y);
hold on, trimesh(tri,x,y,z), hold off
hidden off; grid on
xlabel('Longitude'); ylabel('Latitude')
```



Here’s a contour plot:

```
[xi,yi] = meshgrid(210.8:.01:211.8,–48.5:.01:–47.9);
zi = griddata(x,y,z,xi,yi,'cubic');
[c,h] = contour(xi,yi,zi,'c–'); clabel(c,h)
```



The arguments for `meshgrid` encompass the largest and smallest `x` and `y` values in the original `seamount` data. To obtain these values, use

```
min(min(x))
max(max(x))
```

and

```
min(min(y))
max(max(y))
```

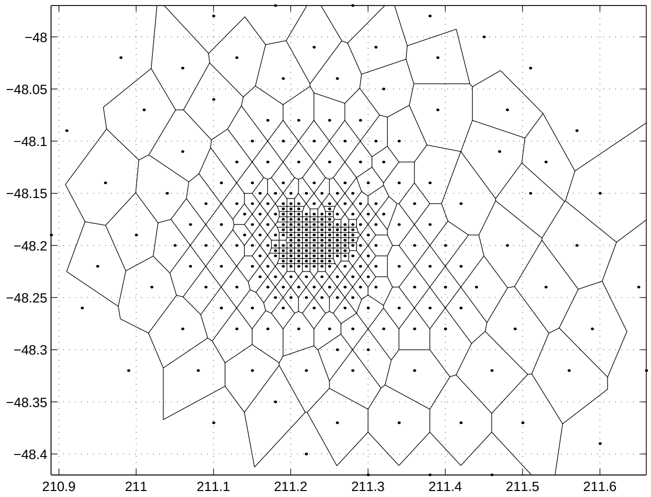
Closest-Point Searches. You can search through the Delaunay triangulation data with two functions:

- `dsearch` finds the point closest to a point you specify.
- `tsearch`, given a point `(xi,yi)`, returns an index into the `delaunay` output that specifies the enclosing triangle for the point.

Voronoi Diagrams

Vornoi diagrams are a closest-point plotting technique related to Delaunay triangulation. The Voronoi diagram for the `seamount` data is

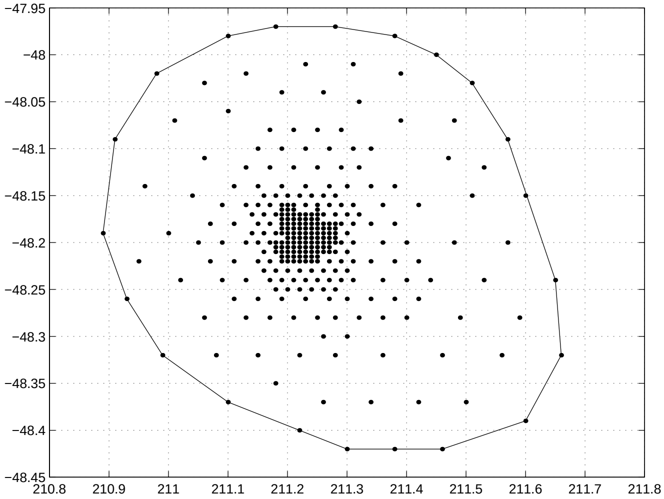
```
load seamount
voronoi(x,y)
grid on
```



Convex Hulls

The convhull function returns the indices of the points in a data set that comprise the convex hull for the set. For example, to view the convex hull for the seamount data:

```
load seamount
plot(x,y,'.','markersize',10)
k = convhull(x,y);
hold on, plot(x(k),y(k)), hold off
grid on
```



Data Analysis and Statistics

This chapter introduces MATLAB's data analysis capabilities. It discusses how to organize arrays for data analysis, how to use simple descriptive statistics functions, and how to perform data pre-processing tasks in MATLAB. It also discusses other data analysis topics, including regression, curve fitting, data filtering, and fast Fourier transforms (FFTs).

The data analysis and statistics functions are in the directory `datafun` in the MATLAB Toolbox. Use online help to get a complete list of functions.

A number of related toolboxes provide advanced functionality for specialized data analysis applications.

Toolbox	**Data Analysis Application**
Optimization	Nonlinear curve fitting and regression.
Signal Processing	Signal processing, filtering, and frequency analysis.
Spline	Curve fitting and regression.
Statistics	Advanced statistical analysis, nonlinear curve fitting, and regression.
System Identification	Parametric / ARMA modeling.
Wavelet	Wavelet analysis.

Column-Oriented Data Sets

Univariate statistical data is typically stored in individual vectors. The vectors can be either 1-by-*n* or *n*-by-1. For multivariate data, a matrix is the natural representation but there are, in principle, two possibilities for orientation. By MATLAB convention, however, the different variables are put into columns, allowing observations to vary down through the rows. Therefore, a data set consisting of twenty four samples of three variables is stored in a matrix of size 24-by-3.

Consider a sample data set comprising vehicle traffic count observations at three locations over a twenty-four hour period.

Time	Location 1	Location 2	Location 3
01h00	11	11	9
02h00	7	13	11
03h00	14	17	20
04h00	11	13	9
05h00	43	51	69
06h00	38	46	76
07h00	61	132	186
08h00	75	135	180
09h00	38	88	115
10h00	28	36	55
11h00	12	12	14
12h00	18	27	30
13h00	18	19	29
14h00	17	15	18
15h00	19	36	48

Time	Location 1	Location 2	Location 3
16h00	32	47	10
17h00	42	65	92
18h00	57	66	151
19h00	44	55	90
20h00	114	145	257
21h00	35	58	68
22h00	11	12	15
23h00	13	9	15
24h00	10	9	7

The raw data is stored in the file, count.dat.

```
     11    11     9
      7    13    11
     14    17    20
     11    13     9
     43    51    69
     38    46    76
     61   132   186
     75   135   180
     38    88   115
     28    36    55
     12    12    14
     18    27    30
     18    19    29
     17    15    18
     19    36    48
     32    47    10
     42    65    92
     57    66   151
     44    55    90
    114   145   257
     35    58    68
     11    12    15
     13     9    15
     10     9     7
```

Use the load command to import the data.

```
load count.dat
```

This creates a matrix count in the workspace.

For this example, there are 24 observations of three variables. This is confirmed by

```
[n,p] = size(count)
n =
    24
p =
     3
```

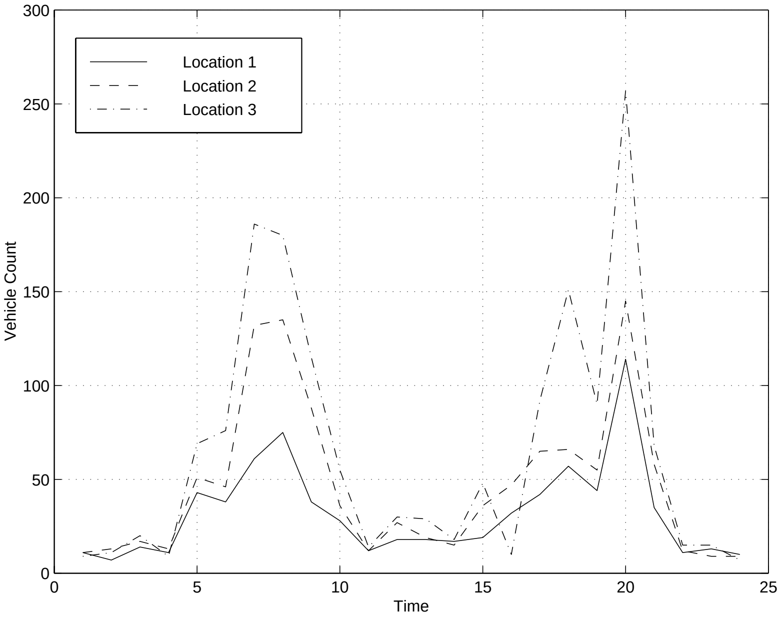
Create a time vector, t, of integers from 1 to n.

```
t = 1:n;
```

Now plot the counts versus time and annotate the plot.

```
set(0,'defaultaxeslinestyleorder’,’-|--|-.’)
set(0,'defaultaxescolororder’,[0 0 0])
plot(t,count), legend('Location 1','Location 2','Location 3',0)
xlabel('Time'), ylabel('Vehicle Count'), grid on
```

The plot shows the vehicle counts at three locations over a 24-hour period.



Basic Data Analysis Functions

A collection of functions provides basic column-oriented data analysis capabilities.

Function	Description
`cumprod`	Cumulative product of elements.
`cumsum`	Cumulative sum of elements.
`cumtrapz`	Cumulative trapezoidal numerical integration.
`diff`	Difference function and approximate derivative.
`max`	Largest component.
`mean`	Average or mean value.
`median`	Median value.
`min`	Smallest component.
`prod`	Product of elements.
`sort`	Sort in ascending order.
`sortrows`	Sort rows in ascending order.
`std`	Standard deviation.
`sum`	Sum of elements.
`trapz`	Trapezoidal numerical integration.

For vector input arguments to these functions, it does not matter whether the vectors are oriented in row or column direction. For array arguments, however, the functions operate column by column on the data in the array. This means, for example, that if you apply max to an array, the result is a row vector containing the maximum values over each column.

Note You can add more functions to this list using M-files, but when doing so, you must exercise care to handle the row-vector case. If you are writing your own column-oriented M-files, check other M-files; for example, mean.m and diff.m.

Continuing with the vehicle traffic count example, the statements

```
mx = max(count)
mu = mean(count)
sigma = std(count)
```

result in

```
mx =
          114          145          257

mu =
      32.0000      46.5417      65.5833

sigma =
      25.3703      41.4057      68.0281
```

To locate the index at which the minimum or maximum occurs, a second output parameter can be specified. For example,

```
[mx,indx] = min(count)

mx =
      7     9     7

indx =
      2    23    24
```

shows that the lowest vehicle count is recorded at 02h00 for the first observation point (column one) and at 23h00 and 24h00 for the other observation points.

You can subtract the mean from each column of the data using an outer product involving a vector of n ones.

```
[n,p] = size(count)
e = ones(n,1)
x = count – e*mu
```

Rearranging the data may help you evaluate a vector function over an entire data set. For example, to find the smallest value in the entire data set, use:

```
min(count(:))
```

which produces:

```
ans =
      7
```

The syntax count(:)) rearranges the 24-by-3 matrix into a 72-by-1 column vector.

Covariance and Correlation Coefficients

MATLAB's statistical capabilities include two functions for the computation of correlation coefficients and covariance.

Function	Description
cov	Variance of vector – measure of spread or dispersion of sample variable. Covariance of matrix – measure of strength of linear relationships between variables.
corrcoef	Correlation coefficient – normalized measure of linear relationship strength between variables.

cov returns the variance for a vector of data. The variance of the data in the first column of count is

```
cov(count(:,1))

ans =
     643.6522
```

For an array of data, `cov` calculates the covariance matrix. The variance values for the array columns are arranged along the diagonal of the covariance matrix. The remaining entries reflect the covariance between the columns of the original array. For an *m*-by-*n* matrix, the covariance matrix has size *n*-by-*n*. For example, the covariance matrix for `count`, `cov(count)`, is arranged as.

$$\begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \sigma_{13}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \sigma_{23}^2 \\ \sigma_{21}^2 & \sigma_{32}^2 & \sigma_{33}^2 \end{bmatrix}$$

$$\sigma_{ij}^2 = \sigma_{ji}^2$$

`corrcoef` produces a matrix of correlation coefficients for an array of data where each row is an observation and each column is a variable. The *correlation coefficient* is a normalized measure of the strength of the linear relationship between two variables. Uncorrelated data results in a correlation coefficient of 0; equivalent data sets have a correlation coefficient of 1.

For an *m*-by-*n* matrix, the correlation coefficient matrix has size *n*-by-*n*. The arrangement of the elements in the correlation coefficient matrix corresponds to the location of the elements in the covariance matrix described above.

For our traffic count example

```
corrcoef(count)
```

results in

```
ans =
    1.0000    0.9331    0.9599
    0.9331    1.0000    0.9553
    0.9599    0.9553    1.0000
```

Clearly there is a strong linear correlation between the three traffic counts observed at the three locations, as the results are close to 1.

Finite Differences

MATLAB provides three functions for finite difference calculations.

Function	Description
`diff`	Difference between successive elements of a vector. Numerical partial derivatives of a vector.
`gradient`	Numerical partial derivatives a matrix.
`del2`	Discrete Laplacian of a matrix.

The `diff` function computes the difference between successive elements in a numeric vector. That is, `diff(X)` is `[X(2)-X(1)   X(3)-X(2) ... X(n)-X(n-1)]`. So, for a vector `A`,

```
A = [9 -2 3 0 1 5 4];
diff(A)

ans =
   -11     5    -3     1     4    -1
```

Besides computing the first difference, `diff` is useful for determining certain characteristics of vectors. For example, you can use `diff` to determine if a vector is monotonic (elements are always either increasing or decreasing), or if a vector has equally spaced elements. This table describes a few different ways to use `diff` with a vector `x`.

`diff(x)==0`	Tests for repeated elements.
`all(diff(x)>0)`	Tests for monotonicity.
`all(diff(diff(x))==0)`	Tests for equally spaced vector elements.

Data Pre-Processing

Missing Values

The special value, NaN, stands for Not-a-Number in MATLAB. IEEE floating-point arithmetic convention specifies NaN as the result of undefined expressions such as 0/0.

The correct handling of missing data is a difficult problem and often varies in different situations. For data analysis purposes, it is often convenient to use NaNs to represent missing values or data that are *not available*.

MATLAB treats NaNs in a uniform and rigorous way; they propagate naturally through to the final result in any calculation. Any mathematical calculation involving NaNs will produce NaNs in the results.

For example, consider a matrix containing the 3-by-3 magic square with its center element set to NaN.

```
a = magic(3); a(2,2) = NaN

a =
     8     1     6
     3   NaN     7
     4     9     2
```

Compute a sum for each column in the matrix.

```
sum(a)

ans =
    15   NaN    15
```

Any mathematical calculation involving NaNs propagates NaNs through to the final result as appropriate.

You should remove `NaN`s from the data before performing statistical computations. Here are some ways to remove `NaN`s from data.

`i = find(˜isnan(x));`
`x = x(i)`
	Find indices of elements in vector that are not `NaN`s, then keep only the non-`NaN` elements.

`x = x(find(˜isnan(x)))`	Remove `NaN`s from vector.
`x = x(˜isnan(x));`	Remove `NaN`s from vector (faster).
`x(isnan(x)) = [];`	Remove `NaN`s from vector.
`X(any(isnan(X)’),:) = [];`	Remove any rows of matrix `X` containing `NaN`s.

Note You must use the special function `isnan` to find `NaN`s because, by IEEE arithmetic convention, the logical comparison, `NaN == NaN` always produces `0`. You *cannot* use `x(x==NaN) = []` to remove `NaN`s from your data.

If you frequently need to remove `NaN`s, write a short M-file function.

```
function X = excise(X)
X(any(isnan(X)’),:) = [];
```

Now, typing

```
X = excise(X);
```

accomplishes the same thing.

Removing Outliers

You can remove outliers or misplaced data points from a data set in much the same manner as `NaN`s. For the vehicle traffic count data, the mean and standard deviations of each column of the data are

```
mu = mean(count);
sigma = std(count);
```

The number of rows with outliers greater than three standard deviations is obtained with:

```
[n,p] = size(count)
outliers = abs(count – mu(ones(n, 1),:)) > 3*sigma(ones(n, 1),:);
nout = sum(outliers)
nout =
       1    0    0
```

There is one outlier in the first column. Remove this entire observation with

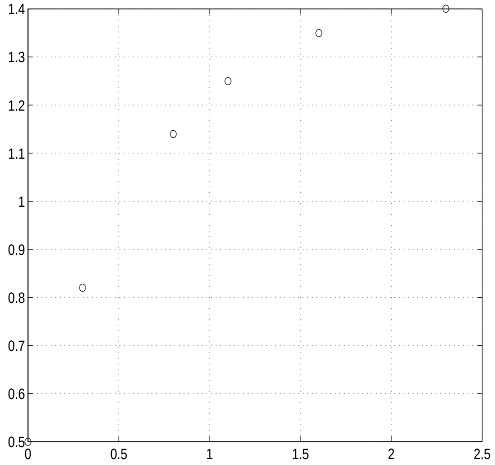
```
count(any(outliers'),:) = [];
```

Regression and Curve Fitting

It is often useful to find functions that describe the relationship between some variables you have observed. Identification of the coefficients of the function often leads to the formulation of an overdetermined system of simultaneous linear equations. These coefficients can be efficiently found using the MATLAB backslash operator.

Suppose you measure a quantity *y* at several values of time *t*.

```
t = [0 .3 .8 1.1 1.6 2.3]';
y = [0.5 0.82 1.14 1.25 1.35 1.40]';
plot(t,y,'o'), grid on
```



Polynomial Regression

Based on the plot, it is possible that the data may be modeled by a polynomial function

$$y = a_0 + a_1 t + a_2 t^2$$

The unknown coefficients a_0, a_1, and a_2, can be computed by doing a *least squares fit*, which minimizes the sum of the squares of the deviations of the data from the model. There are six equations in three unknowns,

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \\ 1 & t_4 & t_4^2 \\ 1 & t_5 & t_5^2 \\ 1 & t_6 & t_6^2 \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

represented by the 6-by-3 matrix

```
X = [ones(size(t))  t  t.^2]

X =
    1.0000         0         0
    1.0000    0.3000    0.0900
    1.0000    0.8000    0.6400
    1.0000    1.1000    1.2100
    1.0000    1.6000    2.5600
    1.0000    2.3000    5.2900
```

The solution is found with the backslash operator.

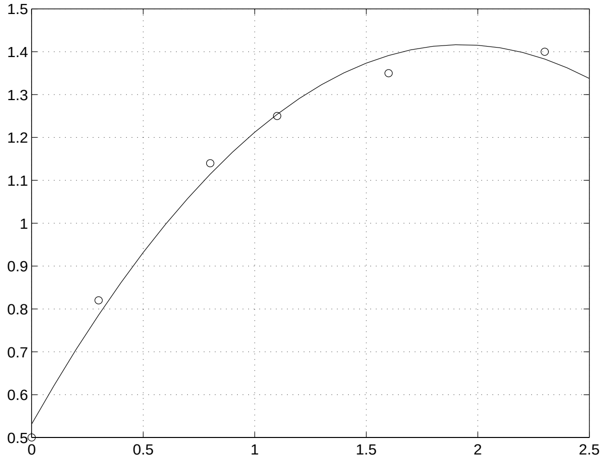
```
a = X\y

a =
    0.5318
    0.9191
  – 0.2387
```

The second order polynomial model of the data is therefore

$$y = 0.5318 + 0.919""1t - 0.2387""t^2$$

Now evaluate the model at regularly spaced points and overlay the original data in a plot.

```
T = (0:0.1:2.5)’;
Y = [ones(size(T))  T  T.^2]*a;
plot(T,Y,'-',t,y,'o',), grid on
```



Clearly this fit does not perfectly approximate the data. We could either increase the order of the polynomial fit, or explore some other functional form to get a better approximation.

Linear-in-the-Parameters Regression

Instead of a polynomial function, we could try using a function that is linear-in-the-parameters. In this case, consider the exponential function

$$y = a_0 + a_1 e^{-t} + a_2 t e^{-t}$$

The unknown coefficients a_0, a_1, and a_2, are computed by performing a *least squares fit*. Construct and solve the set of simultaneous equations by forming the regression matrix, X, and solving for the coefficients using the backslash operator.

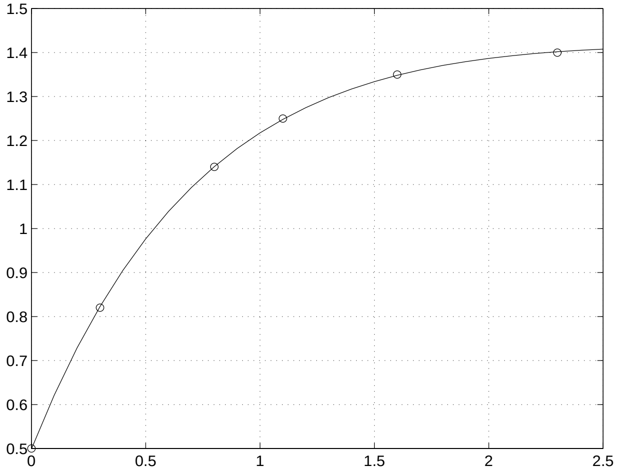
```
X = [ones(size(t))  exp(- t)  t.*exp(- t)];
a = X\y

a =
    1.3974
  - 0.8988
    0.4097
```

The fitted model of the data is therefore

$$y = 1.3974 - 0.8988 \; e^{-t} + 0.4097 \; te^{-t}$$

Now evaluate the model at regularly spaced points and overlay the original data in a plot.

```
T = (0:0.1:2.5)';
Y = [ones(size(T))  exp(- T)  T.exp(- T)]*a;
plot(T,Y,'-',t,y,'o'), grid on
```



This is a much better fit than the second order polynomial function.

Multiple Regression

If y is a function of more than one independent variable, the matrix equations that express the relationships among the variables can be expanded to accommodate the additional data.

Suppose we measure a quantity y for several values of parameters x_1 and x_2. The observations are entered as

```
x1 = [.2 .5 .6 .8 1.0 1.1]';
x2 = [.1 .3 .4 .9 1.1 1.4]';
y  = [.17 .26 .28 .23 .27 .24]';
```

A multivariate model of the data is

$$y = a_0 + a_1x_1 + a_2x_2$$

Multiple regression solves for unknown coefficients a_0, a_1, and a_2, by performing a *least squares fit*. Construct and solve the set of simultaneous equations by forming the regression matrix, X, and solving for the coefficients using the backslash operator.

```
X = [ones(size(x1))  x1  x2];
a = X\y

a =
    0.1018
    0.4844
   –0.2847
```

The least-squares fit model of the data is therefore

$$y = 0.1018 + 0.4844\ x_1 - 0.2847\ x_2$$

To validate the model, find the maximum of the absolute value of the deviation of the data from the model.

```
Y = X*a;
MaxErr = max(abs(Y – y))

MaxErr =
     0.0038
```

This is sufficiently small to be confident the model reasonably fits the data.

Case Study: Curve Fitting

This section provides an overview of some of MATLAB's basic data analysis capabilities in the form of a case study. The examples that follow work with a collection of census data, using MATLAB functions to experiment with fitting curves to the data.

The file census.mat contains U.S. population data for the years 1790 through 1990. Load it into MATLAB

```
load census
```

Your workspace now contains two new variables, cdate and pop.

- cdate is a column vector containing the years from 1790 to 1990 in increments of 10.
- pop is a column vector with the U.S. population figures that correspond to the years in cdate.

Polynomial Fit

A first try in fitting the census data might be a simple polynomial fit. Two MATLAB functions help with this process.

Function	Description
polyfit	Polynomial curve fit.
polyval	Evaluation of polynomial fit.

MATLAB's polyfit function generates a "best fit" polynomial (in the least squares sense) of a specified order for a given set of data. For a polynomial fit of the fourth-order

```
p = polyfit(cdate,pop,4)
Warning: Matrix is close to singular or badly scaled.
         Results may be inaccurate. RCOND = 5.429790e-20
p =
   1.0e+05 *

    0.0000   -0.0000    0.0000   -0.0126 6.0020
```

The warning arises because the polyfit function uses the cdate values as the basis for a matrix with very large values (it creates a Vandermonde matrix in its calculations – see the polyfit M-file for details). The spread of the cdate values results in scaling problems. One way to deal with this is to normalize the cdate data.

Preprocessing: Normalizing the Data

Normalization is a process of scaling the numbers in a data set to improve the accuracy of the subsequent numeric computations. A way to normalize cdate is to scale it for zero mean and unit standard deviation.

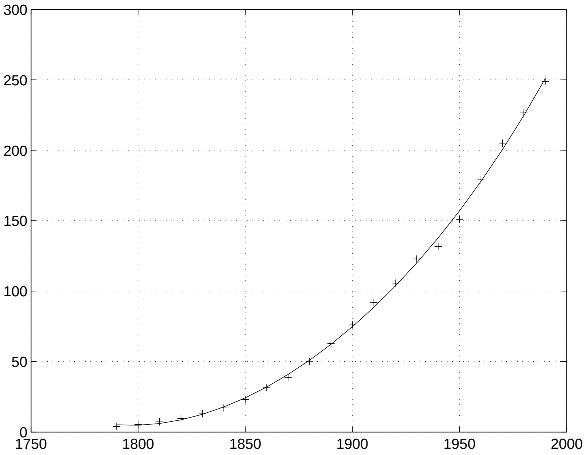
```
sdate = (cdate – mean(cdate))./std(cdate)
```

Now try the fourth-order polynomial model using the normalized data.

```
p = polyfit(sdate,pop,4)

p =
     0.7047    0.9210   23.4706   73.8598   62.2285
```

Evaluate the fitted polynomial at the normalized year values, and plot the fit against the observed data points.

```
pop4 = polyval(p,sdate);
plot(cdate,pop4,'-',cdate,pop,'+'), grid on
```

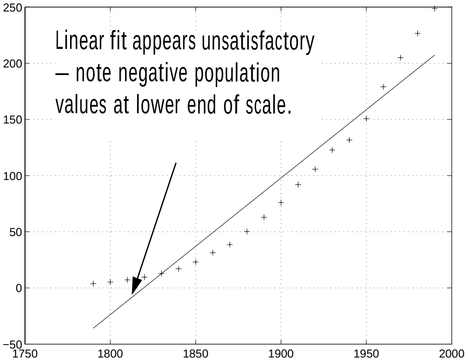
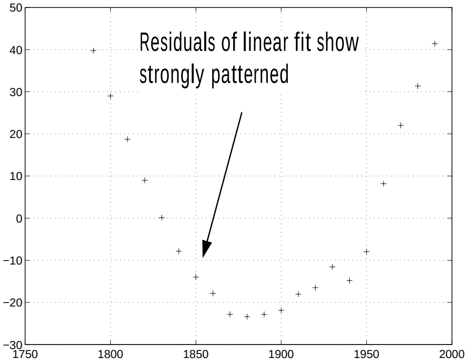


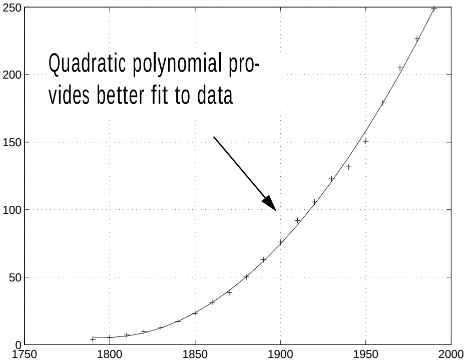
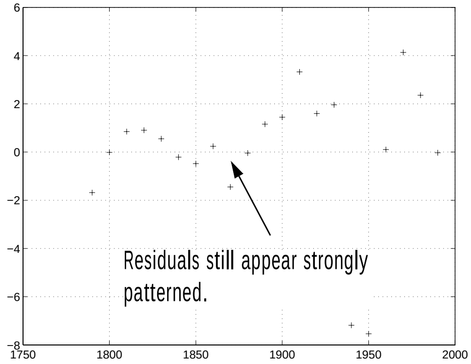
Another way to normalize data is to use some knowledge of the solution and units. For example, with this data set, choosing 1790 to be year zero would also have produced satisfactory results.

Analyzing Residuals

A measure of the “goodness” of fit is the *residual*, the difference between the observed and predicted data. Compare the residuals for the various fits, using normalized `cdate` values.

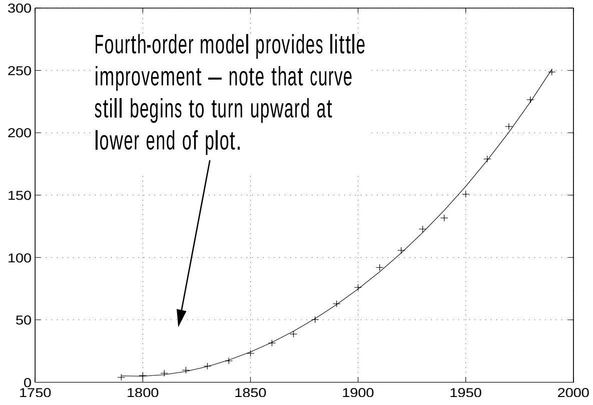
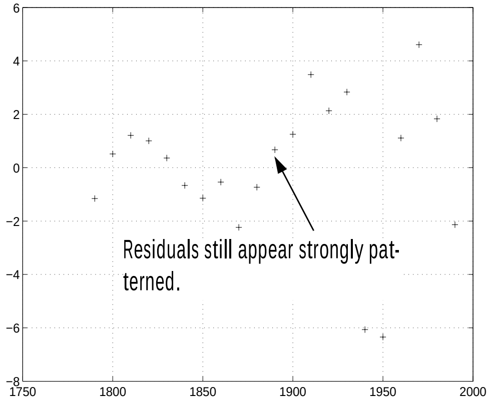
Fit	Residuals

`p1 = polyfit(sdate,pop,1);`
`pop1 = polyval(p1,sdate);`
`plot(cdate,pop1,'-',cdate,pop,'+')`

Linear fit appears unsatisfactory — note negative population values at lower end of scale.
 	
	`res1 = pop – pop1;`
	`figure, plot(cdate,res1,'+')`
	
	Residuals of linear fit show strongly patterned

`p = polyfit(sdate,pop,2);`
`pop2 = polyval(p,sdate);`
`plot(cdate,pop2,'-',cdate,pop,'+')`

Quadratic polynomial provides better fit to data
 	
	`res2 = pop – pop2;`
	`figure, plot(cdate,res2,’+’)`
	
	Residuals still appear strongly patterned.

Fit	Residuals

`p = polyfit(sdate,pop,4);`
`pop4 = polyval(p,sdate);`
`plot(cdate,pop4,'-',cdate,pop,'+')`
	`res4 = pop - pop4;`
	`figure, plot(cdate,res4,'+')`

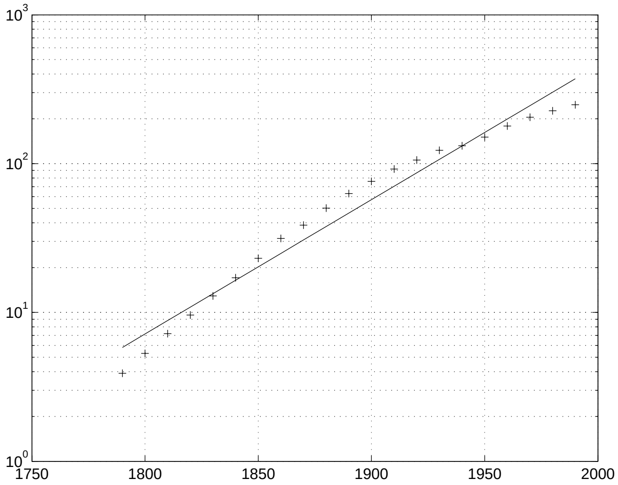
 Fourth-order model provides little improvement – note that curve still begins to turn upward at lower end of plot.	 Residuals still appear strongly patterned.

It's evident from studying the fit plots and residuals that it should be possible to do better than a simple polynomial fit with this data set.

Exponential Fit

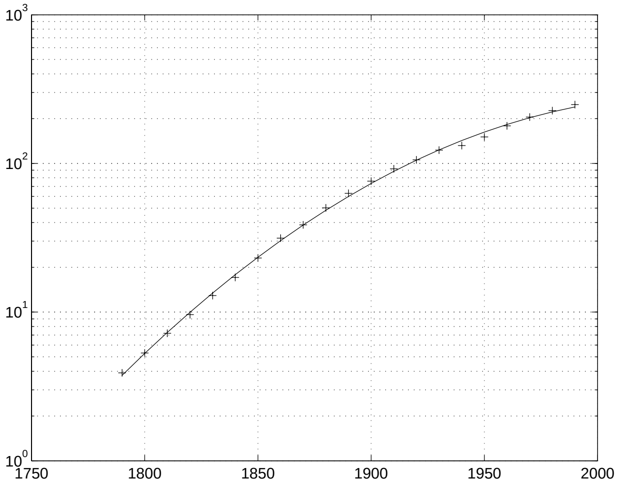
By looking at the population data plots on the previous pages, the population data curve is somewhat exponential in appearance. To take advantage of this, let's try to fit the logarithm of the population values, again working with normalized year values.

```
logp1 = polyfit(sdate,log10(pop),1);
logpred1 = 10.^polyval(logp1,sdate);
semilogy(cdate,logpred1,'-',cdate,pop,'+');
grid on
```



Now try the logarithm analysis with a second-order model.

```
logp2 = polyfit(sdate,log10(pop),2);
logpred2 = 10.^polyval(logp2,sdate);
semilogy(cdate,logpred2,'-',cdate,pop,'+'); grid on
```

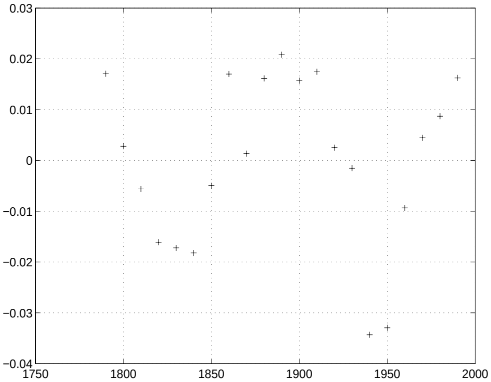


This is a more accurate model. The upper end of the plot appears to taper off, while the polynomial fits in the previous section continue, concave up, to infinity.

Compare the residuals for the second-order logarithmic model.

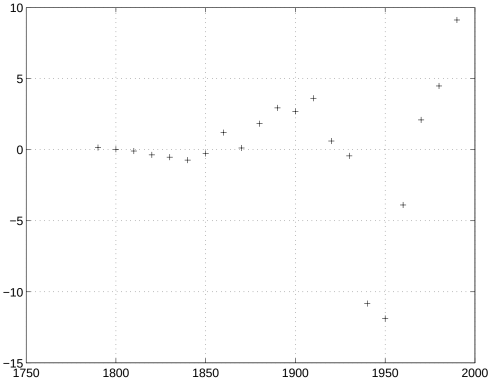
<table>
<tr><th>Residuals in Log Population Scale</th><th>Residuals in Population Scale</th></tr>
<tr><td>

```
logres2 = log10(pop) –
polyval(logp2,sdate);
plot(cdate,logres2,'+')
```



</td><td>

```
r = pop – 10.^(polyval(logp2,sdate));
plot(cdate,r,'+')
```



</td></tr>
</table>

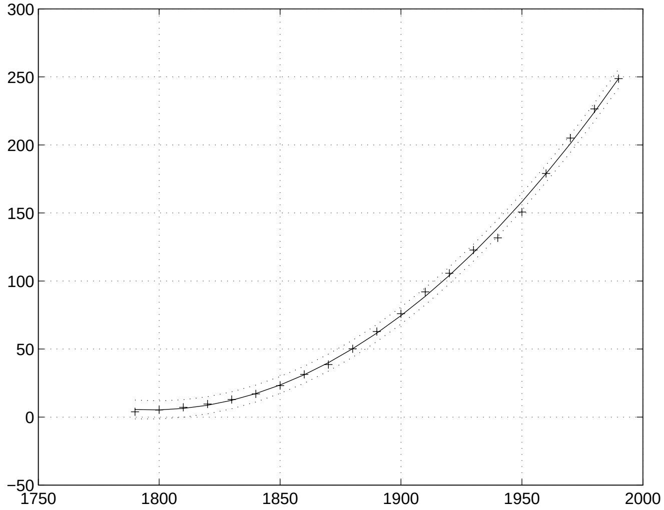
The residuals are more random than for the simple polynomial fit. As might be expected, the residuals tend to get larger in magnitude as the population increases. But overall, the logarithmic model provides a more accurate fit to the population data.

Error Bounds

Error bounds are useful for determining if your data is reasonably modeled by the fit. An optional second output parameter can be obtained from polyfit and passed as an input parameter to polyval in order to obtain the error bounds.

For example, the code below uses polyfit and polyval to produce error bounds for a second-order polynomial model. Year values are normalized. This code uses an interval of ±2Δ, corresponding to a 95% confidence interval.

```
[p2,S2] = polyfit(sdate,pop,2);
[pop2,del2] = polyval(p2,sdate,S2);
plot(cdate,pop,'+',cdate,pop2,'g-',cdate,pop2+2*del2,'r:',...
   cdate,pop2-2*del2,'r:'), grid on
```



Difference Equations and Filtering

MATLAB has functions for working with difference equations and filters. These functions operate primarily on vectors.

Vectors are used to hold sampled-data signals, or sequences, for signal processing and data analysis. For multi-input systems, each row of a matrix corresponds to a sample point with each input appearing as columns of the matrix.

The function

```
y = filter(b, a, x)
```

processes the data in vector `x` with the filter described by vectors `a` and `b`, creating filtered data `y`.

The `filter` command can be thought of as an efficient implementation of the difference equation. The filter structure is the general tapped delay-line filter described by the difference equation below, where *n* is the index of the current sample, *na* is the order of the polynomial described by vector `a` and *nb* is the order of the polynomial described by vector `b`. The output $y(n)$, is a linear combination of current and previous inputs, $x(n)$ $x(n\text{-}1)$ ..., and previous outputs, $y(n\text{-}1)$ $y(n\text{-}2)$ ...

$$a(1)y(n) = b(1)x(n) + b(2)x(n-1) + \ldots + b(nb)x(n-nb+1)$$

$$- a(2)y(n-1) - \ldots - a(na)y(n-na+1)$$

Suppose, for example, we want to smooth our traffic count data with a moving average filter to see the average traffic flow over a 4-hour window. This process is represented by the difference equation

$$y(n) = \frac{1}{4}x(n) + \frac{1}{4}x(n-1) + \frac{1}{4}x(n-2) + \frac{1}{4}x(n-3)$$

The corresponding vectors are

```
a = 1;
b = [1/4 1/4 1/4 1/4];
```

Executing the command

```
load count.dat
```

creates the matrix count in the workspace.

For this example, extract the first column of traffic counts and assign it to the vector x,

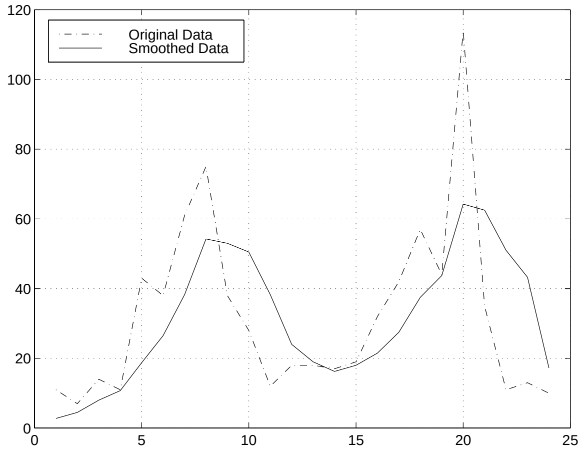
```
x = count(:,1);
```

The four hour moving-average of the data is efficiently calculated with

```
y = filter(b,a,x);
```

Compare the original data and the smoothed data with an overlaid plot of the two curves.

```
t = 1:length(x);
plot(t,x,'-.',t,y,'-'), grid on
legend('Original Data','Smoothed Data',2)
```



The filtered data represented by the solid line is the 4-hour moving average of the observed traffic count data represented by the dashed line.

For practical filtering applications, the Signal Processing Toolbox includes numerous functions for designing and analyzing filters.

Fourier Analysis and the Fast Fourier Transform (FFT)

Fourier analysis is extremely useful for data analysis, as it breaks down a signal into constituent sinusoids of different frequencies. For sampled vector data, Fourier analysis is performed using the discrete Fourier transform (DFT).

The fast Fourier transform (FFT) is an efficient algorithm for computing the DFT of a sequence; it is not a separate transform. It is particularly useful in areas such as signal and image processing, where its uses range from filtering, convolution, and frequency analysis to power spectrum estimation.

MATLAB provides a collection of functions for computing and working with Fourier transforms.

Function	Description
`fft`	Discrete Fourier transform.
`fft2`	Two-dimensional discrete Fourier transform.
`fftn`	N-dimensional discrete Fourier transform.
`ifft`	Inverse discrete Fourier transform.
`ifft2`	Two-dimensional inverse discrete Fourier transform.
`ifftn`	N-dimensional inverse discrete Fourier transform.
`abs`	Magnitude.
`angle`	Phase angle.
`unwrap`	Unwrap phase angle in radians.
`fftshift`	Move zeroth lag to center of spectrum.
`cplxpair`	Sort numbers into complex conjugate pairs.
`nextpow2`	Next higher power of two.

For length N input sequence x, the DFT is a length N vector, X. `fft` and `ifft` implement the relationships

$$X(k) = \sum_{n=1}^{N} x(n)e^{-j2\pi(k-1)\left(\frac{n-1}{N}\right)} \qquad 1 \le k \le N$$

$$x(n) = \frac{1}{N} \sum_{k=1}^{N} X(k)e^{j2\pi(k-1)\left(\frac{n-1}{N}\right)} \qquad 1 \le n \le N$$

Note As the first element of a MATLAB vector has an index 1, the summations in the equations above are from 1 to N. These produce identical results as traditional Fourier equations with summations from 0 to N-1.

If *x(n)* is real, we can rewrite the above equation in terms of a summation of sine and cosine functions with real coefficients

$$x(n) = \frac{1}{N} \sum_{k=1}^{N} a(k)\cos\left(\frac{2\pi(k-1)(n-1)}{N}\right) + b(k)\sin\left(\frac{2\pi(k-1)(n-1)}{N}\right)$$

$$\text{where} \quad a(k) = \text{real}(X(k)),\ b(k) = -\text{imag}(X(k)),\ 1 \le n \le N$$

The FFT of a column vector `x`

```
x = [4 3 7 -9 1 0 0 0]' ;
```

is found with

```
y = fft(x)
```

which results in

```
y =
  6.0000
  11.4853  –2.7574i
  –2.0000 –12.0000i
  –5.4853 +11.2426i
  18.0000
  –5.4853 –11.2426i
  –2.0000 +12.0000i
  11.4853 + 2.7574i
```

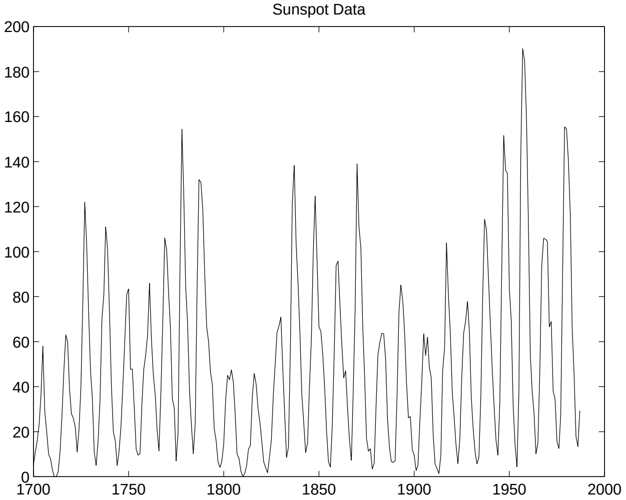
Notice that although the sequence x is real, y is complex. The first component of the transformed data is the constant contribution and the fifth element corresponds to the Nyquist frequency. The last three values of y correspond to negative frequencies and, for the real sequence x, they are complex conjugates of three components in the first half of y.

Suppose, we want to analyze the variations in sunspot activity over the last 300 years. You are probably aware that sunspot activity is cyclical, reaching a maximum about every 11 years. Let's confirm that.

Astronomers have tabulated a quantity called the Wolfer number for almost 300 years. This quantity measures both number and size of sunspots.

Load and plot the sunspot data

```
load sunspot.dat
year = sunspot(:,1);
wolfer = sunspot(:,2);
plot(year,wolfer)
title('Sunspot Data')
```

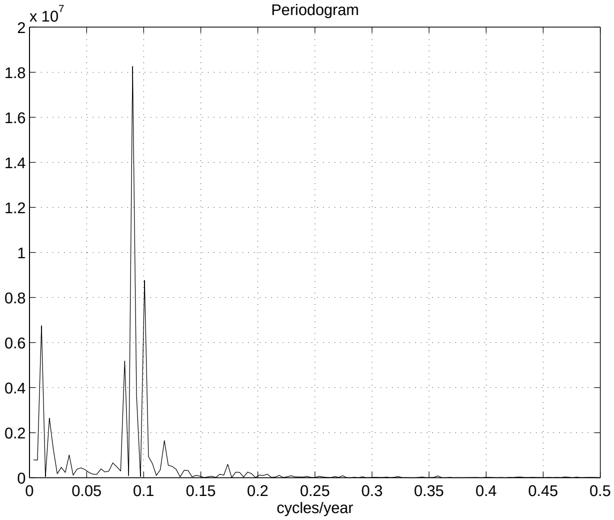


Now take the FFT of the sunspot data

```
Y = fft(wolfer);
```

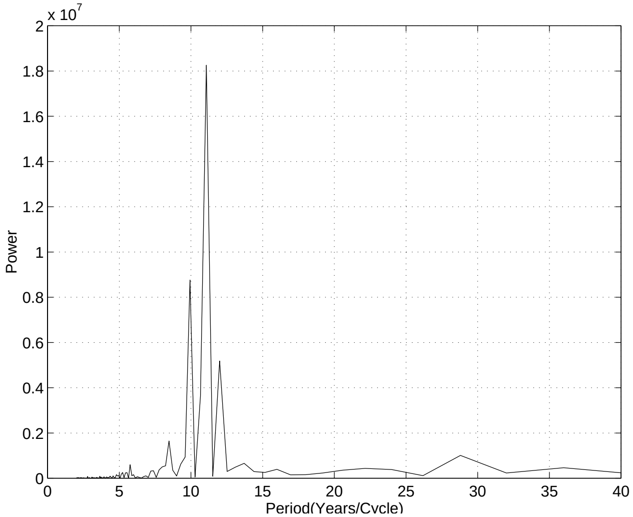
The result of this transform is the complex vector, Y. The magnitude of Y squared is called the power and a plot of power versus frequency is a "periodogram." Remove the first component of Y, which is simply the sum of the data, and plot the results.

```
N = length(Y);
Y(1) = [];
power = abs(Y(1:N/2)).^2;
nyquist = 1/2;
freq = (1:N/2)/(N/2)*nyquist;
plot(freq,power), grid on
xlabel('cycles/year')
title('Periodogram')
```



The scale in cycles/year is somewhat inconvenient. Let's plot in years/cycle and estimate what one cycle is. For convenience, plot the power versus period (where `period = 1./freq`) from 0 to 40 years/cycle.

```
period = 1./freq;
plot(period,power), axis([0 40 0 2e7]), grid on
ylabel('Power')
xlabel('Period(Years/Cycle)')
```



In order to determine the cycle more precisely,

```
[mp index] = max(power);
period(index)

ans =
      11.0769
```

Magnitude and Phase of Transformed Data

Important information about a transformed sequence includes its magnitude and phase. The MATLAB functions abs and angle calculate this information.

To try this, create a time vector t, and use this vector to create a sequence x consisting of two sinusoids at different frequencies.

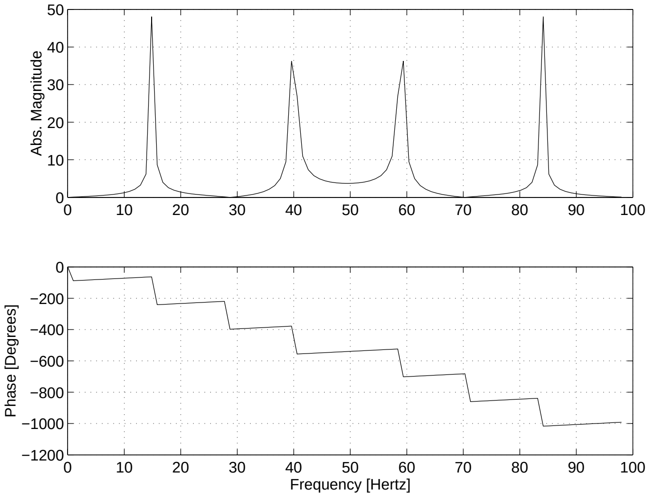
```
t = 0:1/99:1;
x = sin(2*pi*15*t) + sin(2*pi*40*t);
```

Now use the fft function to compute the DFT of the sequence. The code below calculates the magnitude and phase of the transformed sequence. It uses the abs function to obtain the magnitude of the data, the angle function to obtain the phase information, and unwrap to remove phase jumps greater than pi to their 2*pi complement.

```
y = fft(x);
m = abs(y);
p = unwrap(angle(y));
```

Now create a frequency vector for the *x*-axis and plot the magnitude and phase.

```
f = (0:length(y)-1)'*99/length(y);
subplot(2,1,1), plot(f,m),
ylabel('Abs. Magnitude'), grid on
subplot(2,1,2), plot(f,p*180/pi)
ylabel('Phase [Degrees]'), grid on
xlabel('Frequency [Hertz]')
```



The magnitude plot is perfectly symmetrical about the Nyquist frequency of 50 Hertz. The useful information in the signal is found in the range 0 to 50 Hertz.

FFT Length Versus Speed

You can add a second argument to `fft` to specify a number of points `n` for the transform:

```
y = fft(x,n)
```

With this syntax, `fft` pads `x` with zeros if it is shorter than `n`, or truncates it if it is longer than `n`. If you do not specify `n`, `fft` defaults to the length of the input sequence.

The execution time for `fft` depends on the length of the transform.

- For any `n` that is a power of two, `fft` uses the high-speed radix-2 algorithm. This results in the fastest execution time. Additionally, the algorithm for power of two `n` is highly optimized for real `x`, providing a 40% increase in speed over the complex case.
- For any composite number `n` that is not a power of two, `fft` uses a prime factor algorithm. The speed of this algorithm depends on both the size of `n` and the number of prime factors it has. Although 1013 and 1000 are close in

magnitude, `fft` transforms a sequence of length 1000 much more quickly than a sequence of length 1013.

- For a prime number `n`, `fft` cannot use an FFT algorithm. It instead performs the slower, computation-intensive DFT directly.

The inverse FFT function `ifft` also accepts a transform length argument.

For practical application of the FFT, the Signal Processing Toolbox includes numerous functions for spectral analysis.

7

Function Functions

This chapter describes *function functions*, MATLAB functions that work with mathematical functions instead of numeric arrays. These *function functions* include:

- Plotting
- Optimization and zero finding
- Numerical integration (quadrature)

The function functions are located in the MATLAB `funfun` directory.

This table provides a brief description of the functions discussed in this chapter. Related functions are grouped by category.

Category	Function	Description
Plotting	`fplot`	Plot function.
Optimization and zero finding	`fminbnd`	Minimize function of one variable with bound constraints.
	`fminsearch`	Minimize function of several variables.
	`fzero`	Find zero of function of one variable.
Numerical integration	`quad`	Numerically evaluate integral, low order method.
	`quad8`	Numerically evaluate integral, higher order method.
	`dblquad`	Numerically evaluate double integral.

Note For details on another set of function functions, ordinary differential equation solvers, see Chapter 8.

Representing Functions in MATLAB

MATLAB represents mathematical functions by expressing them in M-files. For example, consider the function:

$$f(x)= \frac{1}{(x-0.3)^2+0.01}+\frac{1}{(x-0.9)^2+0.04}-6$$

This function can be used as input to any of the function functions. You can find it in the M-file named `humps.m`:

```
function y = humps(x)
y = 1./((x – 0.3).^2 + 0.01) + 1./((x – 0.9).^2 + 0.04) – 6;
```

A second way to represent a mathematical function at the command line is by creating an inline object from a string expression. For example, you can create an inline object of the `humps` function:

```
f = inline(‘1./((x–0.3).^2 + 0.01) + 1./((x–0.9).^2 + 0.04)–6’);
```

You can then evaluate `f` at 2.0:

```
f(2.0)
ans =
   –4.8552
```

You can also create functions of more than one argument with `inline` by specifying the names of the input arguments along with the string expression. For example, the following function has two input arguments `x` and `y`:

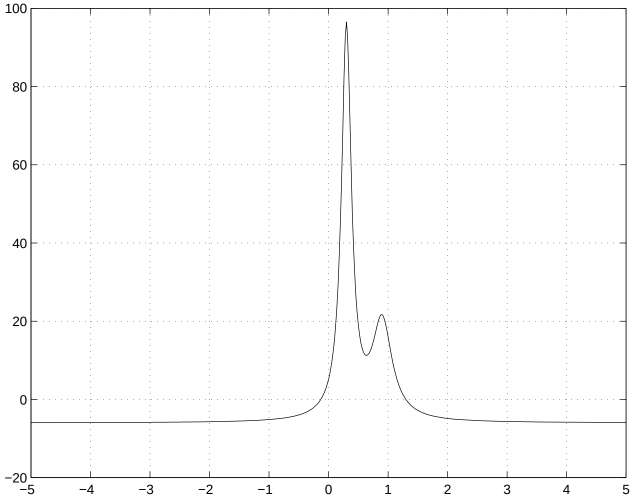
```
f= inline('y*sin(x)+x*cos(y)','x','y')
f(pi,2*pi)
ans =
    3.1416
```

All of the functions described in this chapter are called *function functions* because they accept, as one of their arguments, either the name of an M-file like `humps` or an inline object that defines a mathematical function.

Plotting Mathematical Functions

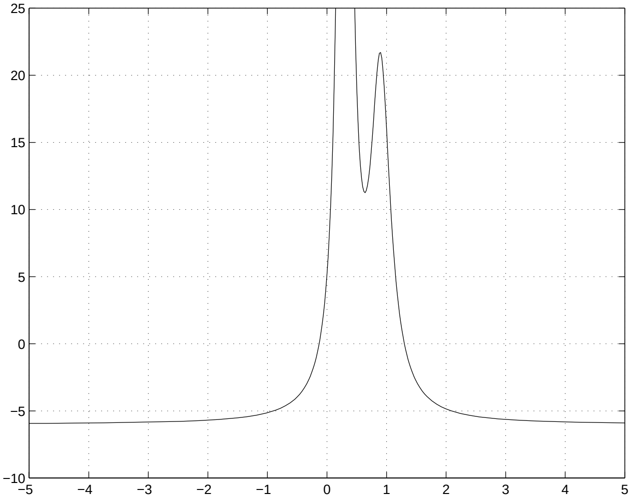
The fplot function plots a mathematical function between a given set of axes limits. You can control the *x*-axis limits only, or both the *x*- and *y*-axis limits. For example, to plot the humps function over the *x*-axis range [–5 5], use

```
fplot('humps',[–5 5])
grid on
```



You can zoom in on the function by selecting *y*-axis limits of `–10` and `25`, using

```
fplot('humps',[–5 5 –10 25])
grid on
```



You can also pass an expression for `fplot` to graph, as in

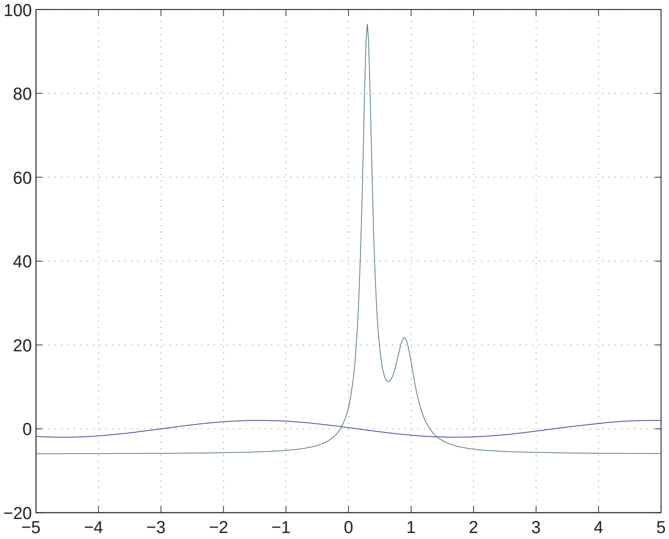
```
fplot('2*sin(x+3)',[–1 1])
```

You can plot more than one function on the same graph with one call to `fplot`. If you use this with a M-file function, then the M-file must take a column vector `x` and return a matrix where each column corresponds to each function, evaluated at each value of `x`.

If you pass an expression of several functions to `fplot`, it also must return a matrix where each column corresponds to each function evaluated at each value of `x`, as in

```
fplot('[2*sin(x+3), humps(x)]',[–1 1])
```

which plots the first and second expressions on the same graph.



Note that the expression

```
[2*sin(x+3), humps(x)]
```

evaluates to a matrix of two columns, one for each function, when x is a column vector.

Minimizing Functions and Finding Zeros

MATLAB provides a number of high-level function functions that perform optimization-related tasks. This section describes:

- Minimizing a function of one variable
- Minimizing a function of several variables
- Setting minimization options
- Finding a zero of a function of one variable

For more sophisticated optimization capabilities, see the Optimization Toolbox.

Minimizing Functions of One Variable

Given a mathematical function of a single variable coded in an M-file, you can use the fminbnd function to find a local minimizer of the function in a given interval. For example, to find a minimum of the humps function in the range (0.3, 1), use

```
x = fminbnd('humps',0.3,1)
```

which returns

```
x =

    0.6370
```

You can ask for a tabular display of output by passing a fourth argument created by the optimset command to fminbnd

```
x = fminbnd('humps',0.3,1,optimset('Display','iter'))
```

which gives the output

```
Func-count      x          f(x)         Procedure
    1       0.567376      12.9098        initial
    2       0.732624      13.7746        golden
    3       0.465248      25.1714        golden
    4       0.644416      11.2693        parabolic
    5         0.6413      11.2583        parabolic
    6       0.637618      11.2529        parabolic
    7       0.636985      11.2528        parabolic
    8       0.637019      11.2528        parabolic
    9       0.637052      11.2528        parabolic

x =
     0.6370
```

This shows the current value of x and the function value at f(x) each time a function evaluation occurs. For fminbnd, one function evaluation corresponds to one iteration of the algorithm. The last column shows what procedure is being used at each iteration, either a golden section search or a parabolic interpolation.

Minimizing Functions of Several Variables

The fminsearch function is similar to fminbnd except that it handles functions of many variables, and you specify a starting vector x_0 rather than a starting interval. fminsearch attempts to return a vector x that is a local minimizer of the mathematical function near this starting vector.

To try fminsearch, create an M-file three_var.m that defines a function of three variables, x, y, and z:

```
function b = three_var(v)
x = v(1);
y = v(2);
z = v(3);
b = x.^2 + 2.5*sin(y) – z^2*x^2*y^2;
```

Now find a minimum for this function using `x = –0.6`, `y = –1.2`, and `z = 0.135` as the starting values:

```
v = [–0.6 –1.2 0.135];
a = fminsearch('three_var',v)

a =
    0.0000   –1.5708    0.1803
```

Setting Minimization Options

You can specify a vector of control options that sets some minimization parameters by calling `fminbnd` with the syntax

```
x = fminbnd(fun,x1,x2,options)
```

or `fminsearch` with the syntax

```
x = fminsearch(fun,x0,options)
```

`options` is a structure used by specialized Optimization Toolbox functions. To set values as needed, use

```
options = optimset('Display','iter');
```

to generate output at each iteration.

`fminbnd` and `fminsearch` use only four of the `options` parameters:

- `options.Display` is a flag that determines if intermediate steps in the minimization appear on the screen. If set to `’iter’`, intermediate steps are displayed; if set to `’off’`, no intermediate solutions are displayed, if set to `final`, displays just the final output.
- `options.TolX` is the termination tolerance for *x*. Its default value is `1.e–4`.
- `options.TolFun` is the termination tolerance for the function value. The default value is `1.e–4`. This parameter is used by `fminsearch` but not `fminbnd`.
- `options.MaxFunEvals` is the maximum number of function evaluations allowed. The default value is `500` for `fminbnd` and `200*length(x0)` for `fminsearch`.

The number of function evaluations, the number of iterations, and the algorithm are returned in the structure `output` when you provide `fminbnd` or `fminsearch` with a fourth output argument, as in

```
[x,fval,exitflag,output] = fminbnd('humps',0.3,1);
```

or

```
[x,fval,exitflag,output] = fminsearch('three_var',v);
```

Finding Zeros of Functions

The `fzero` function attempts to find a zero of one equation with one variable. This function can be called with either a one-element starting point or a two-element vector that designates a starting interval. If you give `fzero` a starting point x_0, `fzero` first searches for an interval around this point where the function changes sign. If the interval is found, then `fzero` returns a value near where the function changes sign. If no such interval is found, `fzero` returns `NaN`. Alternatively, if you know two points where the function value differs in sign, you can specify this starting interval using a two-element vector; `fzero` is guaranteed to narrow down the interval and return a value near a sign change.

Use `fzero` to find a zero of the `humps` function near `-0.2`

```
a = fzero('humps',-0.2)

a =

-0.1316
```

For this starting point, `fzero` searches in the neighborhood of –0.2 until it finds a change of sign between –0.10949 and –0.264. This interval is then narrowed down to –0.1316. You can verify that –0.1316 has a function value very close to zero using

```
humps(a)

ans =
   8.8818e -16
```

Suppose you know two places where the function value of humps differs in sign such as x = 1 and x = –1. You can use

```
humps(1)

ans =

    16

humps(–1)

ans =

   –5.1378
```

Then you can give fzero this interval to start with and fzero then returns a point near where the function changes sign. You can display information as fzero progresses with

```
options = optimset('Display','iter');
a = fzero('humps',[–1 1],options)

 Func-count      x            f(x)          Procedure
    1              –1      –5.13779        initial
    1               1            16        initial
    2       –0.513876      –4.02235        interpolation
    3        0.243062       71.6382        bisection
    4       –0.473635      –3.83767        interpolation
    5       –0.115287      0.414441        bisection
    6       –0.150214     –0.423446        interpolation
    7       –0.132562    –0.0226907        interpolation
    8       –0.131666    –0.0011492        interpolation
    9       –0.131618   1.88371e–07        interpolation
   10       –0.131618   –2.7935e–11        interpolation
   11       –0.131618   8.88178e–16        interpolation
   12       –0.131618  –9.76996e–15        interpolation

a =

   –0.1316
```

The steps of the algorithm include both bisection and interpolation under the `Procedure` column. If the example had started with a scalar starting point instead of an interval, the first steps after the initial function evaluations would have included some search steps while `fzero` searched for an interval containing a sign change.

You can specify a relative error tolerance using `optimset`. In the call above, passing in the empty matrix causes the default relative error tolerance of `eps` to be used.

Tips

Optimization problems may take many iterations to converge. Most optimization problems benefit from good starting guesses. Providing good starting guesses improves the execution efficiency and may help locate the global minimum instead of a local minimum.

Sophisticated problems are best solved by an evolutionary approach whereby a problem with a smaller number of independent variables is solved first. Solutions from lower order problems can generally be used as starting points for higher order problems by using an appropriate mapping.

The use of simpler cost functions and less stringent termination criteria in the early stages of an optimization problem can also reduce computation time. Such an approach often produces superior results by avoiding local minima.

Troubleshooting

Below is a list of typical problems and recommendations for dealing with them.

Problem	Recommendation
The solution found by `fminbnd` or `fminsearch` does not appear to be a global minimum.	There is no guarantee that you have a global minimum unless your problem is continuous and has only one minimum. Starting the optimization from a number of different starting points (or intervals in the case of `fminbnd`) may help to locate the global minimum or verify that there is only one minimum. Use different methods, where possible, to verify results.
Sometimes an optimization problem has values of `x` for which it is impossible to evaluate `f`.	Modify your function to include a penalty function to give a large positive value to `f` when infeasibility is encountered.
The minimization routine appears to enter an infinite loop or returns a solution that is not a minimum (or not a zero in the case of `fzero`).	Your objective function may be returning `Inf`, `NaN`, or complex values. The optimization routines expect only real numbers to be returned. Any other values may cause unexpected results. Insert code into your objective function M-file to verify that only real numbers are returned (use the functions `isreal` and `isfinite`).

Numerical Integration (Quadrature)

The area beneath a section of a function $F(x)$ can be determined by numerically integrating $F(x)$, a process referred to as *quadrature*. The two MATLAB functions for one-dimensional quadrature are:

- quad – Use Adaptive Simpson's rule
- quad8 – Use Adaptive Newton Cotes 8 panel rule

To integrate the function defined by humps.m from 0 to 1, use

```
q = quad('humps',0,1)

q =
    29.8583
```

Both quad and quad8 operate recursively. If either method reaches the maximum number of 10 recursive calls, the method returns a value of Inf indicating possible singularity.

You can include a fourth argument for quad or quad8 that specifies a relative error tolerance for the integration. If this fourth argument is a two-element vector, its first element specifies a relative tolerance and its second an absolute tolerance. If a nonzero fifth argument is passed to quad or quad8, the function evaluations are traced with a point plot of the integrand.

Example: Computing the Length of a Curve

You can use quad or quad8 to compute the length of a curve. Consider the curve parameterized by the equations

$$x(t) = \sin(2t), \qquad y(t) = \cos(t), \qquad z(t) = t$$

where $t \in [0, 3\pi]$.

A three-dimensional plot of this curve is

```
t = 0:0.1:3*pi;
plot3(sin(2*t),cos(t),t)
```

The arc length formula says the length of the curve is the integral of the norm of the derivatives of the parameterized equations

$$\int_0^{3\pi} \sqrt{4\cos(2t)^2 + \sin(t)^2 + 1}\ \ dt$$

The function hcurve computes the integrand

```
function f = hcurve(t)
f = sqrt(4*cos(2*t).^2 + sin(t).^2 + 1);
```

Integrate this function with a call to quad

```
len = quad('hcurve',0,3*pi)
len =
   1.7222e+01
```

The length of this curve is about 17.2.

Example: Double Integration

Consider the numerical solution of

$$\int_{ymin}^{ymax} \int_{xmin}^{xmax} f(x,y)\ \ dx dy$$

For this example $f(x,y) = y\sin(x) + x\cos(y)$. The first step is to build the function to be evaluated. The function must be capable of returning a vector output when given a vector input. You must also consider which variable is in the inner integral, and which goes in the outer integral. In this example, the inner variable is *x* and the outer variable is *y* (the order in the integral is *dxdy*). In this case, the integrand function is

```
function out = integrnd(x,y)
out = y*sin(x) + x*cos(y);
```

To perform the integration, two functions are available in the funfun directory. The first, dblquad, is called directly from the command line. This M-file evaluates the outer loop using quad. At each iteration, quad calls the second helper function that evaluates the inner loop.

To evaluate the double integral, use

```
result = dblquad('integrnd',xmin,xmax,ymin,ymax);
```

The first argument is a string with the name of the integrand function; the second to fifth arguments are

xmin lower limit of inner integral

xmax upper limit of the inner integral

ymin lower limit of outer integral

ymax upper limit of the outer integral

Here is a numeric example that illustrates the use of dblquad.

```
xmin = pi;
xmax = 2*pi;
ymin = 0;
ymax = pi;
result = dblquad('integrnd',xmin,xmax,ymin,ymax)
```

The result is –9.8698.

By default, dblquad calls quad. To integrate the previous example using quad8 (with the default values for the tolerance and trace arguments), use

```
result = dblquad('integrnd',xmin,xmax,ymin,ymax,[],'quad8');
```

Alternatively, any user-defined quadrature function name can be passed to dblquad as long as the quadrature function has the same calling and return arguments as quad.

Ordinary Differential Equations

This chapter describes how to use MATLAB to solve initial value problems of ordinary differential equations (ODEs) and differential algebraic equations (DAEs). It discusses how to represent initial value problems (IVPs) in MATLAB and how to apply MATLAB's ODE solvers to such problems. It explains how to select a solver, and how to specify solver options for efficient, customized execution. This chapter also includes a troubleshooting guide in the *Questions and Answers* section and extensive examples in the *Examples: Applying the ODE Solver* section.

Category	**Function**	**Description**
Ordinary differential equation solvers	`ode45`	Nonstiff differential equations, medium order method.
	`ode23`	Nonstiff differential equations, low order method.
	`ode113`	Nonstiff differential equations, variable order method.
	`ode15s`	Stiff differential equations and DAEs, variable order method.
	`ode23s`	Stiff differential equations, low order method.
	`ode23t`	Moderately stiff differential equations and DAEs, trapezoidal rule.
	`ode23tb`	Stiff differential equations, low order method.
ODE option handling	`odeset`	Create/alter ODE `OPTIONS` structure.
	`odeget`	Get ODE `OPTIONS` parameters.
ODE output functions	`odeplot`	Time series plots.
	`odephas2`	Two-dimensional phase plane plots.
	`odephas3`	Three-dimensional phase plane plots.
	`odeprint`	Print to command window.

Quick Start

1 Write the ordinary differential equation $y^{(n)} = f\ (t, y, y', \ldots., y^{(n-1)})$ as a system of first-order equations by making the substitutions

$$y_1 = y, y_2 = y', \ldots, y_n = y^{(n-1)}$$

Then

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= y_3 \\ &\vdots \\ y_n' &= f(t, y_1, y_2, \ldots, y_n) \end{aligned}$$

is a system of n first-order ODEs. For example, consider the initial value problem

$$y''' - 3y'' - y'y = 0 \qquad y(0) = 0 \qquad y'(0) = 1 \qquad y''(0) = -1$$

Solve the differential equation for its highest derivative, writing y''' in terms of t and its lower derivatives $y''' = 3y'' + y'y$. If you let $y_1 = y, y_2 = y'$, and $y_3 = y''$, then

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= y_3 \\ y_3' &= 3y_3 + y_2 y_1 \end{aligned}$$

is a system of three first-order ODEs with initial conditions

$$y_1(0) = 0$$
$$y_2(0) = 1$$
$$y_3(0) = -1$$

Note that the IVP now has the form $Y' = F(t, Y), Y(0) = Y_0$, where $Y = [y_1; y_2; y_3]$.

2 Code the first-order system in an M-file that accepts two arguments, `t` and `y`, and returns a column vector:

```
function dy = F(t,y)
dy = [y(2); y(3); 3*y(3)+y(2)*y(1)];
```

This ODE file must accept the arguments `t` and `y`, although it does not have to use them. Here, the vector `dy` must be a column vector.

3 Apply a solver function to the problem. The general calling syntax for the ODE solvers is

```
[T,Y] = solver('F',tspan,y0)
```

where *solver* is a solver function like `ode45`. The input arguments are:

`F`	String containing the ODE file name
`tspan`	Vector of time values where `[t0 tfinal]` causes the solver to integrate from `t0` to `tfinal`
`y0`	Column vector of initial conditions at the initial time `t0`

For example, to use the `ode45` solver to find a solution of the sample IVP on the time interval `[0 1]`, the calling sequence is

```
[T,Y] = ode45('F',[0 1],[0; 1; -1])
```

Each row in solution array `Y` corresponds to a time returned in column vector `T`. Also, in the case of the sample IVP, `Y(:,1)` is the solution, `Y(:,2)` is the derivative of the solution, and `Y(:,3)` is the second derivative of the solution.

Representing Problems

This section describes how to represent ordinary differential equations as systems for the MATLAB ODE solvers.

The MATLAB ODE solvers are designed to handle *ordinary differential equations*. These are differential equations containing one or more derivatives of a dependent variable *y* with respect to a single independent variable *t*, usually referred to as *time*. The derivative of *y* with respect to *t* is denoted as y', the second derivative as y'', and so on. Often *y(t)* is a vector, having elements *y1, y2, ... yn*.

ODEs often involve a number of dependent variables, as well as derivatives of order higher than one. To use the MATLAB ODE solvers, you must rewrite such equations as an equivalent system of first-order differential equations in terms of a vector *y* and its first derivative.

$$y' = F(t, y)$$

Once you represent the equation in this way, you can code it as an ODE M-file that a MATLAB ODE solver can use.

Initial Value Problems and Initial Conditions

Generally there are many functions $y(t)$ that satisfy a given ODE, and additional information is necessary to specify the solution of interest. In an *initial value problem*, the solution of interest has a specific *initial condition*, that is, *y* is equal to y_0 at a given initial time t_0. An initial value problem for an ODE is then

$$y' = F(t, y)$$
$$y(t_0) = y_0$$

If the function $F(t, y)$ is sufficiently smooth, this problem has one and only one solution. Generally there is no analytic expression for the solution, so it is necessary to approximate $y(t)$ by numerical means, such as one of the solvers of the MATLAB ODE suite.

Example: The van der Pol Equation

An example of an ODE is the van der Pol equation

$$y_1'' - \mu(1 - y_1^2)y_1' + y_1 = 0$$

where $\mu > 0$ is a scalar parameter.

Rewriting the System

To express this equation as a system of first-order differential equations for MATLAB, introduce a variable y_2 such that $y_1' = y_2$. You can then express this system as

$$y_1' = y_2$$

$$y_2' = \mu(1 - y_1^2)y_2 - y_1$$

Writing the ODE File

The code below shows how to represent the van der Pol system in a MATLAB ODE file, an M-file that describes the system to be solved. An ODE file always accepts at least two arguments, `t` and `y`. This simple two line file assumes a value of `1` for μ. y_1 and y_2 become `y(1)` and `y(2)`, elements in a two-element vector.

```
function dy = vdp1(t,y)
dy = [y(2); (1–y(1)^2)*y(2)–y(1)];
```

Note This ODE file does not actually use the `t` argument in its computations. It is not necessary for it to use the `y` argument either – in some cases, for example, it may just return a constant. The `t` and `y` variables, however, must always appear in the input argument list.

Calling the Solver

Once the ODE system is coded in an ODE file, you can use the MATLAB ODE solvers to solve the system on a given time interval with a particular initial condition vector. For example, to use `ode45` to solve the van der Pol equation on time interval `[0 20]` with an initial value of `2` for `y(1)` and an initial value of `0` for `y(2)`.

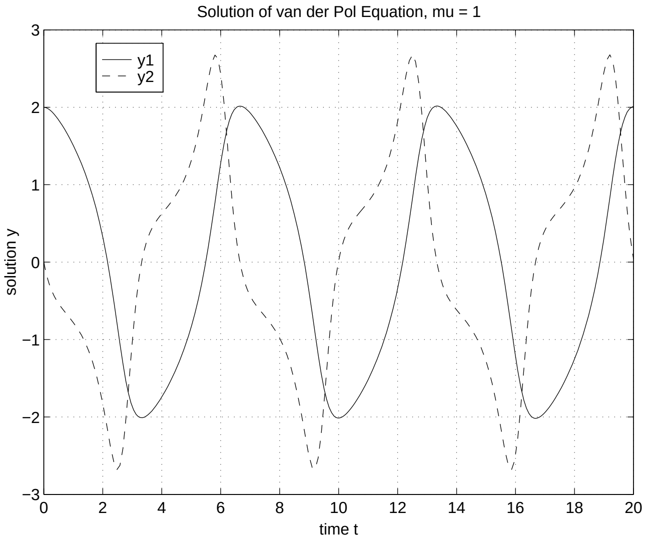
```
[T,Y] = ode45('vdp1',[0 20],[2; 0]);
```

The resulting output [T,Y] is a column vector of time points T and a solution array Y. Each row in solution array Y corresponds to a time returned in column vector T.

Viewing the Results

Use the plot command to view solver output.

```
plot(t,y(:,1),'-',t,y(:,2),'- -')
title('Solution of van der Pol Equation, mu = 1');
xlabel('time t');
ylabel('solution y');
legend('y1','y2')
```



Example: The van der Pol Equation, μ = 1000 (Stiff)

Stiff ODE Problems This section presents a *stiff* problem. For a *stiff* problem, solutions can change on a time scale that is very short compared to the interval of integration, but the solution of interest changes on a much longer time scale. Methods not designed for stiff problems are ineffective on intervals where the solution changes slowly because they use time steps small enough to resolve the fastest possible change.

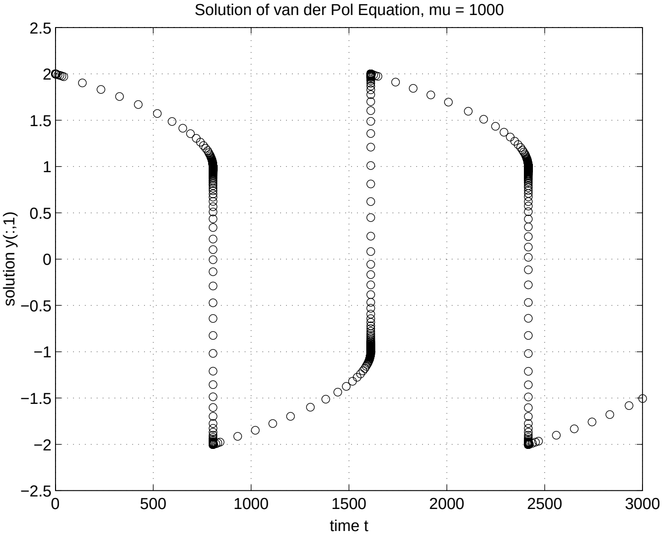
When μ is increased to 1000, the solution to the van der Pol equation changes dramatically and exhibits oscillation on a much longer time scale. Approximating the solution of the initial value problem becomes a more difficult task. Because this particular problem is stiff, a nonstiff solver such as `ode45` is so inefficient that it is impractical. The stiff solver `ode15s` is intended for such problems.

This code shows how to represent the van der Pol system in an ODE file with μ = 1000.

```
function dy = vdp1000(t,y)
dy = [y(2); 1000*(1–y(1)^2)*y(2)–y(1)];
```

Now use the `ode15s` function to solve `vdp1000`. Retain the initial condition vector of `[2; 0]`, but use a time interval of `[0 3000]`. For scaling purposes, plot just the first component of `y(t)`.

```
[t,y] = ode15s('vdp1000',[0 3000],[2; 0]);
plot(t,y(:,1),'o');
title('Solution of van der Pol Equation, mu = 1000');
xlabel('time t');
ylabel('solution y(:,1)');
```


Solution of van der Pol Equation, mu = 1000
solution y(:,1)
time t
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
0
500
1000
1500
2000
2500
3000

ODE Solvers

The MATLAB ODE solver functions implement numerical integration methods. Beginning at the initial time and with initial conditions, they step through the time interval, computing a solution at each time step. If the solution for a time step satisfies the solver's error tolerance criteria, it is a successful step. Otherwise, it is a failed attempt; the solver shrinks the step size and tries again.

This section describes how to represent problems for use with the MATLAB solvers and how to optimize solver performance. You can also use the online help facility to get information on the syntax for any function, as well as information on demo files for these solvers.

Nonstiff Solvers

There are three solvers designed for nonstiff problems:

- `ode45` is based on an explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair. It is a *one-step* solver – in computing $y(t_n)$, it needs only the solution at the immediately preceding time point, $y(t_{n-1})$. In general, `ode45` is the best function to apply as a "first try" for most problems.
- `ode23` is also based on an explicit Runge-Kutta (2,3) pair of Bogacki and Shampine. It may be more efficient than `ode45` at crude tolerances and in the presence of mild stiffness. Like `ode45`, `ode23` is a one-step solver.
- `ode113` is a variable order Adams-Bashforth-Moulton PECE solver. It may be more efficient than `ode45` at stringent tolerances and when the ODE function is particularly expensive to evaluate. `ode113` is a *multistep* solver – it normally needs the solutions at several preceding time points to compute the current solution.

Stiff Solvers

Not all difficult problems are stiff, but all stiff problems are difficult for solvers not specifically designed for them. Stiff solvers can be used exactly like the other solvers. However, you can often significantly improve the efficiency of the stiff solvers by providing them with additional information about the problem. See "Improving Solver Performance" on page 8-17 for details on how to provide this information, and for details on how to change solver parameters such as error tolerances.

There are four solvers designed for stiff (or moderately stiff) problems:

- `ode15s` is a variable-order solver based on the numerical differentiation formulas (NDFs). Optionally it uses the backward differentiation formulas, BDFs, (also known as Gear's method) that are usually less efficient. Like `ode113`, `ode15s` is a multistep solver. If you suspect that a problem is stiff or if `ode45` failed or was very inefficient, try `ode15s`.
- `ode23s` is based on a modified Rosenbrock formula of order 2. Because it is a one-step solver, it may be more efficient than `ode15s` at crude tolerances. It can solve some kinds of stiff problems for which `ode15s` is not effective.
- `ode23t` is an implementation of the trapezoidal rule using a "free" interpolant. Use this solver if the problem is only moderately stiff and you need a solution without numerical damping.
- `ode23tb` is an implementation of TR-BDF2, an implicit Runge-Kutta formula with a first stage that is a trapezoidal rule step and a second stage that is a backward differentiation formula of order two. By construction, the same iteration matrix is used in evaluating both stages. Like `ode23s`, this solver may be more efficient than `ode15s` at crude tolerances.

ODE Solver Basic Syntax

All of the ODE solver functions share a syntax that makes it easy to try any of the different numerical methods if it is not apparent which is the most appropriate. To apply a different method to the same problem, simply change the ODE solver function name. The simplest syntax, common to all the solver functions, is

```
[T,Y] = solver('F',tspan,y0)
```

where *solver* is one of the ODE solver functions listed previously.

The input arguments are:

`'F'` String containing the name of the file that describes the system of ODEs.

`tspan` Vector specifying the interval of integration. For a two-element vector `tspan = [t0 tfinal]`, the solver integrates from `t0` to `tfinal`. For `tspan` vectors with more than two elements, the solver returns solutions at the given time points, as described below. Note that `t0 > tfinal` is allowed.

`y0` Vector of initial conditions for the problem.

The output arguments are:

`T` Column vector of time points

`Y` Solution array. Each row in `Y` corresponds to the solution at a time returned in the corresponding row of `T`.

Obtaining Solutions at Specific Time Points

To obtain solutions at specific time points `t0`, `t1`, ... `tfinal`, specify `tspan` as a vector of the desired times. The time values must be in order, either increasing or decreasing.

Specifying these time points in the `tspan` vector does not affect the internal time steps that the solver uses to traverse the interval from `tspan(1)` to `tspan(end)` and has little effect on the efficiency of computation. All solvers in the MATLAB ODE suite obtain output values by means of continuous extensions of the basic formulas. Although a solver does not necessarily step precisely to a time point specified in `tspan`, the solutions produced at the specified time points are of the same order of accuracy as the solutions computed at the internal time points.

Specifying Solver Options

In addition to the simple syntax, all of the ODE solvers accept a fourth input argument, `options`, which can be used to change the default integration parameters.

```
[t,y] = solver('F',tspan,y0,options)
```

The `options` argument is created with the `odeset` function (see “Creating an Options Structure: The odeset Function” on page 8-20). Any input parameters after the `options` argument are passed to the ODE file every time it is called. For example,

```
[T,Y] = solver('F',tspan,y0,options,p1,p2,...)
```

calls

```
F(t,y,flag,p1,p2,...)
```

Obtaining Statistics About Solver Performance

Use an additional output argument `S` to obtain statistics about the ODE solver’s computations.

```
[T,Y,S] = solver('F',tspan,y0,options,...)
```

`S` is a six-element column vector:

- Element 1 is the number of successful steps.
- Element 2 is the number of failed attempts.
- Element 3 is the number of times the ODE file was called to evaluate $F(t,y)$.
- Element 4 is the number of times that the partial derivatives matrix $\partial F/\partial y$ was formed.
- Element 5 is the number of LU decompositions.
- Element 6 is the number of solutions of linear systems.

The last three elements of the list apply to the stiff solvers only.

The solver automatically displays these statistics if the `Stats` property (see 8-25) is set in the `options` argument.

Creating ODE Files

The van der Pol examples in the previous sections show some simple ODE files. This section provides more detail and describes how to create more advanced ODE files that can accept additional input parameters and return additional information.

ODE File Overview

Look at the simple ODE file vdp1.m from earlier in this chapter.

```
function dy = vdp1(t,y)
dy = [y(2); (1–y(1)^2)*y(2)–y(1)];
```

Although this is a simple example, it demonstrates two important requirements for ODE files:

- The first two arguments must be t and y.
- By default, the ODE file must return a column vector F(t,y).

Defining the Initial Values in the ODE File

It is possible to specify default tspan, y0 and options in the ODE file, defining the entire initial value problem in the one file. In this case, the solver can be called as

```
[T,Y] = solver(’F’,[],[]);
```

The solver extracts the default values from the ODE file. You can also omit empty arguments at the end of the argument list. For example,

```
[T,Y] = solver(’F’);
```

When you call a solver with an empty or missing tspan or y0, the solver calls the specified ODE file to obtain any values not supplied in the solver argument list. It uses the syntax

```
[tspan,y0,options] = F([],[],’init’)
```

The ODE file is then expected to return three outputs:

- Output 1 is the `tspan` vector.
- Output 2 is the initial value, `y0`.
- Output 3 is either an options structure created with the `odeset` function or an empty matrix `[]`.

Coding the ODE File to Return Initial Values

If you use this approach, your ODE file must check the value of the third argument and return the appropriate output. For example, you can modify the van der Pol ODE file `vdp1.m` to check the third argument, `flag`, and return either the default vector `F(t,y)` or `[tspan,y0,options]` depending on the value of `flag`.

```
function [out1,out2,out3] = vdp1(t,y,flag)
if strcmp(flag,’’)

   % Return dy/dt = F(t,y).
   out1 = [y(2); (1–y(1)^2)*y(2)–y(1)];

elseif strcmp(flag,'init')

   % Return [tspan,y0,options].
   out1 = [0; 20];                       % tspan
   out2 = [2; 0];                        % initial conditions
   out3 = odeset('RelTol',1e–4);         % options

end
```

Note The third argument, referred to as the `flag` argument, is a special argument that notifies the ODE file that the solver is expecting a specific kind of information. The `'init'` string, for initial values, is just one possible value for this flag. For complete details on the `flag` argument, see “Special Purpose ODE Files and the flag Argument” on page 8-17.

Passing Additional Parameters to the ODE File

In some cases your ODE system may require additional parameters beyond the required t and y arguments. For example, you can generalize the van der Pol ODE file by passing it a mu parameter, instead of specifying a value for mu explicitly in the code.

```
function [out1,out2,out3] = vdpode(t,y,flag,mu)
if nargin < 4 | isempty(mu)
    mu = 1;

end
if strcmp(flag,’’)

   % Return dy/dt = F(t,y).
   out1 = [y(2); mu*(1–y(1)^2)*y(2)–y(1)];

elseif strcmp(flag,'init')

   % Return [tspan,y0,options].
   out1 = [0; 20];                          % tspan
   out2 = [2; 0];                           % initial conditions
   out3 = odeset('RelTol',1e–4);            % options

end
```

In this example, the parameter mu is an optional argument specific to the van der Pol example. MATLAB and the ODE solvers do not set a limit on the number of parameters you can pass to an ODE file.

Guidelines for Creating ODE Files

- The ode file must have at least two input arguments, t and y. It is not necessary, however, for the function to use either t or y.
- The derivatives returned by F(t,y) must be column vectors.
- Any additional parameters beyond t and y must appear at the end of the argument list and must begin at the fourth input parameter. The third position is reserved for an optional flag, as shown above in “Coding the ODE File to Return Initial Values.” The flag argument is described in more detail in “Special Purpose ODE Files and the flag Argument” on 8-17.

Improving Solver Performance

In some cases, you can improve ODE solver performance by specially coding your ODE file. For instance, you might accelerate the solution of a stiff problem by coding the ODE file to compute the Jacobian matrix analytically.

Another way to improve solver performance, often used in conjunction with a specially coded ODE file, is to tune solver parameters. The default parameters in the ODE solvers are selected to handle common problems. In some cases, however, tuning the parameters for a specific problem can improve performance significantly. You do this by supplying the solvers with one or more property values contained within an options argument.

```
[T,Y] = solver('F',tspan,y0,options)
```

The property values within the options argument are created with the odeset function, in which named properties are given specified values.

Category	Property Name	Page
Error tolerance	RelTol, AbsTol	8-21
Solver output	OutputFcn, OutputSel, Refine, Stats	8-22
Jacobian matrix	Jacobian, JConstant, JPattern, Vectorized	8-25
Step size	InitialStep, MaxStep	8-28
Mass matrix	Mass, MassSingular	8-29
Event location	Events	8-30
ode15s	MaxOrder, BDF	8-32

Special Purpose ODE Files and the flag Argument

The MATLAB ODE solvers are capable of using additional information provided in the ODE file. In this more general use, an ODE file is expected to respond to the arguments odefile(t,y,flag,p1,p2,...) where t and y are the integration variables, flag is a string indicating the type of information that the ODE file should return, and p1,p2,... are any additional parameters that the problem requires. This table shows the currently supported flags.

Flags	Return Values
'' (empty)	$F(t, y)$
'init'	tspan, y0, and options for this problem
'jacobian'	Jacobian matrix $J(t, y) = \partial F / \partial y$
'jpattern'	Matrix showing the Jacobian sparsity pattern
'mass'	Mass matrix M for solving $M(t, y)\, y' = F(t, y)$
'events'	Information to define an event location problem

The template below illustrates how to code an extended ODE file that uses the switch construct and the ODE file's third input argument, flag, to supply additional information. For illustration, the file also accepts two additional input parameters p1 and p2.

Note The example below is only a template. In your own coding you should not include all of the cases shown. For example, ’jacobian’ information is used for evaluating Jacobians analytically, and ’jpattern’ information is used for generating Jacobians numerically.

```
function varargout = odefile(t,y,flag,p1,p2)
switch flag
 case ’’                                % Return dy/dt = f(t,y).
   varargout{1} = f(t,y,p1,p2);
 case ’init’                % Return default [tspan,y0,options].
   [varargout{1:3}] = init(p1,p2);
 case ’jacobian’                 % Return Jacobian matrix df/dy.
   varargout{1} = jacobian(t,y,p1,p2);
 case ’jpattern’             % Return sparsity pattern matrix S.
   varargout{1} = jpattern(t,y,p1,p2);
 case ’mass    ’                         % Return mass matrix.
   varargout{1} = mass(t,y,p1,p2);
 case ’events’             % Return[value,isterminal,direction].
   [varargout{1:3}] = events(t,y,p1,p2);
 otherwise
   error([’Unknown flag ’’’ flag ’’’.’]);
 end
 % ------------------------------------------------------------
 function dydt = f(t,y,p1,p2)
 dydt = < Insert a function of t and/or y, p1, and p2 here. >;
 % ------------------------------------------------------------
 function [tspan,y0,options] = init(p1,p2)
 tspan = < Insert tspan here. >;
 y0 = < Insert y0 here. >;
 options = < Insert options = odeset(...) or [] here. >;
 % ------------------------------------------------------------
 function dfdy = jacobian(t,y,p1,p2)
 dfdy = < Insert Jacobian matrix here. >;
 % ------------------------------------------------------------
 function S = jpattern(t,y,p1,p2)
 S = < Insert Jacobian matrix sparsity pattern here. >;
 % ------------------------------------------------------------
```

```
function M = mass(t,y,p1,p2)
M = < Insert mass matrix here. >;
% -------------------------------------------------------------
function [value,isterminal,direction] = events(t,y,p1,p2)
value = < Insert event function vector here. >;
isterminal = < Insert logical ISTERMINAL vector here. >;
direction = < Insert DIRECTION vector here. >;
```

Creating an Options Structure: The odeset Function

The odeset function creates an options structure that you can supply to any of the ODE solvers. odeset accepts property name/property value pairs using the syntax

```
options = odeset('name1',value1,'name2',value2,...)
```

This creates a structure options in which the named properties have the specified values. Any unspecified properties contain default values in the solvers. For all properties, it is sufficient to type only the leading characters that uniquely identify the property name. odeset ignores case for property names.

With no input arguments, odeset displays all property names and their possible values, indicating defaults with { }.

```
AbsTol: [ positive scalar or vector {1e–6} ]
BDF: [ on | {off} ]
Events: [ on | {off} ]
InitialStep: [positive scalar]
Jacobian: [ on | {off} ]
JConstant: [ on | {off} ]
JPattern: [ on | {off} ]
Mass: [ {none} | M | M(t) | M(t,y) ]
MassSingular: [ yes | no | {maybe} ]
MaxOrder: [ 1 | 2 | 3 | 4 | {5} ]
MaxStep: [ positive scalar ]
OutputFcn: [ string ]
OutputSel: [ vector of integers ]
Refine: [ positive integer ]
RelTol: [ positive scalar {1e–3} ]
Stats: [ on | {off} ]
Vectorized: [on | {off}]
```

Modifying an Existing Options Structure

To modify an existing options argument, use

```
options = odeset(oldopts,'name1',value1,...)
```

This sets options equal to the existing structure oldopts, overwriting any values in oldopts that are respecified using name/value pairs and adding to the structure any new pairs. The modified structure is returned as an output argument. In the same way, the command

```
options = odeset(oldopts,newopts)
```

combines the structures oldopts and newopts. In the output argument, any values in the second argument (other than the empty matrix) overwrite those in the first argument.

Querying Options: The odeget Function

The solvers use the odeget function to extract property values from an options structure created with odeset.

```
o = odeget(options,'name')
```

This returns the value of the specified property, or an empty matrix [] if the property value is unspecified in the options structure.

As with odeset, it is sufficient to type only the leading characters that uniquely identify the property name; case is ignored for property names.

Error Tolerance Properties

The solvers use standard local error control techniques for monitoring and controlling the error of each integration step. At each step, the local error e in the i'th component of the solution is estimated and is required to be less than or equal to the acceptable error, which is a function of two user-defined tolerances RelTol and AbsTol.

```
|e(i)| <= max(RelTol*abs(y(i)),AbsTol(i))
```

- RelTol is the *relative accuracy tolerance*, a measure of the error relative to the size of each solution component. Roughly, it controls the number of correct digits in the answer. The default, 1e–3, corresponds to 0.1% accuracy.

- AbsTol is a scalar or vector of the *absolute error tolerances* for each solution component. AbsTol(i) is a threshold below which the values of the corresponding solution components are unimportant. The absolute error tolerances determine the accuracy when the solution approaches zero. The default value is 1e–6.

Set tolerances using odeset, either at the command line or in the ODE file.

Property	Value	Description
RelTol	Positive scalar {1e–3}	A relative error tolerance that applies to all components of the solution vector y. Default value is 10^(–3) (0.1% accuracy).
AbsTol	Positive scalar or vector {1e–6}	Absolute error tolerances that apply to the corresponding components of the solution vector. If a scalar value is specified, it applies to all components of the solution vector y. Default value is 10^(–6).

The ODE solvers are designed to deliver, for routine problems, accuracy roughly equivalent to the accuracy you request. They deliver less accuracy for problems integrated over “long” intervals and problems that are moderately unstable. Difficult problems may require tighter tolerances than the default values. For relative accuracy, adjust RelTol. For the absolute error tolerance, the scaling of the solution components is important: if |y| is somewhat smaller than AbsTol, the solver is not constrained to obtain any correct digits in y. You might have to solve a problem more than once to discover the scale of solution components.

Solver Output Properties

The solver output properties available with odeset let you control the output that the solvers generate. With these properties, you can specify an *output function*, a function that executes if you call the solver with no output arguments. In addition, the ODE solver output options let you obtain additional solutions at equally spaced points within each time step, or view statistics about the computations.

Property	Value	Description
OutputFcn	String	The name of an output function.
OutputSel	Vector of indices	Indices of solver output components to pass to an output function.
Refine	Positive integer	Produces smoother output, increasing the number of output points by a factor of Refine. If Refine is 1, the solver returns solutions only at the end of each time step. If Refine is n >1, the solver uses continuous extension to subdivide each time step into n smaller intervals, and returns solutions at each time point. Refine is 1 by default in all solvers except ode45 where it is 4 because of the solver's large step sizes. Refine does not apply when length(tspan)>2.
Stats	on | {off}	Specifies whether statistics about the solver's computations should be displayed.

OutputFcn

The OutputFcn property lets you define your own output function and pass the name of this function to the ODE solvers. If no output arguments are specified, the solvers call this function after each successful time step. You can use this feature, for example, to plot results as they are computed.

You must code your output function in a specific way for it to interact properly with the ODE solvers. When the name of an executable M-file function, e.g., myfun, is passed to an ODE solver as the OutputFcn property

```
options = odeset('OutputFcn','myfun')
```

the solver calls it with myfun(tspan,y0,'init') before beginning the integration so that the output function can initialize. Subsequently, the solver calls status = myfun(t,y) after each step. In addition to your intended use of (t,y), code myfun so that it returns a status output value of 0 or 1. If status = 1, integration halts. This might be used, for instance, to implement a **STOP** button. When integration is complete, the solver calls the output function with myfun([],[],'done').

Some example output functions are included with the ODE solvers:

- odeplot – time series plotting
- odephas2 – two-dimensional phase plane plotting
- odephas3 – three-dimensional phase plane plotting
- odeprint – print solution as it is computed

Use these as models for your own output functions. odeplot is the default output function for all the solvers. It is automatically invoked when the solvers are called with no output arguments.

OutputSel

The OutputSel property is a vector of indices specifying which components of the solution vector are to be passed to the output function. For example, if you want to use the odeplot output function, but you want to plot only the first and third components of the solution, you can do this using

```
    options = odeset('OutputFcn','odeplot','OutputSel',[1 3]);
```

Refine

The Refine property, an integer n, produces smoother output by increasing the number of output points by a factor of n. This feature is especially useful when using a medium or high order solver, such as ode45, for which solution components can change substantially in the course of a single step. To obtain smoother plots, increase the Refine property.

Note In all the solvers, the default value of Refine is 1.Within ode45, however, Refine is 4 to compensate for the solver's large step sizes. To override this and see only the time steps chosen by ode45, set Refine to 1.

The extra values produced for `Refine` are computed by means of continuous extension formulas. These are specialized formulas used by the ODE solvers to obtain accurate solutions between computed time steps without significant increase in computation time.

Stats

The `Stats` property specifies whether statistics about the computational cost of the integration should be displayed. By default, `Stats` is `off`. If it is on, after solving the problem the integrator displays:

- The number of successful steps
- The number of failed attempts
- The number of times the ODE file was called to evaluate *F(t,y)*
- The number of times that the partial derivatives matrix $\partial F / \partial y$ was formed
- The number of LU decompositions
- The number of solutions of linear systems

You can obtain the same values by including a third output argument in the call to the ODE solver:

```
[T,Y,S] = ode45('myfun', ...);
```

This statement produces a vector `S` that contains these statistics.

Jacobian Matrix Properties

The stiff ODE solvers often execute faster if you provide additional information about the Jacobian matrix $\partial F / \partial y$, a matrix of partial derivatives of the function defining the differential equation.

$$\begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \cdots \\ \vdots & \vdots & \end{bmatrix}$$

There are two aspects to providing information about the Jacobian:

- You can set up your ODE file to calculate and return the value of the Jacobian matrix for the problem. In this case, you must also use `odeset` to set the `Jacobian` property.
- If you do not calculate the Jacobian in the ODE file, `ode15s` and `ode23s` call the helper function `numjac` to approximate Jacobians numerically by finite differences. In this case, you may be able to use the `JConstant`, `Vectorized`, or `JPattern` properties.

The Jacobian matrix properties pertain only to the stiff solvers `ode15s` and `ode23s` for which the Jacobian matrix $\partial F / \partial y$ is critical to reliability and efficiency.

Property	**Value**	**Description**
`JConstant`	`on` | {`off`}	Set `on` if the Jacobian matrix $\partial F / \partial y$ is constant (does not depend on `t` or `y`).
`Jacobian`	`on` | {`off`}	Set `on` to inform the solver that the ODE file is coded such that `F(t,y,'Jacobian')` returns $\partial F / \partial y$.
`JPattern`	`on` | {`off`}	Set `on` if $\partial F / \partial y$ is a sparse matrix and the ODE file is coded so that `F([],[],'JPattern')` returns a sparsity pattern matrix.
`Vectorized`	`on` | {off}	Set `on` to inform the stiff solver that the ODE file is coded so that `F(t,[y1 y2 ...])` returns `[F(t,y1) F(t,y2) ...]`.

JConstant

Set JConstant on if the Jacobian matrix $\partial F/\partial y$ is constant (does not depend on t or y). Whether computing the Jacobians numerically or evaluating them analytically, the solver takes advantage of this information to reduce solution time. For the stiff van der Pol example, the Jacobian matrix is

```
J = [ 0                       1
      (–2000*y(1)*y(2) – 1)  (1000*(1–y(1)^2)) ]
```

(not constant) so the JConstant property does not apply.

Jacobian

Set Jacobian on to inform the solver that the ODE file is coded such that F(t,y,'Jacobian') returns $\partial F/\partial y$. By default, Jacobian is off, and Jacobians are generated numerically.

Coding the ODE file to evaluate the Jacobian analytically often increases the speed and reliability of the solution for the stiff problem. The Jacobian shown above for the stiff van der Pol problem can be coded into the ODE file as

```
function out1 = vdp1000(t,y,flag)
if strcmp(flag,’’)               % return dy
  out1 = [y(2); 1000*(1–y(1)^2)*y(2)–y(1)];
elseif strcmp(flag,'jacobian')   % return J
  out1 = [ 0                      1
           (–2000*y(1)*y(2) – 1)  (1000*(1–y(1)^2)) ];
end
```

JPattern

Set JPattern on if $\partial F/\partial y$ is a sparse matrix and the ODE file is coded so that F([],[],'JPattern') returns a sparsity pattern matrix. This is a sparse matrix with 1s where there are nonzero entries in the Jacobian. numjac uses the sparsity pattern to generate a sparse Jacobian matrix numerically. If the Jacobian matrix is large (size greater than approximately 100-by-100) and sparse, this can accelerate execution greatly. For an example using the JPattern property, see the brussode example on 8-37.

Vectorized

Set Vectorized on to inform the stiff solver that the ODE file is coded so that F(t,[y1 y2 ...]) returns [F(t,y1) F(t,y2) ...]. When computing Jacobians numerically, the solver passes this information to the numjac routine. This allows numjac to reduce the number of function evaluations required to compute all the columns of the Jacobian matrix, and may reduce solution time significantly.

With MATLAB's array notation, it is typically an easy matter to vectorize an ODE file. For example, the stiff van der Pol example shown previously can be vectorized by introducing colon notation into the subscripts and by using the array power (.^) and array multiplication (.*) operators.

```
function dy = vdp1000(t,y)
dy = [y(2,:); 1000*(1–y(1,:).^2).*y(2,:)–y(1,:)];
```

Step-Size Properties

The step-size properties let you specify the first step size tried by the solver, potentially helping it to recognize better the scale of the problem. In addition, you can specify bounds on the sizes of subsequent time steps.

Property	Value	Description
MaxStep	Positive scalar	Upper bound on solver step size.
InitialStep	Positive scalar	Suggested initial step size.

Generally it is not necessary for you to adjust MaxStep and InitialStep because the ODE solvers implement state-of-the-art variable time step control algorithms. Adjusting these properties without good reason may result in degraded solver performance.

MaxStep

MaxStep has a positive scalar value. This property sets an upper bound on the magnitude of the step size the solver uses. If the differential equation has periodic coefficients or solution, it may be a good idea to set MaxStep to some fraction (such as 1/4) of the period. This guarantees that the solver does not enlarge the time step too much and step over a period of interest.

- Do not reduce MaxStep to produce more output points. This can slow down solution time significantly. Instead, use Refine (8-24) to compute additional outputs by continuous extension at very low cost.
- Do not reduce MaxStep when the solution does not appear to be accurate enough. Instead, reduce the relative error tolerance RelTol, and use the solution you just computed to determine appropriate values for the absolute error tolerance vector AbsTol. (See "Error Tolerance Properties" on page 8-21 for a description of the error tolerance properties.)
- Generally you should not reduce MaxStep to make sure that the solver doesn't step over some behavior that occurs only once during the simulation interval. If you know the time at which the change occurs, break the simulation interval into two pieces and call the solvers twice. If you do not know the time at which the change occurs, try reducing the error tolerances RelTol and AbsTol. Use MaxStep as a last resort.

InitialStep

InitialStep has a positive scalar value. This property sets an upper bound on the magnitude of the first step size the solver tries. Generally the automatic procedure works very well. However, the initial step size is based on the slope of the solution at the initial time tspan(1), and if the slope of all solution components is zero, the procedure might try a step size that is much too large. If you know this is happening or you want to be sure that the solver resolves important behavior at the start of the integration, help the code start by providing a suitable InitialStep.

Mass Matrix Properties

The solvers of the ODE suite can solve problems of the form *M(t, y) y' = F(t, y)* with a mass matrix *M* that is nonsingular and (usually) sparse. Use odeset to set Mass to 'M', 'M(t)', or 'M(t,y)' if the ODE file F.m is coded so that F(t,y,'mass') returns a constant, time-dependent, or time-and-state dependent mass matrix, respectively. The default value of Mass is 'none'. The ode23s solver can only solve problems with a constant mass matrix *M*. For examples of mass matrix problems, see fem1ode, fem2ode, or batonode.

If *M* is singular, then *M(t) * y' = F(t, y)* is a differential algebraic equation (DAE). DAEs have solutions only when *y0* is consistent, that is, if there is a vector *yp0* such that *M(t0) * y0 = f(t0, y0)*. The ode15s and ode23t solvers can solve DAEs of index 1 provided that *M* is not state-dependent and *y0* is sufficiently close to being consistent. If there is a mass matrix, you can use `odeset` to set the `MassSingular` property to `'yes'`, `'no'`, or `'maybe'`. The default value of `'maybe'` causes the solver to test whether the problem is a DAE. If it is, the solver treats *y0* as a guess, attempts to compute consistent initial conditions that are close to `y0`, and continues to solve the problem. When solving DAEs, it is very advantageous to formulate the problem so that *M* is a diagonal matrix (a semi-explicit DAE). For examples of DAE problems, see `hb1dae` or `amp1dae`.

Property	Value	Description

`Mass`	`{none} | M`
	`| M(t)`
	`| M(t,y)`
		Indicate whether the ODE file returns a mass matrix.

`MassSingular`	`yes | no`
	`| {maybe}`
		Indicate whether the mass matrix is singular.

Mass

Change this property from `'none'` if the ODE file is coded so that `F(t,y,'mass')` returns a mass matrix. `'M'` indicates a constant mass matrix, `'M(t)'` indicates a time-dependent mass matrix, and `'M(t,y)'` indicates a time- and state-dependent mass matrix.

MassSingular

Set this property to `'no'` if the mass matrix is not singular.

For an example of an ODE file with a mass matrix, see "Example 4: Finite Element Discretization" on page 8-40.

Event Location Property

In some ODE problems the times of specific events are important, such as the time at which a ball hits the ground, or the time at which a spaceship returns to the earth, or the times at which the ODE solution reaches certain values.

While solving a problem, the MATLAB ODE solvers can locate transitions to, from, or through zeros of a vector of user-defined functions.

String	Value	Description
`Events`	`on | {off}`	Set this on if the ODE file evaluates and returns the event functions, and returns information about the events.

Events

Set this parameter on to inform the solver that the ODE file is coded so that `F(t,y,'events')` returns appropriate event function information. By default, `'events'` is `off`.

For example, the statement

```
[T,Y,TE,YE,IE] = solver('F',tspan,y0,options)
```

with the `Events` property in `options` set `on` solves an ODE problem while also locating zero crossings of an events function defined in the ODE file. In this case, the solver returns three additional outputs:

- `TE` is a column vector of times at which events occur.
- Rows of `YE` are solutions corresponding to times in `TE`.
- Indices in vector `IE` specify which event occurred at the time in `TE`.

The ODE file must be coded to return three values in response to the `'events'` flag.

```
[value,isterminal,direction] = F(t,y,'events');
```

The first output argument `value` is the vector of event functions evaluated at `(t,y)`. The `value` vector may be any length. It is evaluated at the beginning and end of each integration step, and if any elements make transitions to, from, or through zero (with the directionality specified in constant vector `direction`), the solver uses the continuous extension formulas to determine the time when the transition occurred.

Terminal events halt the integration. The argument `isterminal` is a logical vector of `1`s and `0`s that specifies whether a zero-crossing of the corresponding value element is terminal. 1 corresponds to a terminal event, halting the integration; 0 corresponds to a nonterminal event.

The direction vector specifies a desired directionality: positive (1), negative (–1), or don't care (0), for each value element.

The time an event occurs is located to machine precision within an interval of [t– t+]. Nonterminal events are reported at t+. For terminal events, both t– and t+ are reported.

For an example of an ODE file with an event location, see "Example 5: Simple Event Location" on page 8-44.

ode15s Properties

The ode15s solver is a variable-order stiff solver based on the numerical differentiation formulas (NDFs). The NDFs are generally more efficient than the closely related family of backward differentiation formulas (BDFs), also known as Gear's methods. The ode15s properties let you choose between these formulas, as well as specifying the maximum order for the solver.

Property	Value	Description
MaxOrder	1 | 2 | 3 | 4 |{5}	The maximum order formula used.
BDF	on | {off}	Specifies whether the backward differentiation formulas are to be used instead of the default numerical differentiation formulas.

MaxOrder

MaxOrder is an integer 1 through 5 used to set an upper bound on the order of the formula that computes the solution. By default, the maximum order is 5.

BDF

Set BDF on to have ode15s use the BDFs. By default, BDF is off, and the solver uses the NDFs.

For both the NDFs and BDFs, the formulas of orders 1 and 2 are A-stable (the stability region includes the entire left half complex plane). The higher order formulas are not as stable, and the higher the order the worse the stability.

There is a class of stiff problems (stiff oscillatory) that is solved more efficiently if `MaxOrder` is reduced (for example to `2`) so that only the most stable formulas are used.

Examples: Applying the ODE Solvers

This section contains several examples of ODE files. These examples illustrate the kinds of problems you can solve in MATLAB. For more examples, see MATLAB's `demos` directory.

Example 1: Simple Nonstiff Problem

`rigidode` is a nonstiff example that can be solved with all five solvers of the ODE suite. It is a standard test problem, proposed by Krogh, for nonstiff solvers. The analytical solutions are Jacobian elliptic functions accessible in MATLAB. The interval here is about 1.5 periods.

The `rigidode` system consists of the Euler equations of a rigid body without external forces as proposed by Krogh. `rigidode` is a system of three equations

$$y'_1 = y_2\,y_3$$

$$y'_2 = -y_1\,y_3$$

$$y'_3 = -0.51\,y_1\,y_2$$

`rigidode([],[],'init')` returns the default `tspan`, `y0`, and `options` values for this problem. These values are retrieved by an ODE solver if the solver is invoked with empty `tspan` or `y0` arguments. This example uses the default solver options, so the third output argument is set to empty, `[]`, instead of an options structure created with `odeset`. By means of the `'init'` flag, the entire initial value problem is defined in one file.

Reference Shampine, L. F. and M. K. Gordon, *Computer Solution of Ordinary Differential Equations*, W.H. Freeman & Co., 1975.

```
function varargout = rigidode(t,y,flag)
%RIGIDODE Euler equations of a rigid body without external forces.
switch flag
case ’’                     % Return dy/dt = f(t,y).
  varargout{1} = f(t,y);
case ’init’                  % Return default [tspan,y0,options].
  [varargout{1:3}] = init;
otherwise
  error([’Unknown flag ’’’ flag ’’’.’]);
end
% ------------------------------------------------------------
function dydt = f(t,y)
dydt = [y(2)*y(3); -y(1)*y(3); -0.51*y(1)*y(2)];
% ------------------------------------------------------------
function [tspan,y0,options] = init
tspan = [0; 12];
y0 = [0; 1; 1];
options = [];
```

Example 2: van der Pol Equation

vdpode is a more general version of the van der Pol example that has been used in various forms throughout this chapter. For illustrative purposes, it is coded for both fast numerical Jacobian computation (Vectorized property) and for analytical Jacobian evaluation (Jacobian property). In practice you would supply only one or the other of these options. It is not necessary to supply either.

The van der Pol equation is written as a system of two equations.

$$y'_1 = y_2$$

$$y'_2 = \mu(1 - y_1^2)y_2 - y_1$$

vdpode(t,y) or vdpode(t,y,[],mu) returns the derivatives vector for the van der Pol equation. By default, mu is 1 and the problem is not stiff. Optionally, pass in the mu parameter as an additional input argument to an ODE solver. The problem becomes more stiff as mu is increased and the period of oscillation becomes larger.

When `mu` is `1000` the equation is in relaxation oscillation and the problem is very stiff. The limit cycle has portions where the solution components change slowly and the problem is quite stiff, alternating with regions of very sharp change where it is not stiff (quasi-discontinuities).

This example sets `Vectorized` `on` with `odeset` because `vdpode` is coded so that `vdpode(t,[y1 y2 ...])` returns `[vdpode(t,y1) vdpode(t,y2) ...]` for scalar time `t` and vectors `y1`,`y2`,... The stiff ODE solvers take advantage of this feature only when approximating the columns of the Jacobian numerically.

`vdpode([],[],'init')` returns the default `tspan`, `y0`, and `options` values for this problem. The entire initial value problem is defined in this one file.

`vdpode(t,y,'jacobian')` or `vdpode(t,y,'jacobian',mu)` returns the Jacobian matrix $\partial F/\partial y$ evaluated analytically at `(t,y)`. By default, the stiff solvers of the ODE suite approximate Jacobian matrices numerically. However, if `Jacobian` is set `on` with `odeset`, a solver calls the ODE file with the flag `'jacobian'` to obtain $\partial F/\partial y$. Providing the solvers with an analytic Jacobian is not necessary, but it can improve the reliability and efficiency of integration.

```
function varargout = vdpode(t,y,flag,mu)
%VDPODE Parameterizable van der Pol equation (stiff for large mu).

if nargin < 4 | isempty(mu)
  mu = 1;
end
switch flag
case ''                      % Return dy/dt = f(t,y).
  varargout{1} = f(t,y,mu);
case 'init'                   % Return default [tspan,y0,options].
  [varargout{1:3}] = init(mu);
case 'jacobian'              % Return Jacobian matrix df/dy.
  varargout{1} = jacobian(t,y,mu);
otherwise
  error(['Unknown flag ''' flag '''.']);
end
% ------------------------------------------------------------
function dydt = f(t,y,mu)
dydt = [y(2,:); (mu*(1-y(1,:).^2).*y(2,:) - y(1,:))]; %
Vectorized
% ------------------------------------------------------------
function [tspan,y0,options] = init(mu)
tspan = [0; max(20,3*mu)];  % several periods
y0 = [2; 0];
options = odeset('Vectorized','on');
% ------------------------------------------------------------
function dfdy = jacobian(t,y,mu)
dfdy = [ 0                      1
         (-2*mu*y(1)*y(2) - 1)  (mu*(1-y(1)^2)) ];
```

Example 3: Large, Stiff Sparse Problem

This is an example of a (potentially) large stiff sparse problem. Like vdpode, the file is coded to use both the Vectorized and Jacobian properties, but only one is used during the course of a simulation. Like both previous examples, brussode responds to the 'init' flag.

The brussode example is the classic "Brusselator" system (Hairer and Wanner) modeling diffusion in a chemical reaction.

$$u'_i = 1 + u_i^2 v_i - 4u_i + \alpha(N+1)^2(u_{i-1} - 2u_i + u_{i+1})$$

and is solved on the time interval `[0,10]` with α = 1/50 and

$$\left.\begin{aligned} u_i(0) &= 1 + \sin(2\pi x_i) \\ v_i(0) &= 3 \end{aligned}\right\} \quad \text{with } x_i = i/(N+1) \text{for } i = 1, \ldots, N$$

There are $2N$ equations in the system, but the Jacobian is banded with a constant width 5 if the equations are ordered as u_1, v_1, u_2, v_2, ...

`brussode(t,y)` or `brussode(t,y,[],n)` returns the derivatives vector for the Brusselator problem. The parameter `n` ≥ `2` is used to specify the number of grid points; the resulting system consists of `2n` equations. By default, `n` is `2`. The problem becomes increasingly stiff and the Jacobian increasingly sparse as `n` is increased.

`brussode([],[],'jpattern')` or `brussode([],[],'jpattern',n)` returns a sparse matrix of `1`s and `0`s showing the locations of nonzeros in the Jacobian $\partial F/\partial y$. By default, the stiff ODE solvers generate Jacobians numerically as full matrices. However, if `JPattern` is set `on` with `odeset`, a solver calls the ODE file with the flag `'jpattern'`. This provides the solver with a sparsity pattern that it uses to generate the Jacobian numerically as a sparse matrix. Providing a sparsity pattern can significantly reduce the number of function evaluations required to generate the Jacobian and can accelerate integration. For the Brusselator problem, if the sparsity pattern is not supplied, `2n` evaluations of the function are needed to compute the `2n`-by-`2n` Jacobian matrix. If the sparsity pattern is supplied, only four evaluations are needed, regardless of the value of `n`.

Reference Hairer, E. and G. Wanner, *Solving Ordinary Differential Equations II, Stiff and Differential-Algebraic Problems*, Springer-Verlag, Berlin, 1991, pp. 5-8.

```
function varargout = brussode(t,y,flag,N)
%BRUSSODE Stiff problem modeling a chemical reaction.

if nargin < 4 | isempty(N)
  N = 2;
end
switch flag
case ’’                     % Return dy/dt = f(t,y).
  varargout{1} = f(t,y,N);
case ’init’                  % Return default [tspan,y0,options].
  [varargout{1:3}] = init(N);
case ’jpattern’              % Return sparsity pattern of df/dy.
  varargout{1} = jpattern(t,y,N);
case ’jacobian’             % Return Jacobian matrix df/dy.
  varargout{1} = jacobian(t,y,N);
otherwise
  error([’Unknown flag ’’’ flag ’’’.’]);
end
% ------------------------------------------------------------
function dydt = f(t,y,N)
c = 0.02 * (N+1)^2;
dydt = zeros(2*N,size(y,2));% preallocate dy/dt
% Evaluate the 2 components of the function at one edge of the grid
% (with edge conditions).
i = 1;
dydt(i,:) = 1 + y(i+1,:).*y(i,:).^2 - 4*y(i,:) + ...
c*(1-2*y(i,:)+y(i+2,:));
dydt(i+1,:) = 3*y(i,:) - y(i+1,:).*y(i,:).^2 + ...
c*(3-2*y(i+1,:)+y(i+3,:));
% Evaluate the 2 components of the function at all interior grid
% points.
i = 3:2:2*N-3;

dydt(i,:) = 1 + y(i+1,:).*y(i,:).^2 - 4*y(i,:) + ...
    c*(y(i-2,:)-2*y(i,:)+y(i+2,:));
dydt(i+1,:) = 3*y(i,:) - y(i+1,:).*y(i,:).^2 + ...
    c*(y(i-1,:)-2*y(i+1,:)+y(i+3,:));
% Evaluate the 2 components of the function at the other edge of
% the grid (with edge conditions).
i = 2*N-1;
```

```
dydt(i,:) = 1 + y(i+1,:).*y(i,:).^2 - 4*y(i,:) + ...
c*(y(i-2,:)-2*y(i,:)+1);
dydt(i+1,:) = 3*y(i,:) - y(i+1,:).*y(i,:).^2 + ...
c*(y(i-1,:)-2*y(i+1,:)+3);
% -------------------------------------------------------------
function [tspan,y0,options] = init(N)
tspan = [0; 10];
y0 = [1+sin((2*pi/(N+1))*(1:N)); 3+zeros(1,N)];
y0 = y0(:);
options = odeset('Vectorized','on');
% -------------------------------------------------------------
function dfdy = jacobian(t,y,N)
c = 0.02 * (N+1)^2;
B = zeros(2*N,5);
B(1:2*(N-1),1) = B(1:2*(N-1),1) + c;
i = 1:2:2*N-1;
B(i,2) = 3 - 2*y(i).*y(i+1);
B(i,3) = 2*y(i).*y(i+1) - 4 - 2*c;
B(i+1,3) = -y(i).^2 - 2*c;
B(i+1,4) = y(i).^2;
B(3:2*N,5) = B(3:2*N,5) + c;
dfdy = spdiags(B,-2:2,2*N,2*N);  % Note this is a SPARSE Jacobian.
% -------------------------------------------------------------
function S = jpattern(t,y,N)
B = ones(2*N,5);
B(2:2:2*N,2) = zeros(N,1);
B(1:2:2*N-1,4) = zeros(N,1);
S = spdiags(B,-2:2,2*N,2*N);
if nargin < 4 | isempty(N)
  N = 2;
end
```

Example 4: Finite Element Discretization

fem1ode(t,y) or fem1ode(t,y,[],n) returns the derivatives vector for a finite element discretization of a partial differential equation. The parameter n controls the discretization, and the resulting system consists of n equations. By default, n is 9.

This example involves a mass matrix. The system of ODE's comes from a method of lines solution of the partial differential equation

$$e^{-t}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

with initial condition $u(0, x) = \sin(x)$ and boundary conditions $u(t, 0) = u(t, \pi) = 0$. An integer N is chosen, h is defined as $1/(N+1)$, and the solution of the partial differential equation is approximated at $x_k = k\pi h$ for $k = 0, 1, , ..., N+1$ by

$$u(t,x_k) \approx \sum_{k=1}^{N} c_k(t)\phi_k(x)$$

Here $\phi_k(x)$ is a piecewise linear function that is 1 at x_k and 0 at all the other x_j. A Galerkin discretization leads to the system of ODEs

$$A(t)c' = Rc \quad \text{where} \quad c(t) = \begin{bmatrix} c_1(t) \\ \vdots \\ c_N(t) \end{bmatrix}$$

and the tridiagonal matrices $A(t)$ and R are given by

$$A_{ij} = \begin{cases} \exp(-t)2h/3 & \text{if } i = j \\ \exp(-t)h/6 & \text{if } i = j \pm 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad R_{ij} = \begin{cases} -2/h & \text{if } i = j \\ 1/h & \text{if } i = j \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

The initial values $c(0)$ are taken from the initial condition for the partial differential equation. The problem is solved on the time interval $[0, \pi]$.

`fem1ode(t,[],'mass')` or `fem1ode(t,[],'mass',n)` returns the time-dependent mass matrix M evaluated at time `t`. By default, `ode15s` solves systems of the form $y' = F(t, y)$. However, if the `Mass` property is changed from `'none'` to `'M'`, `'M(t)'`, or `'M(t,y)'` with `odeset`, the solver calls the ODE file with the flag `'mass'`. The ODE file returns a mass matrix, which the solver uses to solve $M(t, y)\, y' = F(t, y)$. If the mass matrix is a constant M, the problem can be also be solved with `ode23s`.

For example, to solve a system of 20 equations, use

```
[T,Y] = ode15s('fem1ode',[],[],odeset('Mass','M(t)'),20);
```

fem1ode also responds to the flag ’init’ (see the rigidode example for details).

```
function varargout = fem1ode(t,y,flag,N)
%FEM1ODE Stiff problem with a time-dependent mass matrix.
if nargin == 0
  flag = ’demo’;
end
if nargin < 4 | isempty(N)
  N = 9;
end
switch flag
case ’’                                 % Return dy/dt = f(t,y).
  varargout{1} = f(t,y,N);
case ’init’                             % Return default.
[tspan,y0,options].
  [varargout{1:3}] = init(N);
case ’mass’                           % Return mass matrix M(t).
  varargout{1} = mass(t,y,N);
case ’demo’                             % Run a demo.
  demo;
otherwise
  error([’Unknown flag ’’’ flag ’’’.’]);
end
%---------------------------------------------------------------
function dydt = f(t,y,N)
e = ((N+1)/pi) + zeros(N,1);   % h=pi/(N+1); e=(1/h)+zeros(N,1);
R = spdiags([e -2*e e], -1:1, N, N);
dydt = R*y;
%---------------------------------------------------------------
function [tspan,y0,options] = init(N)
tspan = [0; pi];
y0 = sin((pi/(N+1))*(1:N)’);
options = odeset(’Mass’,’M(t)’,’Vectorized’,’on’);




%---------------------------------------------------------------
```

```
function M = mass(t,y,N)
e = (exp(-t)*pi/(6*(N+1))) + zeros(N,1); % h=pi/(N+1);
e=exp(-t)*h/6+zeros
M = spdiags([e 4*e e], -1:1, N, N);
%-----------------------------------------------------------------
function demo
[t,y] = ode15s('fem1ode');
surf(1:9,t,y);
set(gca,'ZLim',[0 1]);
view(142.5,30);
title(['Finite element problem with time-dependent mass ' ...
      'matrix, solved by ODE15S']);
xlabel('space');
ylabel('time');
zlabel('solution');
```

Example 5: Simple Event Location

ballode(t,y) returns the derivatives vector for the equations of motion of a bouncing ball. This ODE file illustrates the event location capabilities of the ODE solvers.

The equations for the bouncing ball are:

$$y'_1 = y_2$$
$$y'_2 = -9.8$$

ballode(t,y,'events') returns a zero-crossing vector value evaluated at (t,y), as well as two constant vectors isterminal and direction. By default, the ODE solvers do not locate zero-crossings. However, if the Events property is set on with odeset, a solver calls the ODE file with the flag 'events'. This provides the solver with information that it uses to locate zero-crossings of the elements in the value vector. The value vector may be any length. It is evaluated at the beginning and end of a step, and if any elements change sign (with the directionality specified in direction), the zero-crossing point is located. The isterminal vector consists of logical 1s and 0s, enabling you to specify whether or not a zero-crossing of the corresponding value element halts the integration. The direction vector enables you to specify a desired directionality, positive (1), negative (–1), or don’t care (0) for each value element.

ballode also responds to the flag ’init’ (see the rigidode example for details).

```
function varargout = ballode(t,y,flag)
%BALLODE Equations of motion for a bouncing ball.
switch flag
case ’’                   % Return dy/dt = f(t,y).
  varargout{1} = f(t,y);
case ’init’               % Return default [tspan,y0,options].
  [varargout{1:3}] = init;
case ’events’             % Return [value,isterminal,direction].
  [varargout{1:3}] = events(t,y);
otherwise
  error([’Unknown flag ’’’ flag ’’’.’]);
end
% --------------------------------------------------------------
function dydt = f(t,y)
dydt = [y(2); -9.8];
% --------------------------------------------------------------
function [tspan,y0,options] = init
tspan = [0; 10];
y0 = [0; 20];
options = odeset(’Events’,’on’);
% --------------------------------------------------------------
function [value,isterminal,direction] = events(t,y)
% Locate the time when height passes through zero in a decreasing
% direction and stop integration. Also locate both decreasing and
% increasing zero-crossings of velocity,and don’t stop
% integration.
value = y;                % [height; velocity]
isterminal = [1; 0];
direction = [-1; 0];
```

Example 6: Advanced Event Location

orbitode is a standard test problem for nonstiff solvers presented in Shampine and Gordon, (see reference that follows).

The orbitode problem is a system of four equations.

$$y'_1 = y_3$$

$$y'_2 = y_4$$

$$y'_3 = 2y_4 + y_1 - \frac{\mu^*(y_1 + \mu)}{r_1^3} - \frac{\mu(y_1 - \mu^*)}{r_2^3}$$

$$y'_4 = -2y_3 + y_2 - \frac{\mu^* y_2}{r_1^3} - \frac{\mu y_2}{r_2^3}$$

where

$$\mu = 1/82.45$$

$$\mu^* = 1 - \mu$$

$$r_1 = \sqrt{(y_1 + \mu)^2 + y_2^2}$$

$$r_2 = \sqrt{(y_1 - \mu^*)^2 + y_2^2}$$

The first two solution components are coordinates of the body of infinitesimal mass, so plotting one against the other gives the orbit of the body around the other two bodies. The initial conditions have been chosen so as to make the orbit periodic. This corresponds to a spaceship traveling around the moon and returning to the earth. Moderately stringent tolerances are necessary to reproduce the qualitative behavior of the orbit. Suitable values are 1e–5 for RelTol and 1e–4 for AbsTol.

The event functions implemented in this example locate the point of maximum distance from the earth and the time the spaceship returns to earth.

Reference Shampine, L. F. and M. K. Gordon, *Computer Solution of Ordinary Differential Equations*, W.H. Freeman & Co., 1975, p. 246.

```
function varargout = orbitode(t,y,flag)
%ORBITODE Restricted three body problem.
y0 = [1.2; 0; 0; -1.04935750983031990726];
switch flag
case ''% Return dy/dt = f(t,y).
  varargout{1} = f(t,y);
case 'init'              % Return default [tspan,y0,options].
  [varargout{1:3}] = init(y0);
case 'events'             % Return [value,isterminal,direction].
  [varargout{1:3}] = events(t,y,y0);
otherwise
  error(['Unknown flag ''' flag '''.']);
end
% --------------------------------------------------------------
function dydt = f(t,y)
mu = 1 / 82.45;
mustar = 1 - mu;
r13 = ((y(1) + mu)^2 + y(2)^2) ^ 1.5;
r23 = ((y(1) - mustar)^2 + y(2)^2) ^ 1.5;
dydt = [ y(3)
         y(4)
         (2*y(4) + y(1) - mustar*((y(1)+mu)/r13) -
mu*((y(1)-mustar)/r23))
         (-2*y(3) + y(2) - mustar*(y(2)/r13) - mu*(y(2)/r23)) ];
% --------------------------------------------------------------
function [tspan,y0,options] = init(y)
tspan = [0; 6.19216933131963970674];
y0 = y;
options = odeset('RelTol',1e-5,'AbsTol',1e-4);
% --------------------------------------------------------------
function [value,isterminal,direction] = events(t,y,y0)
% Locate the time when the object returns closest to the initial
% point y0 and starts to move away, and stop integration.  Also
% locate the time when the object is farthest from the initial
% point y0 and starts to move closer.
%
% The current distance of the body is
%
%   DSQ = (y(1)-y0(1))^2 + (y(2)-y0(2))^2 = <y(1:2)-y0,y(1:2)-y0>
%
```

```
% A local minimum of DSQ occurs when d/dt DSQ crosses zero heading
% in the positive direction. We can compute d/dt DSQ as
%
%   d/dt DSQ = 2*(y(1:2)-y0)'*dy(1:2)/dt = 2*(y(1:2)-y0)'*y(3:4)
%
dDSQdt = 2 * ((y(1:2)-y0(1:2))' * y(3:4));
value = [dDSQdt; dDSQdt];
isterminal = [1; 0];% stop at local minimum
direction = [1; -1];% [local minimum, local maximum]
```

Questions and Answers

This section contains a number of tables that answer questions about the use and operation of the MATLAB ODE solvers. This section also contains a troubleshooting table. The question and answer tables cover the following categories:

- General ODE Solver Questions
- Problem Size, Memory Use, and Computation Speed
- Time Steps for Integration
- Error Tolerance and Other Options
- Solving Different Kinds of Systems

General ODE Solver Questions

Question	Answer
How do the ODE solvers differ from quad or quad8?	quad and quad8 solve problems of the form $y' = F(t)$. The ODE suite solves more general problems of the form $y' = F(t, y)$.
Can I solve ODE systems in which there are more equations than unknowns, or vice-versa?	No.

Problem Size, Memory Use, and Computation Speed

Question	Answer
How large a problem can I solve with the ODE suite?	The primary constraints are memory and time. At each time step, the nonstiff solvers allocate vectors of length n, where n is the number of equations in the system. The stiff solvers allocate vectors of length n, but also an n-by-n Jacobian matrix. For these solvers it may be advantageous to use the sparse option. If the problem is nonstiff, or if you are using the sparse option, it may be possible to solve a problem with thousands of unknowns. In this case, however, storage of the result can be problematic.

Problem Size, Memory Use, and Computation Speed

Question	Answer
I'm solving a very large system, but only care about a couple of the components of y. Is there any way to avoid storing all of the elements?	Yes. The user-installable output function capability is designed specifically for this purpose. When an output function is installed and the solver call does not include output arguments, the solver does not allocate storage to hold the entire solution history. Instead, the solver calls `OutputFcn(t,y)` at each time step. To keep the history of specific elements, write an output function that stores or plots only the elements you care about.
How many time steps is too many?	If your integration uses more than 200 time steps, it's likely that your `tspan` is too long, or your problem is stiff. Divide `tspan` into pieces or try `ode15s`.
What is the startup cost of the integration and how can I reduce it?	The biggest startup cost occurs as the solver attempts to find a step size appropriate to the scale of the problem. If you happen to know an appropriate step size, use the `InitialStep` property. For example, if you repeatedly call the integrator in an event location loop, the last step that was taken before the event is probably on scale for the next integration. See `ballode` for an example.

Time Steps for Integration

Question	Answer
The first step size that the integrator takes is too large, and it misses important behavior.	You can specify the first step size with the `InitialStep` property. The integrator tries this value, then reduces it if necessary.
Can I integrate with fixed step sizes?	No.

Error Tolerance and Other Options	
Question	**Answer**
How do I choose RelTol and AbsTol?	RelTol, the relative accuracy tolerance, controls the number of correct digits in the answer. AbsTol, the absolute error tolerance, controls the difference between the answer and the solution. A relative error tolerance gets into trouble when a solution component vanishes. An absolute error tolerance gets into trouble when a solution component is unexpectedly large. The solvers require nonzero tolerances and use a mixed test to avoid these problems. At each step the error e in the i'th component of the solution is required to satisfy this condition `|e(i)| <= max(RelTol*abs(y(i)),AbsTol(i))` The use of RelTol is clear – to obtain p correct digits let RelTol = 10^(-p), or slightly smaller. The use of AbsTol depends on the problem scale. AbsTol is a threshold – the solver does not guarantee correct digits for solution components smaller than AbsTol(i). If the problem has a natural threshold, use it as AbsTol. A small value of AbsTol does not adversely affect the computation, but be aware that the problem's scaling might mean that an important component is smaller than the specified AbsTol. You might think that you computed the component with the relative accuracy of RelTol, when in fact it is below the AbsTol threshold, and you have few if any correct digits. Even if you are not interested in correct digits in this component, failing to compute it accurately may harm the accuracy of components you do care about. Generally the solvers handle this situation automatically, but not always.
I want answers that are correct to the precision of the computer. Why can't I simply set RelTol to eps?	You can get close to machine precision, but not that close. The solvers do not allow RelTol near eps because they try to approximate a continuous function. At tolerances comparable to eps, the machine arithmetic causes all functions to look discontinuous.

Error Tolerance and Other Options	
Question	**Answer**
How do I tell the solver that I don't care about getting an accurate answer for one of the solution components?	You can increase the absolute error tolerance corresponding to this solution component. If the tolerance is bigger than the component, this specifies no correct digits for the component. The solver may have to get some correct digits in this component to compute other components accurately, but it generally handles this automatically.

Solving Different Kinds of Systems	
Question	**Answer**
Can the solvers handle PDEs that have been discretized by the Method of Lines?	Yes. What you obtain is a system of ODEs. Depending on the discretization, you might have a form involving mass matrices – `ode15s`, `ode23s`, `ode23t`, and `ode23tb` provide for this. Often the system is stiff. This is to be expected when the PDE is parabolic and when there are phenomena that happen on very different time scales such as a chemical reaction in a fluid flow. In such cases, use one of the four solvers mentioned above. If, as usual, there are many equations, set the `JPattern` property. This is easy and might make the difference between success and failure due to the computation being too expensive. When the system is not stiff, or not very stiff, `ode23` or `ode45` will be more efficient than `ode15s`, `ode23s`, `ode23t`, or `ode23tb`.
Can I solve differential algebraic equation (DAE) systems?	Yes. The solvers `ode15s` and `ode23t` can solve some DAEs of the form $M(t)y' = f(t,y)$ where $M(t)$ is singular (the DAEs must be index 1). For examples, see `amp1dae` and `hb1dae`.

Solving Different Kinds of Systems

Question	Answer
Can I integrate a set of sampled data?	Not directly. You have to represent the data as a function by interpolation or some other scheme for fitting data. The smoothness of this function is critical. A piecewise polynomial fit like a spline can look smooth to the eye, but rough to a solver; the solver will take small steps where the derivatives of the fit have jumps. Either use a smooth function to represent the data or use one of the lower order solvers (`ode23`, `ode23s`, `ode23t`, `ode23tb`) that is less sensitive to this.
Can I solve delay-differential equations?	Not directly. In some cases it is possible to use the initial value problem solvers to solve delay-differential equations by breaking the simulation interval into smaller intervals the length of a single delay. For more information about this approach, see Shampine, L. F., *Numerical Solution of Ordinary Differential Equations*, Chapman & Hall Mathematics, 1994.
What do I do when I have the final and not the initial value?	`ode45` and the other solvers that are available in this version of the MATLAB ODE suite allow you to solve backwards or forwards in time. The syntax for the solvers is `[T,Y] = ode45('ydot',[t0 tfinal],y0);` and the syntax accepts `t0 > tfinal`.

Troubleshooting

The following table provides troubleshooting questions and answers.

Troubleshooting

Question	Answer
The solution doesn't look like what I expected.	If you're right about its appearance, you need to reduce the error tolerances from their default values. A smaller relative error tolerance is needed to compute accurately the solution of problems integrated over "long" intervals, as well as solutions of problems that are moderately unstable. You should check whether there are solution components that stay smaller than their absolute error tolerance for some time. If so, you are not asking for any correct digits in these components. This may be acceptable for these components, but failing to compute them accurately may degrade the accuracy of other components that depend on them.
My plots aren't smooth enough.	Increase the value of `Refine` from its default of `4` in `ode45` and `1` in the other solvers. The bigger the value of `Refine`, the more output points. Execution speed is not affected much by the value of `Refine`.
I'm plotting the solution as it is computed and it looks fine, but the code gets stuck at some point.	First verify that the ODE function is smooth near the point where the code gets stuck. If it isn't, the solver must take small steps to deal with this. It may help to break `tspan` into pieces on which the ODE function is smooth. If the function is smooth and the code is taking extremely small steps, you are probably trying to solve a stiff problem with a solver not intended for this purpose. Switch to `ode15s`, `ode23s`, or `ode23tb`.

Troubleshooting

Question	Answer
My integration proceeds very slowly, using too many time steps.	First, check that your `tspan` is not too long. Remember that the solver will use as many time points as necessary to produce a smooth solution. If the ODE function changes on a time scale that is very short compared to the `tspan`, then the solver will use a lot of time steps. Long-time integration is a hard problem. Break `tspan` into smaller pieces. If the ODE function does not change noticeably on the `tspan` interval, it could be that your problem is stiff. Try using `ode15s` or `ode23s`. Finally, make sure that the ODE function is written in an efficient way. The solvers evaluate the derivatives in the ODE function many times. The cost of numerical integration depends critically on the expense of evaluating the ODE function. Rather than recompute complicated constant parameters every evaluation, store them in globals or calculate them once outside the function and pass them in as additional parameters.
I know that the solution undergoes a radical change at time `t` where `t0 ≤ t ≤ tfinal` but the integrator steps past without “seeing” it.	If you know there is a sharp change at time `t`, it might help to break the `tspan` interval into two pieces, `[t0 t]` and `[t tfinal]`, and call the integrator twice. If the differential equation has periodic coefficients or solution, you might restrict the maximum step size to the length of the period so the integrator won’t step over periods.

Sparse Matrices

MATLAB supports *sparse matrices*, matrices that contain a small proportion of nonzero elements. This characteristic provides advantages in both matrix storage space and computation time.

This chapter explains how to create sparse matrices in MATLAB, and how to use them in both specialized and general mathematical operations.

The sparse matrix functions are located in the `sparfun` directory in the MATLAB `toolbox` directory.

Category	Function	Description
Elementary sparse matrices	`speye`	Sparse identity matrix.
	`sprand`	Sparse uniformly distributed random matrix.
	`sprandn`	Sparse normally distributed random matrix.
	`sprandsym`	Sparse random symmetric matrix.
	`spdiags`	Sparse matrix formed from diagonals.
Full to sparse conversion	`sparse`	Create sparse matrix.
	`full`	Convert sparse matrix to full matrix.
	`find`	Find indices of nonzero elements.
	`spconvert`	Import from sparse matrix external format.
Working with sparse matrices	`nnz`	Number of nonzero matrix elements.
	`nonzeros`	Nonzero matrix elements.
	`nzmax`	Amount of storage allocated for nonzero matrix elements.
	`spones`	Replace nonzero sparse matrix elements with ones.
	`spalloc`	Allocate space for sparse matrix.
	`issparse`	True for sparse matrix.

Category	Function	Description
	`spfun`	Apply function to nonzero matrix elements.
	`spy`	Visualize sparsity pattern.
	`gplot`	Plot graph, as in “graph theory.”
Reordering algorithms	`colmmd`	Column minimum degree permutation.
	`symmmd`	Symmetric minimum degree permutation.
	`symrcm`	Symmetric reverse Cuthill-McKee permutation.
	`colperm`	Column permutation.
	`randperm`	Random permutation.
	`dmperm`	Dulmage-Mendelsohn permutation.
Linear algebra	`eigs`	A few eigenvalues.
	`svds`	A few singular values.
	`luinc`	Incomplete LU factorization.
	`cholinc`	Incomplete Cholesky factorization.
	`normest`	Estimate the matrix 2-norm.
	`condest`	1-norm condition number estimate.
	`sprank`	Structural rank.
Linear equations (iterative methods)	`pcg`	Preconditioned Conjugate Gradients Method.
	`bicg`	BiConjugate Gradients Method.
	`bicgstab`	BiConjugate Gradients Stabilized Method.
	`cgs`	Conjugate Gradients Squared Method.
	`gmres`	Generalized Minimum Residual Method.

Category	Function	Description
	qmr	Quasi-Minimal Residual Method.
Miscellaneous	symbfact	Symbolic factorization analysis.
	spparms	Set parameters for sparse matrix routines.
	spaugment	Form least squares augmented system.

Introduction

Sparse matrices are a special class of matrices that contain a significant number of zero-valued elements. This property allows MATLAB to:

- Store only the nonzero elements of the matrix, together with their indices.
- Reduce computation time by eliminating operations on zero elements.

Sparse Matrix Storage

For full matrices, MATLAB stores internally every matrix element. Zero-valued elements require the same amount of storage space as any other matrix element. For sparse matrices, however, MATLAB stores only the nonzero elements and their indices. For large matrices with a high percentage of zero-valued elements, this scheme significantly reduces the amount of memory required for data storage.

MATLAB uses three arrays internally to store sparse matrices with real elements. Consider an *m*-by-*n* sparse matrix with nnz nonzero entries:

- The first array contains all the nonzero elements of the array in floating-point format. The length of this array is equal to nnz.
- The second array contains the corresponding integer row indices for the nonzero elements. This array also has length equal to nnz.
- The third array contains integer pointers to the start of each column. This array has length equal to n.

This matrix requires storage for nnz floating-point numbers and nnz+n integers. At 8 bytes per floating-point number and 4 bytes per integer, the total number of bytes required to store a sparse matrix is

```
8*nnz + 4*(nnz+n)
```

Sparse matrices with complex elements are also possible. In this case, MATLAB uses a fourth array with nnz elements to store the imaginary parts of the nonzero elements. An element is considered nonzero if either its real or imaginary part is nonzero.

Creating Sparse Matrices

MATLAB never creates sparse matrices automatically. Instead, you must determine if a matrix contains a large enough percentage of zeros to benefit from sparse techniques.

The *density* of a matrix is the number of non-zero elements divided by the total number of matrix elements. Matrices with very low density are often good candidates for use of the sparse format.

Converting Full to Sparse

You can convert a full matrix to sparse storage using the sparse function with a single argument.

```
S = sparse(A)
```

For example

```
A = [ 0   0   0   5
      0   2   0   0
      1   3   0   0
      0   0   4   0];

S = sparse(A)
```

produces

```
 S =

   (3,1)        1
   (2,2)        2
   (3,2)        3
   (4,3)        4
   (1,4)        5
```

The printed output lists the nonzero elements of S, together with their row and column indices. The elements are sorted by columns, reflecting the internal data structure.

You can convert a sparse matrix to full storage using the full function, provided the matrix order is not too large. For example A = full(S) reverses the example conversion.

Converting a full matrix to sparse storage is not the most frequent way of generating sparse matrices. If the order of a matrix is small enough that full storage is possible, then conversion to sparse storage rarely offers significant savings.

Creating Sparse Matrices Directly

You can create a sparse matrix from a list of nonzero elements using the `sparse` function with five arguments.

```
S = sparse(i,j,s,m,n)
```

`i` and `j` are vectors of row and column indices, respectively, for the nonzero elements of the matrix. `s` is a vector of nonzero values whose indices are specified by the corresponding `(i,j)` pairs. `m` is the row dimension for the resulting matrix, and `n` is the column dimension.

The matrix `S` of the previous example can be generated directly with

```
S = sparse([3 2 3 4 1],[1 2 2 3 4],[1 2 3 4 5],4,4)

S =

   (3,1)        1
   (2,2)        2
   (3,2)        3
   (4,3)        4
   (1,4)        5
```

The `sparse` command has a number of alternate forms. The example above uses a form that sets the maximum number of nonzero elements in the matrix to `length(s)`. If desired, you can append a sixth argument that specifies a larger maximum, allowing you to add nonzero elements later without changing storage requirements.

Example: The Second Difference Operator

The matrix representation of the second difference operator is a good example of a sparse matrix. It is a tridiagonal matrix with –2s on the diagonal and 1s on the super- and subdiagonal. There are many ways to generate it – here's one possibility.

```
D = sparse(1:n,1:n,–2*ones(1,n),n,n);
E = sparse(2:n,1:n–1,ones(1,n–1),n,n);
S = E+D+E'
```

For `n = 5`, MATLAB responds with

```
S =

   (1,1)       –2
   (2,1)        1
   (1,2)        1
   (2,2)       –2
   (3,2)        1
   (2,3)        1
   (3,3)       –2
   (4,3)        1
   (3,4)        1
   (4,4)       –2
   (5,4)        1
   (4,5)        1
   (5,5)       –2
```

Now `F = full(S)` displays the corresponding full matrix.

```
F = full(S)

F =

    –2     1     0     0     0
     1    –2     1     0     0
     0     1    –2     1     0
     0     0     1    –2     1
     0     0     0     1    –2
```

Creating Sparse Matrices from Their Diagonal Elements

Creating sparse matrices based on their diagonal elements is a common operation, so the function `spdiags` handles this task. Its syntax is

```
S = spdiags(B,d,m,n)
```

To create an output matrix `S` of size *m*-by-*n* with elements on `p` diagonals:

- `B` is a matrix of size `min(`*m*`,`*n*`)`-by-*p*. The columns of `B` are the values to populate the diagonals of `S`.
- `d` is a vector of length `p` whose integer elements specify which diagonals of `S` to populate.

That is, the elements in column `j` of `B` fill the diagonal specified by element `j` of `d`. As an example, consider the matrix `B` and the vector `d`.

```
B =

    41    11     0
    52    22     0
    63    33    13
    74    44    24

d =

    -3
     0
     2
```

Use these matrices to create a 7-by-4 sparse matrix `A`.

```
A = spdiags(B,d,7,4)

A =

   (1,1)       11
   (4,1)       41
   (2,2)       22
   (5,2)       52
   (1,3)       13
   (3,3)       33
   (6,3)       63
   (2,4)       24
   (4,4)       44
   (7,4)       74
```

In its full form, A looks like this.

```
full(A)

ans =

    11     0    13     0
     0    22     0    24
     0     0    33     0
    41     0     0    44
     0    52     0     0
     0     0    63     0
     0     0     0    74
```

spdiags can also extract diagonal elements from a sparse matrix, or replace matrix diagonal elements with new values. Type help spdiags for details.

Importing Sparse Matrices from Outside MATLAB

You can import sparse matrices from computations outside MATLAB. Use the spconvert function in conjunction with the load command to import text files containing lists of indices and nonzero elements. For example, consider a three-column text file T.dat whose first column is a list of row indices, second column is a list of column indices, and third column is a list of nonzero values. These statements load T.dat into MATLAB and convert it into a sparse matrix S:

```
load T.dat
S = spconvert(T)
```

The save and load commands can also process sparse matrices stored as binary data in MAT-files. Finally, a Fortran utility routine hbo2mat is available to convert a file containing a sparse matrix in the Harwell-Boeing format into a MAT-file that load can process. The Harwell-Boeing data is available through anonymous ftp or the World Wide Web from ftp.mathworks.com in the directory pub/mathworks/toolbox/matlab/sparfun.

Viewing Sparse Matrices

MATLAB provides a number of functions that let you get quantitative or graphical information about sparse matrices.

General Storage Information

The whos command provides high-level information about matrix storage, including size and storage class. For example, this whos listing shows information about sparse and full versions of the same matrix.

```
whos
  Name            Size         Bytes  Class

  M_full      1100x1100      9680000  double array
  M_sparse    1100x1100         4404  sparse array

Grand total is 1210000 elements using 9684404 bytes
```

Notice that the number of bytes used is much less in the sparse case, because zero-valued elements are not stored. In this case, the density of the sparse matrix is 4404/9680000, or approximately .00045%.

Information About Nonzero Elements

There are several commands that provide high-level information about the nonzero elements of a sparse matrix:

- nnz returns the number of nonzero elements in a sparse matrix.
- nonzeros returns a column vector containing all the nonzero elements of a sparse matrix.
- nzmax returns the amount of storage space allocated for the nonzero entries of a sparse matrix.

To try some of these, load the supplied sparse matrix west0479, one of the Harwell-Boeing collection.

```
load west0479
whos
  Name            Size          Bytes  Class

  west0479     479x479         24576  sparse array
```

This matrix models an eight-stage chemical distillation column.

Try these commands.

```
nnz(west0479)

ans =

        1887

format short e
west0479

west0479 =

  (25,1)      1.0000e+00
  (31,1)     -3.7648e-02
  (87,1)     -3.4424e-01
  (26,2)      1.0000e+00
  (31,2)     -2.4523e-02
  (88,2)     -3.7371e-01
  (27,3)      1.0000e+00
  (31,3)     -3.6613e-02
  (89,3)     -8.3694e-01
  (28,4)      1.3000e+02
    .
    .
    .

nonzeros(west0479);
```

```
ans =

   1.0000e+00
  –3.7648e–02
  –3.4424e–01
   1.0000e+00
  –2.4523e–02
  –3.7371e–01
   1.0000e+00
  –3.6613e–02
  –8.3694e–01
   1.3000e+02
      .
      .
      .
```

Note Use **Ctrl-C** to stop the nonzeros listing at any time.

Note that initially nnz has the same value as nzmax by default. That is, the number of nonzero elements is equivalent to the number of storage locations allocated for nonzeros. However, MATLAB does not dynamically release memory if you zero out additional array elements. Changing the value of some matrix elements to zero changes the value of nnz, but not that of nzmax.

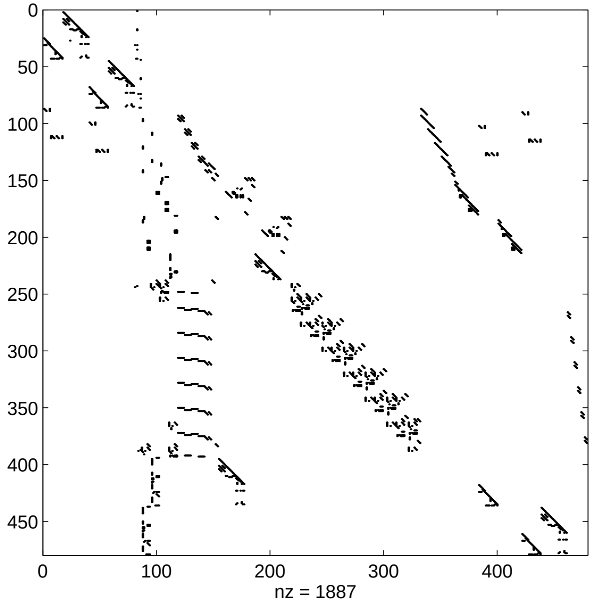
You can add as many nonzero elements to the matrix as desired, however; you are not constrained by the original value of nzmax.

Viewing Sparse Matrices Graphically

It is often useful to use a graphical format to view the distribution of the nonzero elements within a sparse matrix. MATLAB's spy function produces a template view of the sparsity structure, where each point on the graph represents the location of a nonzero array element.

For example,

```
spy(west0479)
```



The find Function and Sparse Matrices

For any matrix, full or sparse, the `find` function returns the indices and values of nonzero elements. Its syntax is:

```
[i,j,s] = find(S)
```

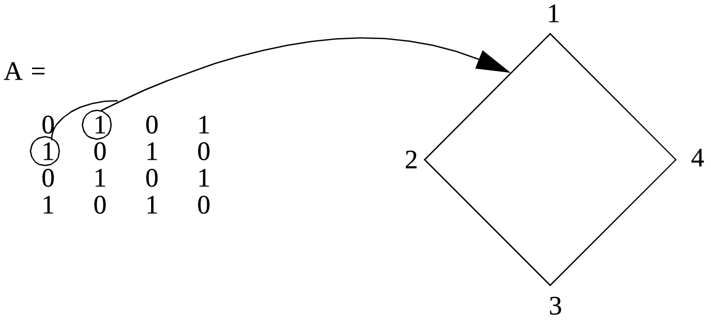
`find` returns the row indices of nonzero values in vector `i`, the column indices in vector `j`, and the nonzero values themselves in the vector `s`. The example below uses `find` to locate the indices and values of the nonzeros in a sparse matrix. The `sparse` function uses the `find` output, together with the size of the matrix, to recreate the matrix.

```
[i,j,s] = find(S)
[m,n] = size(S)
S = sparse(i,j,s,m,n)
```

Example: Adjacency Matrices and Graphs

The formal mathematical definition of a *graph* is a set of points, or nodes, with specified connections between them. An economic model, for example, is a graph with different industries as the nodes and direct economic ties as the connections. The computer software industry is connected to the computer hardware industry, which, in turn, is connected to the semiconductor industry, and so on.

This definition of a graph lends itself to matrix representation. The *adjacency matrix* of an *undirected* graph is a matrix whose `(i,j)`-th and `(j,i)`-th entries are 1 if node `i` is connected to node `j`, and 0 otherwise. For example, the adjacency matrix for a diamond-shaped graph looks like



Since most graphs have relatively few connections per node, most adjacency matrices are sparse. The actual locations of the nonzero elements depend on how the nodes are numbered. A change in the numbering leads to permutation of the rows and columns of the adjacency matrix, which can have a significant effect on both the time and storage requirements for sparse matrix computations.

Graphing Using Adjacency Matrices

MATLAB's `gplot` function creates a graph based on an adjacency matrix and a related array of coordinates. To try `gplot`, create the adjacency matrix shown above by entering

```
A = [0 1 0 1; 1 0 1 0; 0 1 0 1; 1 0 1 0];
```

The columns of gplot's coordinate array contain the Cartesian coordinates for the corresponding node. For the diamond example, create the array by entering

```
xy = [1 3; 2 1; 3 3; 2 5];
```

This places the first node at location (1,3), the second at location (2,1), the third at location (3,3), and the fourth at location (2,5). To view the resulting graph, enter

```
gplot(A,xy)
```

The Bucky Ball

One interesting construction for graph analysis is the *Bucky ball*. This is composed of 60 points distributed on the surface of a sphere in such a way that the distance from any point to its nearest neighbors is the same for all the points. Each point has exactly three neighbors. The Bucky ball models four different physical objects:

- The geodesic dome popularized by Buckminster Fuller
- The C_{60} molecule, a form of pure carbon with 60 atoms in a nearly spherical configuration
- In geometry, the truncated icosahedron
- In sports, the seams in a soccer ball

The Bucky ball adjacency matrix is a 60-by-60 symmetric matrix B. B has three nonzero elements in each row and column, for a total of 180 nonzero values. This matrix has important applications related to the physical objects listed earlier. For example, the eigenvalues of B are involved in studying the chemical properties of C_{60}.

To obtain the Bucky ball adjacency matrix, enter

```
B = bucky;
```

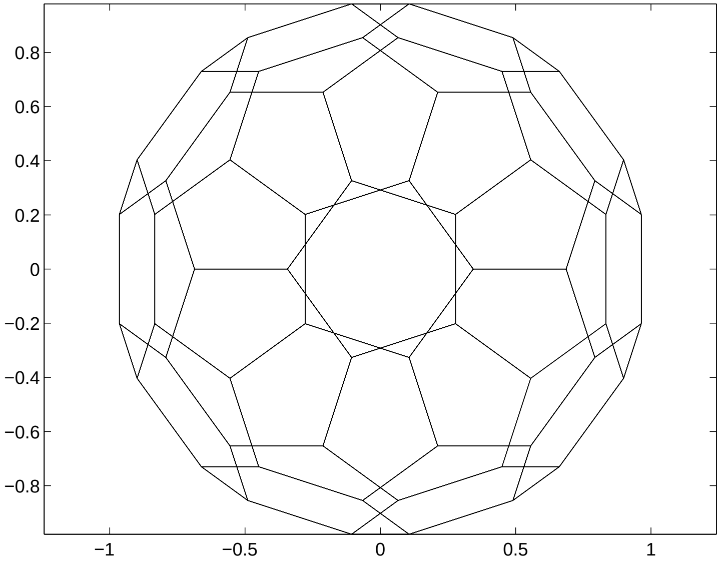
At order 60, and with a density of 5%, this matrix does not require sparse techniques, but it does provide an interesting example.

You can also obtain the coordinates of the Bucky ball graph using

```
[B,v] = bucky;
```

This statement generates `v`, a list of *xyz*-coordinates of the 60 points in 3-space equidistributed on the unit sphere. The function `gplot` uses these points to plot the Bucky ball graph.

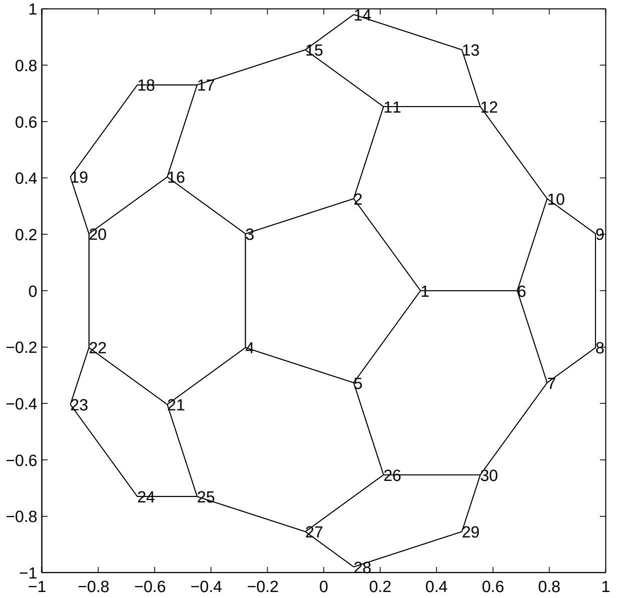
```
gplot(B,v)
axis equal
```



It is not obvious how to number the nodes in the Bucky ball so that the resulting adjacency matrix reflects the spherical and combinatorial symmetries of the graph. The numbering used by `bucky.m` is based on the pentagons inherent in the ball's structure.

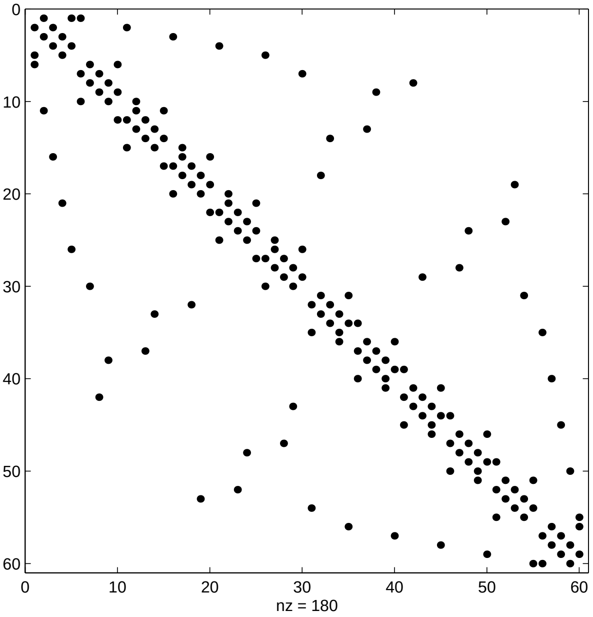
The vertices of one pentagon are numbered 1 through 5, the vertices of an adjacent pentagon are numbered 6 through 10, and so on. The picture on the following page shows the numbering of half of the nodes (one hemisphere); the numbering of the other hemisphere is obtained by a reflection about the equator. Use gplot to produce a graph showing half the nodes. You can add the node numbers using a for loop.

```
k = 1:30;
gplot(B(k,k),v);
axis square
for j = 1:30, text(v(j,1),v(j,2), int2str(j)); end
```



To view a template of the nonzero locations in the Bucky ball's adjacency matrix, use the spy function:

```
spy(B)
```

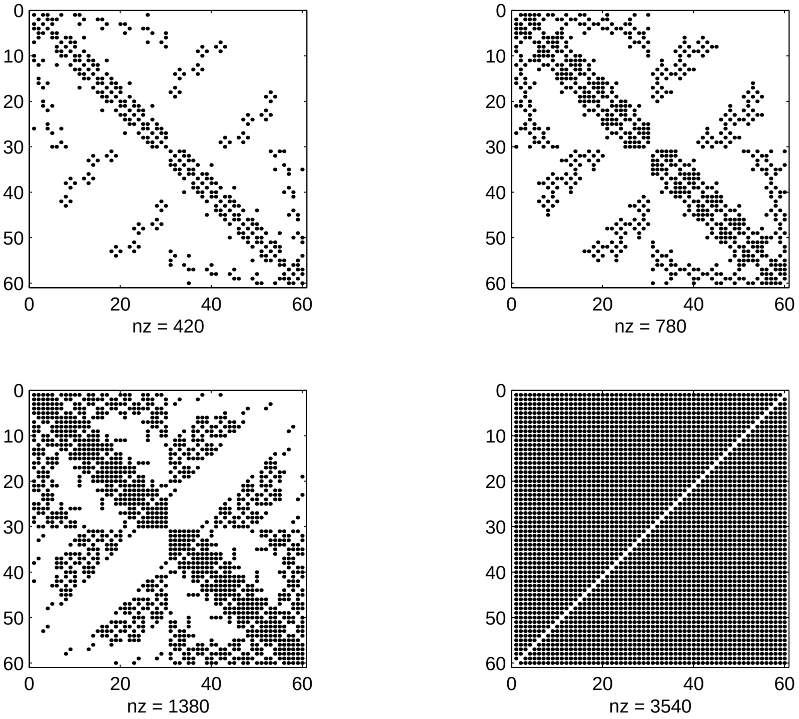


The node numbering that this model uses generates a spy plot with twelve groups of five elements, corresponding to the twelve pentagons in the structure. Each node is connected to two other nodes within its pentagon and one node in some other pentagon. Since the nodes within each pentagon have consecutive numbers, most of the elements in the first super- and sub-diagonals of B are nonzero. In addition, the symmetry of the numbering about the equator is apparent in the symmetry of the spy plot about the antidiagonal.

Graphs and Characteristics of Sparse Matrices

Spy plots of the matrix powers of `B` illustrate two important concepts related to sparse matrix operations, fill-in and distance. spy plots help illustrate these concepts.

```
spy(B^2)
spy(B^3)
spy(B^4)
spy(B^8)
```

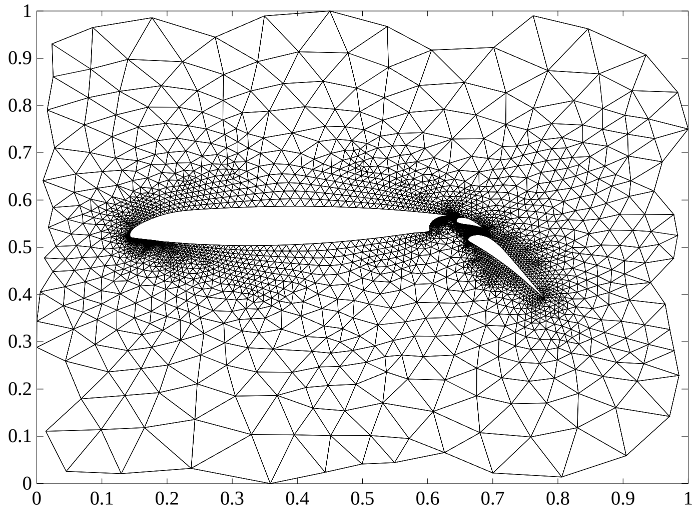


Fill-in is generated by operations like matrix multiplication. The product of two or more matrices usually has more nonzero entries than the individual terms, and so requires more storage. As `p` increases, `B^p` fills in and `spy(B^p)` gets more dense.

The *distance* between two nodes in a graph is the number of steps on the graph necessary to get from one node to the other. The spy plot of the p-th power of B shows the nodes that are a distance p apart. As p increases, it is possible to get to more and more nodes in p steps. For the Bucky ball, B^8 is almost completely full. Only the antidiagonal is zero, indicating that it is possible to get from any node to any other node, except the one directly opposite it on the sphere, in eight steps.

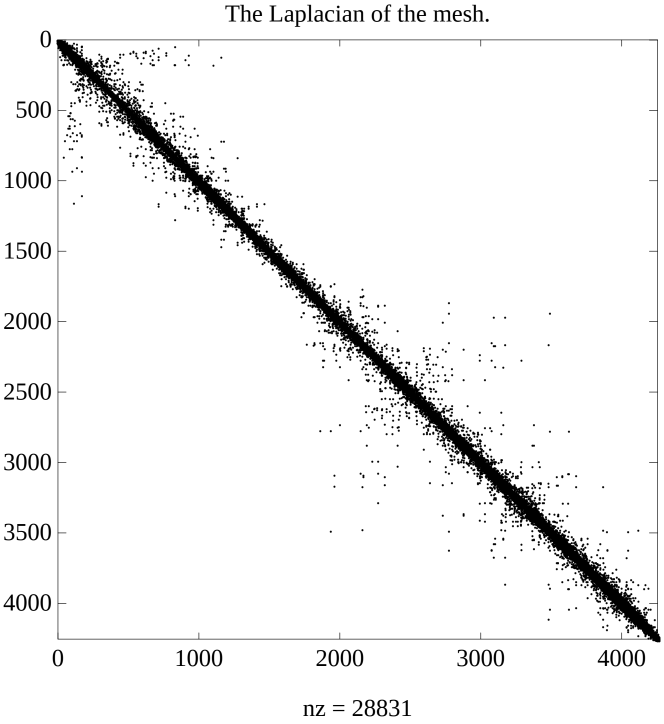
An Airflow Model

A calculation performed at NASA's Research Institute for Applications of Computer Science involves modeling the flow over an airplane wing with two trailing flaps.



In a two-dimensional model, a triangular grid surrounds a cross section of the wing and flaps. The partial differential equations are nonlinear and involve several unknowns, including hydrodynamic pressure and two components of velocity. Each step of the nonlinear iteration requires the solution of a sparse linear system of equations. Since both the connectivity and the geometric location of the grid points are known, the gplot function can produce the graph shown above.

In this example, there are 4253 grid points, each of which is connected to between 3 and 9 others, for a total of 28831 nonzeros in the matrix, and a density equal to 0.0016. This spy plot shows that the node numbering yields a definite band structure.



Sparse Matrix Operations

Most of MATLAB's standard mathematical functions work on sparse matrices just as they do on full matrices. In addition, MATLAB provides a number of functions that perform operations specific to sparse matrices. This section discusses:

- Computational Considerations
- Standard Mathematical Operations
- Permutation and Reordering
- Factorization
- Simultaneous Linear Equations
- Eigenvalues and Singular Values

Computational Considerations

The computational complexity of sparse operations is proportional to nnz, the number of nonzero elements in the matrix. Computational complexity also depends linearly on the row size m and column size n of the matrix, but is independent of the product m*n, the total number of zero and nonzero elements.

The complexity of fairly complicated operations, such as the solution of sparse linear equations, involves factors like ordering and fill-in, which are discussed in the previous section. In general, however, the computer time required for a sparse matrix operation is proportional to the number of arithmetic operations on nonzero quantities. This is the "time is proportional to flops" rule.

Standard Mathematical Operations

Sparse matrices propagate through computations according to these rules:

- Functions that accept a matrix and return a scalar or vector always produce output in full storage format. For example, the size function always returns a full vector, whether its input is full or sparse.
- Functions that accept scalars or vectors and return matrices, such as zeros, ones, rand, and eye, always return full results. This is necessary to avoid introducing sparsity unexpectedly. The sparse analog of zeros(m,n) is simply sparse(m,n). The sparse analogs of rand and eye are sprand and speye, respectively. There is no sparse analog for the function ones.

- Unary functions that accept a matrix and return a matrix or vector preserve the storage class of the operand. If S is a sparse matrix, then chol(S) is also a sparse matrix, and diag(S) is a sparse vector. Columnwise functions such as max and sum also return sparse vectors, even though these vectors may be entirely nonzero. Important exceptions to this rule are the sparse and full functions.
- Binary operators yield sparse results if both operands are sparse, and full results if both are full. For mixed operands, the result is full unless the operation preserves sparsity. If S is sparse and F is full, then S+F, S*F, and F\S are full, while S.*F and S&F are sparse. In some cases, the result might be sparse even though the matrix has few zero elements.
- Matrix concatenation using either the cat function or square brackets produces sparse results for mixed operands.
- Submatrix indexing on the right side of an assignment preserves the storage format of the operand. T = S(i,j) produces a sparse result if S is sparse whether i and j are scalars or vectors. Submatrix indexing on the left, as in T(i,j) = S, does not change the storage format of the matrix on the left.

Permutation and Reordering

A permutation of the rows and columns of a sparse matrix S can be represented in two ways:

- A permutation matrix P acts on the rows of S as P*S or on the columns as S*P′.
- A permutation vector p, which is a full vector containing a permutation of 1:n, acts on the rows of S as S(p,:), or on the columns as S(:,p).

For example, the statements

```
p = [1 3 4 2 5]
I = eye(5,5);
P = I(p,:);
e = ones(4,1);
S = diag(11:11:55) + diag(e,1) + diag(e,–1)
```

produce

```
p =

     1     3     4     2     5

P =

     1     0     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     1     0     0     0
     0     0     0     0     1

S =

    11     1     0     0     0
     1    22     1     0     0
     0     1    33     1     0
     0     0     1    44     1
     0     0     0     1    55
```

You can now try some permutations using the permutation vector p and the permutation matrix P. For example, the statements S(p,:) and P*S produce

```
ans =

    11     1     0     0     0
     0     1    33     1     0
     0     0     1    44     1
     1    22     1     0     0
     0     0     0     1    55
```

Similarly, S(:,p) and S*P' produce

```
ans =

    11     0     0     1     0
     1     1     0    22     0
     0    33     1     1     0
     0     1    44     0     1
     0     0     1     0    55
```

If P is a sparse matrix, then both representations use storage proportional to n and you can apply either to S in time proportional to nnz(S). The vector representation is slightly more compact and efficient, so the various sparse matrix permutation routines all return full row vectors with the exception of the pivoting permutation in LU (triangular) factorization, which returns a matrix compatible with earlier versions of MATLAB.

To convert between the two representations, let I = speye(n) be an identity matrix of the appropriate size. Then,

```
P = I(p.:)
P’ = I(:,p).
p = (1:n)*P’
p = (P*(1:n)’)’
```

The inverse of P is simply R = P’. You can compute the inverse of p with r(p) = 1:n.

```
r(p) = 1:5

r =

     1     4     2     3     5
```

Reordering for Sparsity

Reordering the columns of a matrix can often make its Cholesky, LU, or QR factors sparser. The simplest such reordering is to sort the columns by nonzero count. This is sometimes a good reordering for matrices with very irregular structures, especially if there is great variation in the nonzero counts of rows or columns.

The function p = colperm(S) computes this column-count permutation. The colperm M-file has only a single line.

```
[ignore,p] = sort(full(sum(spones(S))));
```

This line performs these steps:

1. The inner call to spones creates a sparse matrix with ones at the location of every nonzero element in S.

2. The sum function sums down the columns of the matrix, producing a vector that contains the count of nonzeros in each column.

3 `full` converts this vector to full storage format.

4 `sort` sorts the values in ascending order. The second output argument from `sort` is the permutation that sorts this vector.

Reordering to Reduce Bandwidth

The reverse Cuthill-McKee ordering is intended to reduce the profile or bandwidth of the matrix. It is not guaranteed to find the smallest possible bandwidth, but it usually does. The function `symrcm(A)` actually operates on the nonzero structure of the symmetric matrix `A + A'`, but the result is also useful for asymmetric matrices. This ordering is useful for matrices that come from one-dimensional problems or problems that are in some sense "long and thin."

Minimum Degree Ordering

The degree of a node in a graph is the number of connections to that node, which is the same as the number of nonzero elements in the corresponding row of the adjacency matrix. The minimum degree algorithm generates an ordering based on how these degrees are altered during Gaussian elimination. It is a complicated and powerful algorithm that usually leads to sparser factors than most other orderings, including column count and reverse Cuthill-McKee. MATLAB has two versions, `symmd` for symmetric matrices and `colmmd` for nonsymmetric matrices. You can change various parameters associated with details of the algorithm using the `spparms` function.

For more details on the algorithm and MATLAB's version of it, see Gilbert, John R., Cleve Moler, and Robert Schreiber, "Sparse Matrices in MATLAB: Design and Implementation," *SIAM J. Matrix Anal. Appl.*, Vol. 13, No. 1. January 1992: pp. 333-356.

Factorization

This section discusses four important factorization techniques for sparse matrices:

- LU, or triangular, factorization
- Cholesky factorization
- QR, or orthogonal factorization
- Incomplete factorizations

LU Factorization

If S is a sparse matrix, the statement below returns three sparse matrices L, U, and P such that P*S = L*U.

```
[L,U,P] = lu(S)
```

lu obtains the factors by Gaussian elimination with partial pivoting. The permutation matrix P has only n nonzero elements. As with dense matrices, the statement [L,U] = lu(S) returns a permuted unit lower triangular matrix and an upper triangular matrix whose product is S. By itself, lu(S) returns L and U in a single matrix without the pivot information.

The sparse LU factorization does not pivot for sparsity, but it does pivot for numerical stability. In fact, both the sparse factorization (line 1) and the full factorization (line 2) below produce the same L and U, even though the time and storage requirements might differ greatly:

```
[L,U] = lu(S)                % sparse factorization

[L,U] = sparse(lu(full(S)))  % full factorization
```

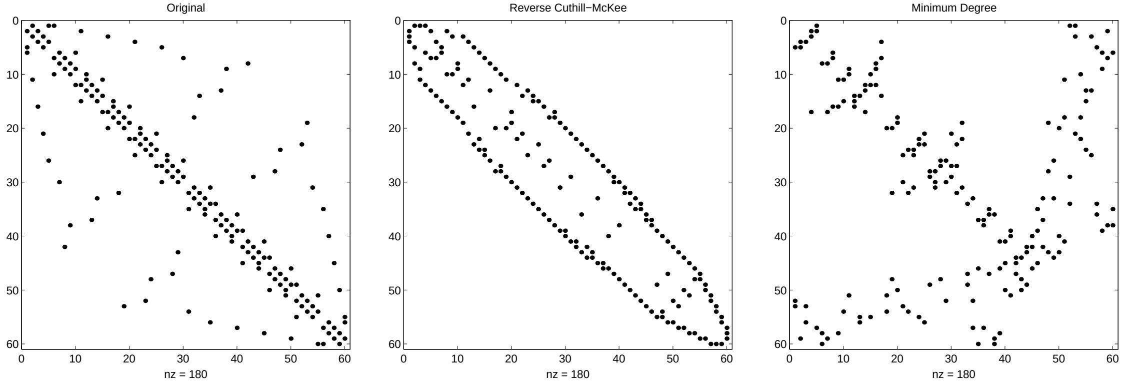
MATLAB automatically allocates the memory necessary to hold the sparse L and U factors during the factorization. MATLAB does not use any symbolic LU prefactorization to determine the memory requirements and set up the data structures in advance.

Reordering and factorization. If you obtain a good column permutation p that reduces fill-in, perhaps from symrcm or colmmd, then computing lu(S(:,p)) will take less time and storage than computing lu(S). Two permutations are the symmetric reverse Cuthill-McKee ordering and the symmetric minimum degree ordering.

```
r = symrcm(B);
m = symmmd(B);
```

The three spy plots produced by the lines below show the three adjacency matrices of the Bucky Ball graph with these three different numberings. The local, pentagon-based structure of the original numbering is not evident in the other three.

```
spy(B)
spy(B(r,r))
spy(B(m,m))
```



The reverse Cuthill-McGee ordering, `r`, reduces the bandwidth and concentrates all the nonzero elements near the diagonal. The minimum degree ordering, `m`, produces a fractal-like structure with large blocks of zeros.

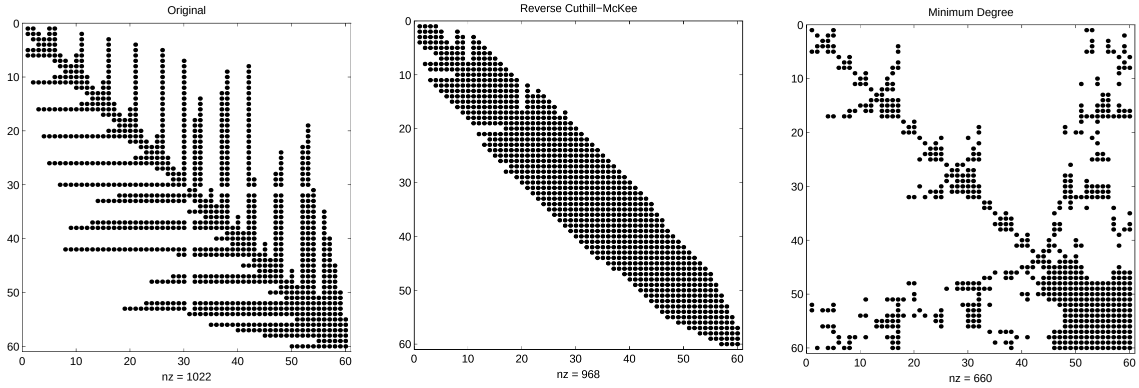
To see the fill-in generated in the LU factorization of the Bucky ball, use `speye(n,n)`, the sparse identity matrix, to insert -3s on the diagonal of `B`.

```
B = B - 3*speye(n,n)
```

Since each row sum is now zero, this new `B` is actually singular, but it is still instructive to compute its LU factorization. When called with only one output argument, `lu` returns the two triangular factors, `L` and `U`, in a single sparse matrix. The number of nonzeros in that matrix is a measure of the time and storage required to solve linear systems involving `B`. Here are the nonzero counts for the three permutations being considered.

Original	`lu(B)`	1022
Reverse Cuthill-McKee	`lu(B(r,r))`	968
Minimum degree	`lu(B(m,m))`	660

Even though this is a small example, the results are typical. The original numbering scheme leads to the most fill-in. The fill-in for the reverse Cuthill-McKee ordering is concentrated within the band, but it is almost as extensive as the first two orderings. For the minimum degree ordering, the relatively large blocks of zeros are preserved during the elimination and the amount of fill-in is significantly less than that generated by the other orderings. The spy plots below reflect the characteristics of each reordering.



Cholesky Factorization

If `S` is a symmetric (or Hermitian), positive definite, sparse matrix, the statement below returns a sparse, upper triangular matrix `R` so that `R'*R = S`.

```
R = chol(S)
```

`chol` does not automatically pivot for sparsity, but you can compute minimum degree and profile limiting permutations for use with `chol(S(p,p))`.

Since the Cholesky algorithm does not use pivoting for sparsity and does not require pivoting for numerical stability, it is possible to do a quick calculation of the amount of memory required and allocate all the memory at the start of the factorization.

QR Factorization

MATLAB will compute the complete QR factorization of a sparse matrix `S` with

```
[Q,R] = qr(S)
```

but this is usually impractical. The orthogonal matrix `Q` often fails to have a high proportion of zero elements. A more practical alternative, sometimes known as “the Q-less QR factorization,” is available.

With one sparse input argument and one output argument

```
R = qr(S)
```

returns just the upper triangular portion of the QR factorization. The matrix `R` provides a Cholesky factorization for the matrix associated with the normal equations,

```
R'*R = S'*S
```

However, the loss of numerical information inherent in the computation of `S'*S` is avoided.

With two input arguments having the same number of rows, and two output arguments, the statement

```
[C,R] = qr(S,B)
```

applies the orthogonal transformations to `B`, producing `C = Q'*B` without computing `Q`.

The Q-less QR factorization allows the solution of sparse least squares problems

$$\text{minimize}\|Ax - b\|$$

with two steps

```
[c,R] = qr(A,b)
x = R\c
```

If `A` is sparse, but not square, MATLAB uses these steps for the linear equation solving backslash operator

```
x = A\b
```

Or, you can do the factorization yourself and examine `R` for rank deficiency.

It is also possible to solve a sequence of least squares linear systems with different right-hand sides, `b`, that are not necessarily known when `R = qr(A)` is computed. The approach solves the “semi-normal equations”

```
R'*R*x = A'*b
```

with

```
x = R\(R'\(A'*b))
```

and then employs one step of iterative refinement to reduce roundoff error

```
r = b – A*x
e = R\(R'\(A'*r))
x = x + e
```

Incomplete Factorizations

The `luinc` and `cholinc` functions provide approximate, *incomplete* factorizations, which are useful as preconditioners for sparse iterative methods.

The `luinc` function produces two different kinds of incomplete LU factorizations, one involving a drop tolerance and one involving fill-in level. If `A` is a sparse matrix, and `tol` is a small tolerance, then

```
[L,U] = luinc(A,tol)
```

computes an approximate LU factorization where all elements less than `tol` times the norm of the relevant column are set to zero. Alternatively,

```
[L,U] = luinc(A,'0')
```

computes an approximate LU factorization where the sparsity pattern of `L+U` is a permutation of the sparsity pattern of `A`.

For example,

```
load west0479
A = west0479;
nnz(A)
nnz(lu(A))
nnz(luinc(A,1e–6))
nnz(luinc(A,'0'))
```

shows that `A` has 1887 nonzeros, its complete LU factorization has 16777 nonzeros, its incomplete LU factorization with a drop tolerance of 1e–6 has 10311 nonzeros, and its `lu(’0’)` factorization has 1886 nonzeros.

The `luinc` function has a few other options. See the online help for details.

The `cholinc` function provides drop tolerance and level 0 fill-in Cholesky factorizations of symmetric, positive definite sparse matrices. See the online help for more information.

Simultaneous Linear Equations

Systems of simultaneous linear equations can be solved by two different classes of methods:

- Direct methods. These are usually variants of Gaussian elimination and are often expressed as matrix factorizations such as LU or Cholesky factorization. The algorithms involve access to the individual matrix elements.
- Iterative methods. Only an approximate solution is produced after a finite number of steps. The coefficient matrix is involved only indirectly, through a matrix-vector product or as the result of an abstract linear operator.

Direct Methods

Direct methods are usually faster and more generally applicable, if there is enough storage available to carry them out. Iterative methods are usually applicable to restricted cases of equations and depend upon properties like diagonal dominance or the existence of an underlying differential operator. Direct methods are implemented in the core of MATLAB and are made as efficient as possible for general classes of matrices. Iterative methods are usually implemented in MATLAB M-files and may make use of the direct solution of subproblems or preconditioners.

The usual way to access direct methods in MATLAB is not through the `lu` or `chol` functions, but rather with the matrix division operators `/` and `\`. If `A` is square, the result of `X = A\B` is the solution to the linear system `A*X = B`. If `A` is not square, then a least squares solution is computed.

If `A` is a square, full, or sparse matrix, then `A\B` has the same storage class as `B`. Its computation involves a choice among several algorithms:

- If `A` is triangular, perform a triangular solve for each column of `B`.
- If `A` is a permutation of a triangular matrix, permute it and perform a sparse triangular solve for each column of `B`.
- If A is symmetric or Hermitian and has positive real diagonal elements, find a symmetric minimum degree order `p` and attempt to compute the Cholesky factorization of `A(p,p)`. If successful, finish with two sparse triangular solves for each column of `B`.
- Otherwise (if `A` is not triangular, or is not Hermitian with positive diagonal, or if Cholesky factorization fails), find a column minimum degree order `p`.

Compute the LU factorization with partial pivoting of `A(:,p)`, and perform two triangular solves for each column of `B`.

For a square matrix, MATLAB tries these possibilities in order of increasing cost. The tests for triangularity and symmetry are relatively fast and, if successful, allow for faster computation and more efficient memory usage than the general purpose method.

For example, consider the sequence below.

```
[L,U] = lu(A);
y = L\b;
x = U\y;
```

In this case, MATLAB uses triangular solves for both matrix divisions, since `L` is a permutation of a triangular matrix and `U` is triangular.

Use the function `spparms` to turn off the minimum degree preordering if a better preorder is known for a particular matrix.

Iterative Methods

Six functions are available that implement iterative methods for sparse systems of simultaneous linear systems.

Function	Description
`bicg`	Biconjugate gradient.
`bicgstab`	Biconjugate gradient stabilized.
`cgs`	Conjugate gradient squared.
`gmres`	Generalized minimum residual.
`pcg`	Preconditioned conjugate gradient.
`qmr`	Quasiminimal residual.

All six methods are designed to solve $Ax = b$. The preconditioned conjugate gradient method, `pcg`, is restricted to symmetric, positive definite matrix A. The other five can handle nonsymmetric, square matrices.

All six methods can make use of left and right preconditioners. The linear system

$$Ax = b$$

is replaced by the equivalent system

$$M_1^{-1}AM_2^{-1}M_2x = M_1^{-1}b$$

The preconditioners M_1 and M_2 are chosen to accelerate convergence of the iterative method. In many cases, the preconditioners occur naturally in the mathematical model. A partial differential equation with variable coefficients may be approximated by one with constant coefficients, for example. Incomplete matrix factorizations may be used in the absence of natural preconditioners.

The five-point finite difference approximation to Laplace's equation on a square, two-dimensional domain provides an example. The following statements use the preconditioned conjugate gradient method with an incomplete Cholesky factorization as a preconditioner.

```
A = delsq(numgrid(’S’,50));
b = ones(size(A,1),1);
tol = 1.e–3;
maxit = 10;
R = cholinc(A,tol);
[x,flag,err,iter,res] = pcg(A,b,tol,maxit,R',R);
```

Only four iterations are required to achieve the prescribed accuracy.

Background information on these iterative methods and incomplete factorizations is available in:

Saad, Yousef. *Iterative Methods for Sparse Linear Equations*. PWS Publishing Company: 1996.

Barrett, Richard et al. *Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods*. Society for Industrial and Applied Mathematics: 1994.

Eigenvalues and Singular Values

Two functions are available which compute a few specified eigenvalues or singular values.

Function	Description
`eigs`	Few eigenvalues.
`svds`	Few singular values.

These functions are most frequently used with sparse matrices, but they can be used with full matrices or even with linear operators defined by M-files.

The statement

```
[V,lambda] = eigs(A,k,sigma)
```

finds the `k` eigenvalues and corresponding eigenvectors of the matrix `A` which are nearest the “shift” `sigma`. If `sigma` is omitted, the eigenvalues largest in magnitude are found. If `sigma` is zero, the eigenvalues smallest in magnitude are found. A second matrix, `B`, may be included for the generalized eigenvalue problem

$$Av = \lambda Bv$$

The statement

```
[U,S,V] = svds(A,k)
```

finds the `k` largest singular values of `A` and

```
[U,S,V] = svds(A,k,0)
```

finds the `k` smallest singular values.

For example, the statements

```
L = numgrid('L',65);
A = delsq(L);
```

set up the five-point Laplacian difference operator on a 65-by-65 grid in an L-shaped, two-dimensional domain. The statements

```
size(A)
nnz(A)
```

show that A is a matrix of order 2945 with 14,473 nonzero elements.

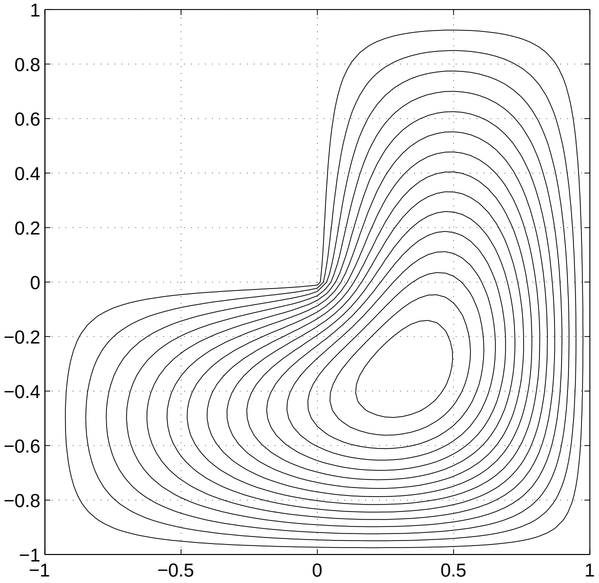
The statement

```
[v,d] = eigs(A,1,0);
```

computes the smallest eigenvalue and eigenvector. Finally,

```
L(L>0) = full(v(L(L>0)));
x = -1:1/32:1;
contour(x,x,L,15)
axis square
```

distributes the components of the eigenvector over the appropriate grid points and produces a contour plot of the result.



The numerical techniques used in eigs and svds are described in a paper by D. C. Sorensen, *Implicitly Restarted Arnoldi/Lanczos Methods for Large Scale*

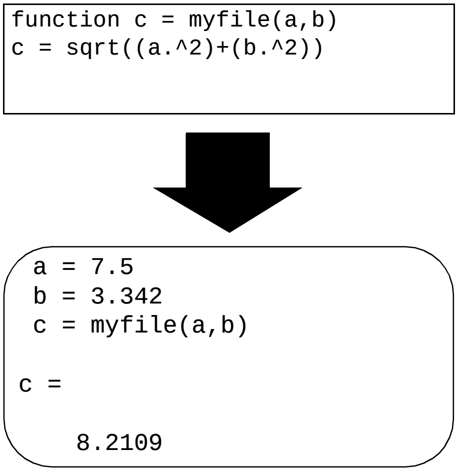
Eigenvalue Calculations. A copy of the paper is available through the MATLAB Help Desk.

M-File Programming

MATLAB Programming: A Quick Start

Files that contain MATLAB language code are called *M-files*. M-files can be *functions* that accept arguments and produce output, or they can be *scripts* that execute a series of MATLAB statements. For MATLAB to recognize a file as an M-file, its name must end in `.m`.

You create M-files using a text editor, then use them as you would any other MATLAB function or command. The process looks like this:



1 Create an M-file using a text editor.

```
function c = myfile(a,b)
c = sqrt((a.^2)+(b.^2))
```

2 Call the M-file from the command line, or from within another M-file.

```
 a = 7.5
 b = 3.342
 c = myfile(a,b)

c =

    8.2109
```

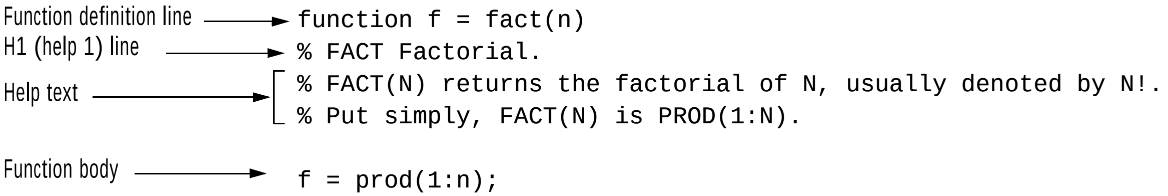
Kinds of M-Files

There are two kinds of M-files.

Script M-Files	Function M-Files
• Do not accept input arguments or return output arguments	• Can accept input arguments and return output arguments
• Operate on data in the workspace	• Internal variables are local to the function by default
• Useful for automating a series of steps you need to perform many times	• Useful for extending the MATLAB language for your application

What's in an M-File?

This section shows you the basic parts of a function M-file, so you can familiarize yourself with MATLAB programming and get started with some examples.



```
function f = fact(n)
% FACT Factorial.
% FACT(N) returns the factorial of N, usually denoted by N!.
% Put simply, FACT(N) is PROD(1:N).

f = prod(1:n);
```

This function has some elements that are common to all MATLAB functions:

- A *function definition line*. This line defines the function name, and the number and order of input and output arguments.
- A *H1 line*. H1 stands for "help 1" line. MATLAB displays the H1 line for a function when you use `lookfor` or request help on an entire directory.
- *Help text*. MATLAB displays the help text entry together with the H1 line when you request help on a specific function.
- The *function body*. This part of the function contains code that performs the actual computations and assigns values to any output arguments.

The “Functions” section coming up provides more detail on each of these parts of a MATLAB function.

Creating M-Files: Accessing Text Editors

M-files are ordinary text files that you create using a text editor. MATLAB provides a built in editor, although you can use any text editor you like.

> **Note** To open the editor on the PC, from the **File** menu choose **New** and then **M-File**.

Another way to edit an M-file is from the MATLAB command line using the `edit` command. For example,

```
edit poof
```

opens the editor on the file `poof.m`. Omitting a filename opens the editor on an untitled file.

You can create the `fact` function shown on the previous page by opening your text editor, entering the lines shown, and saving the text in a file called `fact.m` in your current directory.

Once you’ve created this file, here are some things you can do:

- List the names of the files in your current directory

  ```
  what
  ```

- List the contents of M-file `fact.m`

  ```
  type fact
  ```

- Call the `fact` function

  ```
  fact(5)
  ans =

        120
  ```

Scripts

Scripts are the simplest kind of M-file – they have no input or output arguments. They're useful for automating series of MATLAB commands, such as computations that you have to perform repeatedly from the command line. Scripts operate on existing data in the workspace, or they can create new data on which to operate. Any variables that scripts create remain in the workspace after the script finishes so you can use them for further computations.

Simple Script Example

These statements calculate rho for several trigonometric functions of theta, then create a series of polar plots.

Comment line

Computations

Graphical output commands

```
% An M–file script to produce "flower petal" plots
theta = –pi:0.01:pi;
rho(1,:) = 2*sin(5*theta).^2;
rho(2,:) = cos(10*theta).^3;
rho(3,:) = sin(theta).^2;
rho(4,:) = 5*cos(3.5*theta).^3;
for i = 1:4
    polar(theta,rho(i,:))
    pause
end
```

Try entering these commands in an M-file called petals.m. This file is now a MATLAB script. Typing petals at the MATLAB command line executes the statements in the script.

After the script displays a plot, press **Return** to move to the next plot. There are no input or output arguments; petals creates the variables it needs in the MATLAB workspace. When execution completes, the variables (i, theta, and rho) remain in the workspace. To see a listing of them, enter whos at the command prompt.

Functions

Functions are M-files that accept input arguments and return output arguments. They operate on variables within their own workspace. This is separate from the workspace you access at the MATLAB command prompt.

Simple Function Example

The average function is a simple M-file that calculates the average of the elements in a vector.

```
function y = average(x)
% AVERAGE Mean of vector elements.
% AVERAGE(X), where X is a vector, is the mean of vector elements.
% Non-vector input results in an error.
[m,n] = size(x);
if (~((m == 1) | (n == 1)) | (m == 1 & n == 1))
    error('Input must be a vector')
end
y = sum(x)/length(x);      % Actual computation
```

If you would like, try entering these commands in an M-file called `average.m`. The average function accepts a single input argument and returns a single output argument. To call the average function, enter:

```
z = 1:99;

average(z)

ans =
    50
```

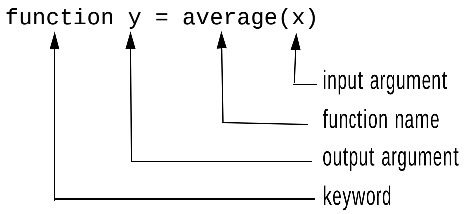
Basic Parts of a Function M-File

A function M-file consists of:

- A function definition line
- A H1 line
- Help text
- The function body
- Comments

Function Definition Line

The function definition line informs MATLAB that the M-file contains a function, and specifies the argument calling sequence of the function. The function definition line for the `average` function is:



```
function y = average(x)
```

All MATLAB functions have a function definition line that follows this pattern.

If the function has multiple output values, enclose the output argument list in square brackets. Input arguments, if present, are enclosed in parentheses. Use commas to separate multiple input or output arguments. Here's a more complicated example.

```
function [x,y,z] = sphere(theta,phi,rho)
```

If there is no output, leave the output blank

```
function printresults(x)
```

or use empty square brackets

```
function [] = printresults(x)
```

The variables that you pass to the function do not need to have the same name as those in the function definition line.

H1 Line

The H1 line, so named because it is the first help text line, is a comment line immediately following the function definition line. Because it consists of comment text, the H1 line begins with a percent sign, "%." For the `average` function, the H1 line is:

```
% AVERAGE Mean of vector elements.
```

This is the first line of text that appears when a user types `help` *function_name* at the MATLAB prompt. Further, the `lookfor` command searches on and displays only the H1 line. Because this line provides important summary information about the M-file, it is important to make it as descriptive as possible.

Help Text

You can create online help for your M-files by entering text on one or more comment lines, beginning with the line immediately following the H1 line. The help text for the average function is:

```
% AVERAGE(X), where X is a vector, is the mean of vector elements.
% Nonvector input results in an error.
```

When you type help *function_name*, MATLAB displays the comment lines that appear between the function definition line and the first non-comment (executable or blank) line. The help system ignores any comment lines that appear after this help block.

For example, typing help sin results in

```
SIN     Sine.
        SIN(X) is the sine of the elements of X.
```

Function Body

The function body contains all the MATLAB code that performs computations and assigns values to output arguments. The statements in the function body can consist of function calls, programming constructs like flow control and interactive input/output, calculations, assignments, comments, and blank lines.

For example, the body of the average function contains a number of simple programming statements.

```
                               [m,n] = size(x);
Flow control              →    if (~((m == 1) | (n == 1)) | (m == 1 & n == 1))
Error message display     →        error('Input must be a vector')
                               end
Computation and assignment →   y = sum(x)/length(x);
```

Comments

As mentioned earlier, comment lines begin with a percent sign (%). Comment lines can appear anywhere in an M-file, and you can append comments to the end of a line of code. For example,

```
% Add up all the vector elements.
y = sum(x)           % Use the sum function.
```

The first comment line immediately following the function definition line is considered the H1 line for the function. The H1 line and any comment lines immediately following it constitute the online help entry for the file.

In addition to comment lines, you can insert blank lines anywhere in an M-file. Blank lines are ignored. However, a blank line can indicate the end of the help text entry for an M-file.

Help for Directories

You can make help entries for an entire directory by creating a file with the special name `Contents.m` that resides in the directory. This file must contain only comment lines; that is, every line must begin with a percent sign. MATLAB displays the lines in a `Contents.m` file whenever you type

`help` *directory_name*

If a directory does not contain a `Contents.m` file, typing `help` *directory_name* displays the first help line (the H1 line) for each M-file in the directory.

Function Names

MATLAB function names have the same constraints as variable names. MATLAB uses the first 31 characters of names. Function names must begin with a letter; the remaining characters can be any combination of letters, numbers, and underscores. Some operating systems may restrict function names to shorter lengths.

The name of the text file that contains a MATLAB function consists of the function name with the extension `.m` appended. For example,

```
average.m
```

If the filename and the function definition line name are different, the internal name is ignored.

Thus, while the function name specified on the function definition line does not have to be the same as the filename, we strongly recommend that you use the same name for both.

How Functions Work

You can call function M-files from either the MATLAB command line or from within other M-files. Be sure to include all necessary arguments, enclosing input arguments in parentheses and output arguments in square brackets.

Function Name Resolution

When MATLAB comes upon a new name, it resolves it into a specific function by following these steps:

1. Checks to see if the name is a variable.
2. Checks to see if the name is a *subfunction*, a MATLAB function that resides in the same M-file as the calling function. Subfunctions are discussed on page 10-38.
3. Checks to see if the name is a *private function*, a MATLAB function that resides in a *private directory*, a directory accessible only to M-files in the directory immediately above it. Private directories are discussed on page 10-39.
4. Checks to see if the name is a function on the MATLAB search path. MATLAB uses the first file it encounters with the specified name.

If you duplicate function names, MATLAB executes the one found first using the above rules. It is also possible to overload function names. This uses additional dispatching rules and is discussed in Chapter 14, “Classes and Objects.”

What Happens When You Call a Function

When you call a function M-file from either the command line or from within another M-file, MATLAB parses the function into pseudocode and stores it in memory. This prevents MATLAB from having to reparse a function each time you call it during a session. The pseudocode remains in memory until you clear it using the `clear` command, or until you quit MATLAB. Variants of the `clear` command that you can use to clear functions from memory include:

`clear function_name`	Remove specified function from workspace.
`clear functions`	Remove all compiled M-functions.
`clear all`	Remove all variables and functions

Creating P-Code Files

You can save a preparsed version of a function or script, called P-code files, for later MATLAB sessions using the `pcode` command. For example,

```
pcode average
```

parses `average.m` and saves the resulting pseudocode to the file named `average.p`. This saves MATLAB from reparsing `average.m` the first time you call it in each session.

MATLAB is very fast at parsing so the `pcode` command rarely makes much of a speed difference.

One situation where `pcode` does provide a speed benefit is for large GUI applications. In this case, many M-files must be parsed before the application becomes visible.

Another situation for `pcode` is when, for proprietary reasons, you want to hide algorithms you've created in your M-file.

How MATLAB Passes Function Arguments

From the programmer's perspective, MATLAB appears to pass all function arguments by value. Actually, however, MATLAB passes by value only those arguments that a function modifies. If a function does not alter an argument but simply uses it in a computation, MATLAB passes the argument by reference to optimize memory use.

Function Workspaces

Each M-file function has an area of memory, separate from MATLAB's base workspace, in which it operates. This area is called the function workspace, with each function having its own workspace context.

While using MATLAB, the only variables you can access are those in the calling context, be it the base workspace or that of another function. The variables that you pass to a function must be in the calling context, and the function returns its output arguments to the calling workspace context. You can however, define variables as global variables explicitly, allowing more than one workspace context to access them.

Checking the Number of Function Arguments

The `nargin` and `nargout` functions let you determine how many input and output arguments a function is called with. You can then use conditional statements to perform different tasks depending on the number of arguments. For example,

```
function c = testarg1(a,b)
if (nargin == 1)
    c = a.^2;
elseif (nargin == 2)
    c = a + b;
end
```

Given a single input argument, this function squares the input value. Given two inputs, it adds them together.

Here's a more advanced example that finds the first token in a character string. A *token* is a set of characters delimited by whitespace or some other character. Given one input, the function assumes a default delimiter of whitespace; given two, it lets you specify another delimiter if desired. It also allows for two possible output argument lists.

Function requires at least one input.

If one input, use white space delimiter.

Determine where non-delimiter characters begin.

Find where token ends.

For two output arguments, count characters after first delimiter (remainder).

```
function [token,remainder] = strtok(string,delimiters)
if nargin < 1, error(’Not enough input arguments.’); end
token = []; remainder = [];
len = length(string);
if len == 0
    return
end
if (nargin == 1)
    delimiters = [9:13 32]; % White space characters
end
i = 1;
while (any(string(i) == delimiters))
    i = i + 1;
    if (i > len), return, end
end
                  start = i;
                  while (~any(string(i) == delimiters))
                      i = i + 1;
                      if (i > len), break, end
                  end
                  finish = i – 1;
token = string(start:finish);
if (nargout == 2)
    remainder = string(finish + 1:end);
end
```

Note `strtok` is a MATLAB M-file in the `strfun` directory.

Note that the order in which output arguments appear in the function declaration line is important. The argument that the function returns in most cases appears first in the list. Additional, optional arguments are appended to the list.

Passing Variable Numbers of Arguments

The `varargin` and `varargout` functions let you pass any number of inputs or return any number of outputs to a function. MATLAB packs all of the specified input or output into a *cell array*, a special kind of MATLAB array that consists of cells instead of array elements. Each cell can hold any size or kind of data – one might hold a vector of numeric data, another in the same array might hold an array of string data, and so on.

Here's an example function that accepts any number of two-element vectors and draws a line to connect them.

Cell array indexing

```
function testvar(varargin)
for i = 1:length(varargin)
   x(i) = varargin{i}(1);
   y(i) = varargin{i}(2);
end
xmin = min(0,min(x));
ymin = min(0,min(y));
axis([xmin fix(max(x))+3 ymin fix(max(y))+3])
plot(x,y)
```

Coded this way, the `testvar` function works with various input lists; for example,

```
testvar([2 3],[1 5],[4 8],[6 5],[4 2],[2 3])
testvar([–1 0],[3 –5],[4 2],[1 1])
```

Unpacking varargin Contents

Because `varargin` contains all the input arguments in a cell array, it's necessary to use cell array indexing to extract the data. For example,

```
y(i) = varargin{i}(2);
```

Cell array indexing has two subscript components:

- The cell indexing expression, in curly braces
- The contents indexing expression(s), in parentheses

In the code above, the indexing expression `{i}` accesses the `i`'th cell of `varargin`. The expression `(2)` represents the second element of the cell contents.

Packing varargout Contents

When allowing any number of output arguments, you must pack all of the output into the varargout cell array. Use nargout to determine how many output arguments the function is called with. For example, this code accepts a two-column input array, where the first column represents a set of x coordinates and the second represents y coordinates. It breaks the array into separate [xi yi] vectors that you can pass into the testvar function on the previous page.

Cell array assignment

```
function [varargout] = testvar2(arrayin)
for i = 1:nargout
    varargout{i} = arrayin(i,:)
end
```

The assignment statement inside the for loop uses cell array assignment syntax. The left side of the statement, the cell array, is indexed using curly braces to indicate that the data goes inside a cell. For complete information on cell array assignment, see “Structures and Cell Arrays” in Chapter 13.

Here’s how to call testvar2.

```
a = {1 2;3 4;5 6;7 8;9 0};
[p1,p2,p3,p4,p5] = testvar2(a);
```

varargin and varargout in Argument Lists

varargin or varargout must appear last in the argument list, following any required input or output variables. That is, the function call must specify the required arguments first. For example, these function declaration lines show the correct placement of varargin and varargout.

```
function [out1,out2] = example1(a,b,varargin)
function [i,j,varargout] = example2(x1,y1,x2,y2,flag)
```

Local and Global Variables

The same guidelines that apply to MATLAB variables at the command line also apply to variables in M-files:

- You do not need to type or declare variables. Before assigning one variable to another, however, you must be sure that the variable on the right-hand side of the assignment has a value.
- Any operation that assigns a value to a variable creates the variable if needed, or overwrites its current value if it already exists.
- MATLAB variable names consist of a letter followed by any number of letters, digits, and underscores. MATLAB distinguishes between uppercase and lowercase characters, so A and a are not the same variable.
- MATLAB uses only the first 31 characters of variable names.

Ordinarily, each MATLAB function, defined by an M-file, has its own local variables, which are separate from those of other functions, and from those of the base workspace. However, if several functions, and possibly the base workspace, all declare a particular name as global, then they all share a single copy of that variable. Any assignment to that variable, in any function, is available to all the other functions declaring it global.

Suppose you want to study the effect of the interaction coefficients, α and β, in the Lotka-Volterra predator-prey model

$$\dot{y}_1 = y_1 - \alpha y_1 y_2$$
$$\dot{y}_2 = -y_2 + \beta y_1 y_2$$

Create an M-file, lotka.m.

```
function yp = lotka(t,y)
%LOTKA   Lotka-Volterra predator-prey model.
global ALPHA BETA
yp = [y(1) – ALPHA*y(1)*y(2); –y(2) + BETA*y(1)*y(2)];
```

Then interactively enter the statements

```
global ALPHA BETA
ALPHA = 0.01
BETA = 0.02
[t,y] = ode23('lotka',0,10,[1; 1]);
plot(t,y)
```

The two global statements make the values assigned to ALPHA and BETA at the command prompt available inside the function defined by lotka.m. They can be modified interactively and new solutions obtained without editing any files.

For your MATLAB application to work with global variables:

- Declare the variable as global in every function that requires access to it. To enable the workspace to access the global variable, also declare it as global from the command line.
- In each function, issue the global command before the first occurrence of the variable name. The top of the M-file is recommended.

MATLAB global variable names are typically longer and more descriptive than local variable names, and sometimes consist of all uppercase characters. These are not requirements, but guidelines to increase the readability of MATLAB code and reduce the chance of accidentally redefining a global variable.

Persistent Variables

A variable may be defined as persistent so that it does not change value from one call to another. Persistent variables may be used within a function only. Persistent variables remain in memory until the M-file is cleared or changed.

persistent is exactly like global, except that the variable name is not in the global workspace, and the value is reset if the M-file is changed or cleared.

Three MATLAB functions support the use of persistent variables.

mlock	Prevents an M-file from being cleared.
munlock	Unlocks an M-file that had previously been locked by mlock.
mislocked	Indicates whether an M-file can be cleared or not.

Special Values

Several functions return important special values that you can use in your M-files.

`ans` Most recent answer (variable). If you do not assign an output variable to an expression, MATLAB automatically stores the result in `ans`.

`eps` Floating-point relative accuracy. This is the tolerance MATLAB uses in its calculations.

`realmax` Largest floating-point number your computer can represent.

`realmin` Smallest floating-point number your computer can represent.

`pi` `3.1415926535897...`

`i`, `j` Imaginary unit.

`inf` Infinity. Calculations like `n/0`, where `n` is any nonzero real value, result in `inf`.

`NaN` Not-a-Number, an invalid numeric value. Expressions like `0/0` and `inf/inf` result in a `NaN`, as do arithmetic operations involving a `NaN`. `n/0`, where `n` is complex, also returns `NaN`.

`computer` Computer type.

`flops` Count of floating-point operations.

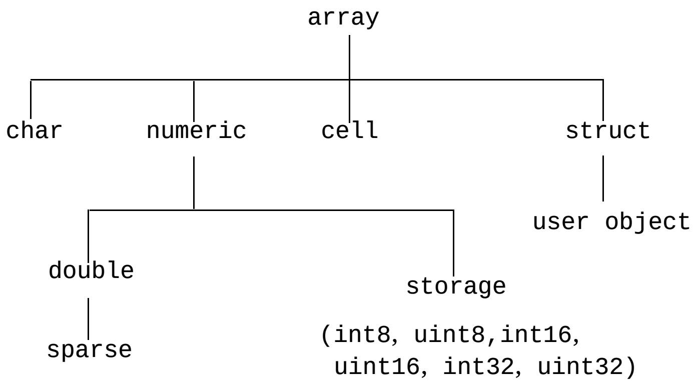
`version` MATLAB version string.

All of these special functions and constants reside in MATLAB's `elmat` directory, and provide online help. Here are several examples that use them in MATLAB expressions.

```
x = 2*pi;
A = [3+2i 7–8i];
tol = 3*eps;
```

Data Types

There are six fundamental data types (classes) in MATLAB, each one a multidimensional array. The six classes are double, char, sparse, storage, cell, and struct. The two-dimensional versions of these arrays are called *matrices* and are where MATLAB gets its name.



You will probably spend most of your time working with only two of these data types: the double precision matrix (double) and the character array (char) or string. This is because all computations are done in double-precision and most of the functions in MATLAB work with arrays of double-precision numbers or strings.

The other data types are for specialized situations like image processing (unit8), sparse matrices (sparse), and large scale programming (cell and struct).

You can't create variables with the types numeric, array, or storage. These *virtual* types serve only to group together types that share some common attributes.

The storage data types are for memory efficient storage only. You can apply basic operations such as subscripting and reshaping to these types of arrays but you can't perform any math with them. You must convert such arrays to double via the double function before doing any math operations.

You can define user classes and objects in MATLAB that are based on the struct data type. For more information about creating classes and objects, see "Classes and Objects: An Overview" in Chapter 14.

This table describes the data types in more detail.

Class	Example	Description
`array`		virtual data type
`cell`	`{17 'hello' eye(2)}`	Cell array. Elements of cell arrays contain other arrays. Cell arrays collect related data and information of a dissimilar size together.
`char`	`'Hello'`	Character array (each character is 16 bits long). Also referred to as a string.

`double`	`[1 2;3 4]`
	`5+6i`
		Double precision numeric array (this is the most common MATLAB variable type).

`numeric`		virtual data type
`sparse`	`speye(5)`	Sparse double precision matrix (2-D only). The sparse matrix stores matrices with only a few nonzero elements in a fraction of the space required for an equivalent full matrix. Sparse matrices invoke special methods especially tailored to solve sparse problems.
`storage`		virtual data type

`struct`	`a.day = 12;`
	`a.color = 'Red';`
	`a.mat = magic(3);`
		Structure array. Structure arrays have field names. The fields contain other arrays. Like cell arrays, structures collect related data and information together.

`uint8`	`uint8(magic(3))`	Unsigned 8 bit integer array. The `unit8` array stores integers in the range from 0 to 255 in 1/8 the memory required for a double precision array. No mathematical operations are defined for `uint8` arrays.
`user object`	`inline('sin(x)')`	User-defined data type.

Operators

MATLAB's operators fall into three categories:

- Arithmetic operators that perform numeric computations, for example, adding two numbers or raising the elements of an array to a given power.
- Relational operators that compare operands quantitatively, using operators like "less than" and "not equal to."
- Logical operators that use the logical operators `AND`, `OR`, and `NOT`.

Arithmetic Operators

MATLAB provides these arithmetic operators:

`+`	Addition
`-`	Subtraction
`.*`	Multiplication
`./`	Right division
`.\`	Left division
`+`	Unary plus
`-`	Unary minus
`:`	Colon operator
`.^`	Power

.’	Transpose
’	Complex conjugate transpose
*	Matrix multiplication
/	Matrix right division
\	Matrix left division
^	Matrix power

Arithmetic Operators and Arrays

Except for some matrix operators, MATLAB’s arithmetic operators work on corresponding elements of arrays with equal dimensions. For vectors and rectangular arrays, both operands must be the same size unless one is a scalar. If one operand is a scalar and the other is not, MATLAB applies the scalar to every element of the other operand – this property is known as *scalar expansion*.

This example uses scalar expansion to compute the product of a scalar operand and a matrix.

```
A = magic(3)

A =

     8     1     6
     3     5     7
     4     9     2

3 * A

ans =

    24     3    18
     9    15    21
    12    27     6
```

Relational Operators

MATLAB provides these relational operators:

<	Less than
<=	Less than or equal to
>	Greater than
>=	Greater than or equal to
==	Equal to
~=	Not equal to

Relational Operators and Arrays

MATLAB's relational operators compare corresponding elements of arrays with equal dimensions. Relational operators always operate element-by-element. In this example, the resulting matrix shows where an element of `A` is equal to the corresponding element of `B`.

```
A = [2 7 6;9 0 5;3 0.5 6];
B = [8 7 0;3 2 5;4 –1 7];
A == B

ans =

     0     1     0
     0     0     1
     0     0     0
```

For vectors and rectangular arrays, both operands must be the same size unless one is a scalar. For the case where one operand is a scalar and the other is not, MATLAB tests the scalar against every element of the other operand. Locations where the specified relation is true receive the value `1`. Locations where the relation is false receive the value `0`.

Relational Operators and Empty Arrays

The relational operators work with arrays for which any dimension has size zero, as long as both arrays are the same size or one is a scalar. However, expressions such as

```
A == []
```

return an error if A is not 0-by-0 or 1-by-1.

To test for empty arrays, use the function

```
isempty(A)
```

Logical Operators

MATLAB provides these logical operators:

&	AND
|	OR
~	NOT

Note In addition to these logical operators, the ops directory contains a number of functions that perform bitwise logical operations. See online help for more information.

Each logical operator has a specific set of rules that determines the result of a logical expression:

- An expression using the AND operator, &, is true if both operands are logically true. In numeric terms, the expression is true if both operands are nonzero.

This example shows the logical AND of the elements in the vector u with the corresponding elements in the vector v

```
u = [1 0 2 3 0 5];
v = [5 6 1 0 0 7];
u & v

ans =

     1     0     1     0     0     1
```

- An expression using the OR operator, |, is true if one operand is logically true, or if both operands are logically true. An OR expression is false only if both operands are false. In numeric terms, the expression is false only if both operands are zero. This example shows the logical OR of the elements in the vector u and with the corresponding elements in the vector v.

```
u | v

ans =

     1     1     1     1     0     1
```

- An expression using the NOT operator, ~, negates the operand. This produces a false result if the operand is true, and true if it is false. In numeric terms, any nonzero operand becomes zero, and any zero operand becomes one. This example shows the negation of the elements in the vector u.

```
~u

ans =

      0     1     0     0     1     0
```

Logical Operators and Arrays

MATLAB's logical operators compare corresponding elements of arrays with equal dimensions. For vectors and rectangular arrays, both operands must be the same size unless one is a scalar. For the case where one operand is a scalar and the other is not, MATLAB tests the scalar against every element of the other operand. Locations where the specified relation is true receive the value 1. Locations where the relation is false receive the value 0.

Logical Functions

In addition to the logical operators, MATLAB provides a number of logical functions.

Function	Description	Examples

`xor`	Performs an exclusive `OR` on its operands. `xor` returns true if one operand is true and the other false. In numeric terms, the function returns `1` if one operand is nonzero and the other operand is zero.	`a = 1;`
		`b = 1;`
		`xor(a,b)`
		
		`ans =`
		
		`     0`

`all`	Returns `1` if all of the elements in a vector are `true` or nonzero. `all` operates columnwise on matrices.	`u = [0 1 2 0];`
		`all(u)`
		
		`ans =`
		
		`     0`
		
		`A = [0 1 2;3 5 0];`
		`all(A)`
		
		`ans =`
		
		`     0     1     0`

`any`	Returns `1` if any of the elements of its argument are true or nonzero; otherwise, it returns `0`. Like `all`, the any function operates columnwise on matrices.	`v = [5 0 8];`
		`any(v)`
		
		`ans =`
		
		`     1`

A number of other MATLAB functions perform logical operations. For example, the `isnan` function returns `1` for `NaN`s; the `isinf` function returns `1` for `Inf`s. See the ops directory for a complete listing of logical functions.

Logical Expressions and Subscripting with the find Function

The `find` function determines the indices of array elements that meet a given logical condition. It's useful for creating masks and index matrices. In its most general form, `find` returns a single vector of indices. This vector can be used to index into arrays of any size or shape. For example,

```
A = magic(4)

A =

    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1

i = find(A > 8);
A(i) = 100

A =

   100     2     3   100
     5   100   100     8
   100     7     6   100
     4   100   100     1
```

You can also use `find` to obtain both the row and column indices for a rectangular matrix, as well as the array values that meet the logical condition. Use the `help` facility for more information on `find`.

Operator Precedence

You can build expressions that use any combination of arithmetic, relational, and logical operators. Precedence levels determine the order in which MATLAB evaluates an expression. Within each precedence level, operators have equal precedence and are evaluated from left to right. The precedence rules for MATLAB operators are shown in this table, ordered from highest precedence level to lowest precedence level

Operator	Precedence Level
( )	Highest precedence
~ (negation)	
.' .^ ' ^ + (unary plus) – (unary minus)	
.* ./ .\ * / \	
+ (addition) – (subtraction)	
: < <= > >= == ~=	
& |	Lowest precedence

Precedence of & and |

MATLAB's left to right execution precedence causes a|b&c to be equivalent to (a|b)&c. However, in most programming languages, a|b&c is equivalent to a|(b&c), that is, & takes precedence over |. To ensure compatibility with future versions of MATLAB, you should use parentheses to explicity specify the intended precedence of statements containing combinations of & and |.

Overriding Default Presedence

The default precedence can be overridden using parentheses, as shown in this example.

```
A = [3 9 5];
B = [2 1 5];
C = A./B.^2

C =

    0.7500    9.0000    0.2000

C = (A./B).^2

C =

    2.2500   81.0000    1.0000
```

Expressions can also include values that you access through subscripts.

```
b = sqrt(A(2)) + 2*B(1)

b =

     7
```

Flow Control

There are six flow control statements in MATLAB:

- `if`, together with `else` and `elseif`, executes a group of statements based on some logical condition.
- `switch`, together with `case` and `otherwise`, executes different groups of statements depending on the value of some logical condition.
- `while` executes a group of statements an indefinite number of times, based on some logical condition.
- `for` executes a group of statements a fixed number of times.
- `break` terminates execution of a `for` or `while` loop.
- `try...catch` changes flow control if an error is detected during execution.
- `return` causes execution to return to the invoking function.

All flow constructs use `end` to indicate the end of the flow control block.

Note You can often speed up the execution of MATLAB code by replacing `for` and `while` loops with vectorized code. See "Optimizing the Performance of MATLAB Code" on page 10-63.

if, else, and elseif

`if` evaluates a logical expression and executes a group of statements based on the value of the expression. In its simplest form, its syntax is:

```
if logical_expression
     statements
end
```

If the logical expression is true (`1`), MATLAB executes all the statements between the `if` and `end` lines. It resumes execution at the line following the `end` statement. If the condition is false (`0`), MATLAB skips all the statements between the `if` and `end` lines, and resumes execution at the line following the `end` statement.

For example,

```
if rem(a,2) == 0
    disp('a is even')
    b = a/2;
end
```

You can nest any number of `if` statements.

If the logical expression evaluates to a nonscalar value, all the elements of the argument must be nonzero. For example, assume `X` is a matrix. Then the statement

```
if X
    statements
end
```

is equivalent to

```
if all(X(:))
    statements
end
```

The `else` and `elseif` statements further conditionalize the `if` statement:

- The `else` statement has no logical condition. The statements associated with it execute if the preceding `if` (and possibly `elseif` condition) is false (`0`).
- The `elseif` statement has a logical condition that it evaluates if the preceding `if` (and possibly `elseif` condition) is false (`0`). The statements associated with it execute if its logical condition is true (`1`). You can have multiple `elseif`s within an `if` block.

```
if n < 0             % If n negative, display error message.
    disp('Input must be positive');
elseif rem(n,2) == 0 % If n positive and even, divide by 2.
    A = n/2;
else
   A = (n+1)/2;     % If n positive and odd, increment and divide.
end
```

if Statements and Empty Arrays

An `if` condition that reduces to an empty array represents a false condition. That is,

```
if A
   S1
else
   S0
end
```

will execute statement `S0` when `A` is an empty array.

switch

`switch` executes certain statements based on the value of a variable or expression. Its basic form is:

```
switch expression (scalar or string)
    case value1
        statements
    case value2
        statements
    .
    .
    .
    otherwise
         statements
end
```

Executes if *expression* is *value1* →

Executes if *expression* is *value2* →

Executes if *expression* does not match any case →

This block consists of:

- The word `switch` followed by an expression to evaluate.
- Any number of `case` groups. These groups consist of the word `case` followed by a possible value for the expression, all on a single line. Subsequent lines contain the statements to execute for the given value of the expression. These can be any valid MATLAB statement including another `switch` block. Execution of a `case` group ends when MATLAB encounters the next `case` statement or the `otherwise` statement. Only the first matching `case` is executed.
- An optional `otherwise` group. This consists of the word `otherwise`, followed by the statements to execute if the expression's value is not handled by any

of the preceding case groups. Execution of the otherwise group ends at the end statement.

- An end statement.

switch works by comparing the input expression to each case value. For numeric expressions, a case statement is true if (value==expression). For string expressions, a case statement is true if strcmp(value,expression).

The code below shows a simple example of the switch statement. It checks the variable input_num for certain values. If input_num is –1, 0, or 1, the case statements display the value on screen as text. If input_num is none of these values, execution drops to the otherwise statement and the code displays the text 'other value'.

```
switch input_num
    case –1
        disp('negative one');
    case 0
        disp('zero');
    case 1
        disp('positive one');
    otherwise
         disp('other value');
end
```

Note for C Programmers Unlike the C language switch construct, MATLAB's switch does not "fall through." That is, if the first case statement is true, other case statements do not execute. Therefore, break statements are not used.

switch can handle multiple conditions in a single case statement by enclosing the case expression in a cell array.

```
switch var
    case 1
        disp('1')
    case {2,3,4}
        disp('2 or 3 or 4')
    case 5
        disp('5')
    otherwise
        disp('something else')
end
```

while

The while loop executes a statement or group of statements repeatedly as long as the controlling expression is true (1). Its syntax is:

```
while expression
    statements
end
```

If the expression evaluates to a matrix, all its elements must be 1 for execution to continue. To reduce a matrix to a scalar value, use the all and any functions.

For example, this while loop finds the first integer n for which n! (n factorial) is a 100-digit number.

```
n = 1;
while prod(1:n) < 1e100
    n = n + 1;
end
```

Exit a while loop at any time using the break statement.

while Statements and Empty Arrays

A while condition that reduces to an empty array represents a false condition. That is,

```
while A, S1, end
```

never executes statement S1 when A is an empty array.

for

The for loop executes a statement or group of statements a predetermined number of times. Its syntax is:

```
for index = start:increment:end
    statements
end
```

The default increment is 1. You can specify any increment, including a negative one. For positive indices, execution terminates when the value of the index exceeds the *end* value; for negative increments, it terminates when the index is less than the end value.

For example, this loop executes five times.

```
for i = 2:6
    x(i) = 2*x(i-1);
end
```

You can nest multiple for loops.

```
for i = 1:m
    for j = 1:n
        A(i,j) = 1/(i + j - 1);
    end
end
```

Note You can often speed up the execution of MATLAB code by replacing for and while loops with vectorized code. See page 10-63 for details.

Using Arrays as Indices

The index of a for loop can be an array. For example, consider an m-by-n array A. The statement

```
for i = A
    statements
end
```

sets i equal to the vector A(:,k). For the first loop iteration, k is equal to 1; for the second k is equal to 2, and so on until k equals n. That is, the loop iterates for a number of times equal to the number of columns in A. For each iteration, i is a vector containing one of the columns of A.

break

The break statement terminates the execution of a for loop or while loop. When a break statement is encountered, execution continues with the next statement outside of the loop. In nested loops, break exits from the innermost loop only.

The example below shows a while loop that reads the contents of the file fft.m into a MATLAB character array. A break statement is used to exit the while loop when the first empty line is encountered. The resulting character array contains the M-file help for the fft program.

```
fid = fopen('fft.m','r');
s = '';
while ~feof(fid)
   line = fgetl(fid);
   if isempty(line), break, end
   s = strvcat(s,line);
end
disp(s)
```

try ... catch

The general form of a try/catch statement sequence is:

try *statement*, ...,*statement*, catch *statement*, ...,*statement* end

In this sequence the statements between try and catch are executed until an error occurs. The statements between catch and end are then executed. Use lasterr to see the cause of the error. If an error occurs between catch and end, MATLAB terminates execution unless another try ... catch sequence has been established.

return

return terminates the current sequence of commands and returns control to the invoking function or to the keyboard. return is also used to terminate keyboard mode. A called function normally transfers control to the function that invoked it when it reaches the end of the function. return may be inserted within the called function to force an early termination and to transfer control to the invoking function.

Subfunctions

Function M-files can contain code for more than one function. The first function in the file is the *primary function*, the function invoked with the M-file name. Additional functions within the file are *subfunctions* that are only visible to the primary function or other subfunctions in the same file.

Each subfunction begins with its own function definition line. The functions immediately follow each other. The various subfunctions can occur in any order, as long as the primary function appears first.

Primary function →

```
function [avg,med] = newstats(u)
% NEWSTATS Find mean and median with internal functions.
n = length(u);
avg = mean(u,n);
med = median(u,n);
```

Subfunction →

```
function a = mean(v,n)
% Calculate average.
a = sum(v)/n;
```

Subfunction →

```
function m = median(v,n)
% Calculate median.
w = sort(v);
if rem(n,2) == 1
    m = w((n+1)/2);
else
    m = (w(n/2)+w(n/2+1))/2;
end
```

The subfunctions `mean` and `median` calculate the average and median of the input list. The primary function `newstats` determines the length of the list and calls the subfunctions, passing to them the list length `n`. Functions within the same M-file cannot access the same variables unless you declare them as global within the pertinent functions, or pass them as arguments. In addition, the help facility can only access the primary function in an M-file.

When you call a function from within an M-file, MATLAB first checks the file to see if the function is a subfunction. It then checks for a private function (described in the following section) with that name, and then for a standard M-file on your search path. Because it checks for a subfunction first, you can supersede existing M-files using subfunctions with the same name, for example, mean in the above code. Function names must be unique within an M-file, however.

Private Functions

Private functions are functions that reside in subdirectories with the special name private. They are visible only to functions in the parent directory. For example, assume the directory newmath is on the MATLAB search path. A subdirectory of newmath called private can contain functions that only the functions in newmath can call. Because private functions are invisible outside of the parent directory, they can use the same names as functions in other directories. This is useful if you want to create your own version of a particular function while retaining the original in another directory. Because MATLAB looks for private functions before standard M-file functions, it will find a private function named test.m before a nonprivate M-file named test.m.

You can create your own private directories simply by creating subdirectories called private using the standard procedures for creating directories or folders on your computer. Do not place these private directories on your path.

Indexing and Subscripting

Subscripts

The element in row `i` and column `j` of `A` is denoted by `A (i,j)`. For example, suppose `A = magic(4)`, Then `A (4,2)` is the number in the fourth row and second column. For our magic square, `A (4,2)` is `15`. So it is possible to compute the sum of the elements in the fourth column of `A` by typing

```
A(1, 4) + A(2, 4) + A(3, 4) + A(4, 4)
```

It is also possible to refer to the elements of a matrix with a single subscript, `A(k)`. This is the usual way of referencing row and columns vectors. But it can also apply to a fully two-dimensional matrix, in which case the array is regarded as one long column vector formed from the columns of the original matrix. So, for our magic square, `A(8)` is another way of referring to the value `14` stored in `A(4,2)`.

If you try to use the value of an element outside of the matrix, it is an error

```
t = A(4,5)
Index exceeds matrix dimensions
```

However, if you store a value in an element outside of the matrix, the size of the matrix increases to accommodate the new element.

```
x = A;
x(4,5) = 17

x =

    16     2     3    13     0
     5    11    10     8     0
     9     7     6    12     0
     4    14    15     1    17
```

Subscript expressions involving colons refer to portions of a matrix.

```
A(1:k,j)
```

is the first `k` elements of the `j`-th column of `A`. So

```
sum(A(1:4,4))
```

computes the sum of the fourth column. But there is a better way. The colon by itself refers to *all* the elements in a row or column of a matrix and the keyword end refers to the *last* row or column. So

```
sum(A(:,end))
```

computes the sum of the elements in the last column of A.

```
ans =
     34
```

Concatenation

Concatenation is the process of joining small matrices together to make bigger ones. In fact, you made your first matrix by concatenating its individual elements. The pair of square brackets, [], is the concatenation operator. For an example, start with the 4-by-4 magic square, A, and form

```
B = [A A+32;A+48 A+16]
```

The result is an 8-by-8 matrix, obtained by joining the four submatrices.

```
B =

    16     2     3    13    48    34    35    45
     5    11    10     8    37    43    42    40
     9     7     6    12    41    39    38    44
     4    14    15     1    36    46    47    33
    64    50    51    61    32    18    19    29
    53    59    58    56    21    27    26    24
    57    55    54    60    25    23    22    28
    52    62    63    49    20    30    31    17
```

This matrix is half way to being another magic square. Its elements are a rearrangement of the integers 1:64. Its column sums are the correct value for an 8-by-8 magic square.

```
sum(B)

ans =

   260   260   260   260   260   260   260   260
```

But its row sums, sum(B’)’, are not all the same. Further manipulation is necessary to make this a valid 8-by-8 magic square.

Deleting Rows and Columns

You can delete rows and columns from a matrix using just a pair of square brackets. Start with

```
X = A;
```

Then, to delete the second column of X, use

```
X(:,2) = []
```

This changes X to

```
X =
   162 13
    511 8
    9 712
    414 1
```

If you delete a single element from a matrix, the result isn’t a matrix anymore. So expressions like

```
X(1,2) = []
```

result in an error. However, using a single subscript deletes a single element, or sequence of elements, and reshapes the remaining elements into a row vector. So

```
X(2:2:10) = []
```

results in

```
X =
   169 27  13121
```

Advanced Indexing

MATLAB stores each array as a column of values regardless of the actual dimensions. This column consists of the array columns, appended end to end. For example, MATLAB stores

```
A = [2 6 9; 4 2 8; 3 0 1]
```

as

```
    2
    4
    3
    6
    2
    0
    9
    8
    1
```

Accessing `A` with a single subscript indexes directly into the storage column. `A(3)` accesses the third value in the column, the number 3. `A(7)` accesses the seventh value, 9, and so on.

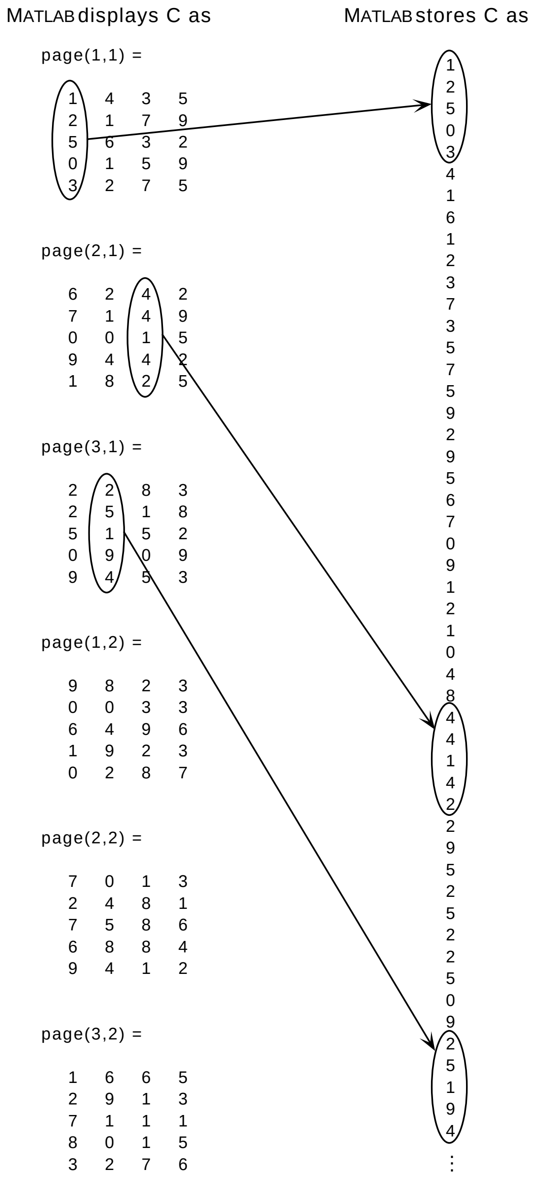
If you supply more subscripts, MATLAB calculates an index into the storage column based on the dimensions you assigned to the array. For example, assume a two-dimensional array like `A` has size `[d1 d2]`, where `d1` is the number of rows in the array and `d2` is the number of columns. If you supply two subscripts `(i,j)` representing row-column indices, the offset is

```
(j–1)*d1+i
```

Given the expression `A(3,2)`, MATLAB calculates the offset into `A`'s storage column as `(2-1)*3+3`, or `6`. Counting down six elements in the column accesses the value `0`.

This storage and indexing scheme also extends to multidimensional arrays. In this case, MATLAB operates on a page-by-page basis to create the storage column, again appending elements columnwise.

For example, consider a 5-by-4-by-3-by-2 array `C`.



MATLAB displays C as

```
page(1,1) =

   1   4   3   5
   2   1   7   9
   5   6   3   2
   0   1   5   9
   3   2   7   5

page(2,1) =

   6   2   4   2
   7   1   4   9
   0   0   1   5
   9   4   4   2
   1   8   2   5

page(3,1) =

   2   2   8   3
   2   5   1   8
   5   1   5   2
   0   9   0   9
   9   4   5   3

page(1,2) =

   9   8   2   3
   0   0   3   3
   6   4   9   6
   1   9   2   3
   0   2   8   7

page(2,2) =

   7   0   1   3
   2   4   8   1
   7   5   8   6
   6   8   8   4
   9   4   1   2

page(3,2) =

   1   6   6   5
   2   9   1   3
   7   1   1   1
   8   0   1   5
   3   2   7   6
```

MATLAB stores C as

```
1
2
5
0
3
4
1
6
1
2
3
7
3
5
7
5
9
2
9
5
6
7
0
9
1
2
1
0
4
8
4
4
1
4
2
2
9
5
2
5
2
2
5
0
9
2
5
1
9
4
⋮
```

Again, a single subscript indexes directly into this column. For example, `C(4)` produces the result

```
ans =

     0
```

If you specify two subscripts `(i,j)` indicating row-column indices, MATLAB calculates the offset as described above. Two subscripts always access the first page of a multidimensional array, provided they are within the range of the original array dimensions.

If more than one subscript is present, all subscripts must conform to the original array dimensions. For example, `C(6,2)` is invalid, because all pages of `C` have only five rows.

If you specify more than two subscripts, MATLAB extends its indexing scheme accordingly. For example, consider four subscripts `(i,j,k,l)` into a four-dimensional array with size `[d1 d2 d3 d4]`. MATLAB calculates the offset into the storage column by

```
(l–1)(d3)(d2)(d1)+(k–1)(d2)(d1)+(j–1)(d1)+i
```

For example, if you index the array `C` using subscripts (3,4,2,1), MATLAB returns the value 5 (index 38 in the storage column).

In general, the offset formula for an array with dimensions $[d_1\ d_2\ d_3\ \ldots\ d_n]$ using any subscripts $(s_1\ s_2\ s_3\ \ldots\ s_n)$ is:

$$(s_n-1)(d_{n-1})(d_{n-2})\ldots(d_1)+(s_{n-1}-1)(d_{n-2})\ldots(d_1)+\ldots+(s_2-1)(d_1)+s_1$$

Because of this scheme, you can index an array using any number of subscripts. You can append any number of 1s to the subscript list because these terms become zero. For example,

```
C(3,2,1,1,1,1,1,1)
```

is equivalent to

```
C(3,2)
```

String Evaluation

String evaluation adds power and flexibility to the MATLAB language, letting you perform operations like executing user-supplied strings and constructing executable strings through concatenation of strings stored in variables.

eval

The eval function evaluates a string that contains a MATLAB expression, statement, or function call. In its simplest form, the eval syntax is:

```
eval(’string’)
```

For example, this code uses eval on an expression to generate a Hilbert matrix of order n.

```
t = '1/(i+j–1)';
for i = 1:n
    for j = 1:n
        a(i,j) = eval(t);
    end
end
```

Here’s an example that uses eval on a statement.

```
eval(’t = clock’)
```

feval

feval differs from eval in that it executes a function whose name is in a string, rather than an entire MATLAB expression. You can use feval and the input function to choose one of several tasks defined by M-files. In this example, the functions have names like sin, cos, and so on.

```
fun = [’sin’; ’cos’; ’log’];
k = input(’Choose function number: ’);
x = input(’Enter value: ’);
feval(fun(k,:),x)
```

Note Use `feval` rather than `eval` whenever possible. M-files that use `feval` execute faster and can be compiled with the MATLAB compiler.

Constructing Strings for Evaluation

You can concatenate strings to create a complete expression for input to `eval`. This code shows how `eval` can create 10 variables named `P1`, `P2`, ...`P10`, and set each of them to a different value.

```
for i=1:10
    eval(['P',int2str(i),'= i.^2'])
end
```

Command/Function Duality

MATLAB commands are statements like

```
load
help
```

Many commands accept modifiers that specify operands.

```
load August17.dat
help magic
type rank
```

An alternate method of supplying the command modifiers makes them string arguments of functions.

```
load('August17.dat')
help('magic')
type('rank')
```

This is MATLAB's "command/function duality." Any command of the form

```
command argument
```

can also be written in the functional form

```
command('argument')
```

The advantage of the functional approach comes when the string argument is constructed from other pieces. The following example processes multiple data files, `August1.dat`, `August2.dat`, and so on. It uses the function `int2str`, which converts an integer to a character string, to help build the filename.

```
for d = 1:31
   s = ['August' int2str(d) '.dat']
   load(s)
% Process the contents of the d-th file
end
```

Empty Matrices

Earlier versions of MATLAB allowed for only one empty matrix, the 0-by-0 array denoted by [ ]. MATLAB 5 provides for matrices and arrays where one, but not all, of the dimensions is zero. For example, 1-by-0, 10-by-0-by-20, and [3 4 0 5 2] are all possible array sizes.

The two-character sequence[ ] continues to denote the 0-by-0 matrix. Empty arrays of other sizes can be created with the functions zeros, ones, rand, or eye. To create a 0-by-5 matrix, for example, use,

```
E = zeros(0,5)
```

The basic model for empty matrices is that any operation that is defined for m-by-n matrices, and that produces a result whose dimension is some function of m and n, should still be allowed when m or n is zero. The size of the result should be that same function, evaluated at zero.

For example, horizontal concatenation

```
C = [A B]
```

requires that A and B have the same number of rows. So if A is m-by-n and B is m-by-p, then C is *m*-by-(*n*+*p*). This is still true if m or n or p is zero.

Many operations in MATLAB produce row vectors or column vectors. It is possible for the result to be the empty row vector.

```
r = zeros(1,0)
```

or the empty column vector

```
C = zeros(0,1)
```

MATLAB 5 retains MATLAB 4 behavior for if and while statements. For example,

```
if A, S1, else, S0, end
```

executes statement S0 when A is an empty array.

Some MATLAB functions, like `sum` and `max`, are *reductions*. For matrix arguments, these functions produce vector results; for vector arguments they produce scalar results. Empty inputs produce the following results with these functions:

- `sum`([ ]) is `0`
- `prod`([ ]) is `1`
- `max`([ ]) is [ ]
- `min`([]) is [ ]

Errors and Warnings

In many cases, it's desirable to take specific actions when different kinds of errors occur. For example, you may want to prompt the user for more input, display extended error or warning information, or repeat a calculation using default values. MATLAB's error handling capabilities let your application check for particular error conditions and execute appropriate code depending on the situation.

Error Handling with eval and lasterr

The basic tools for error-handling in MATLAB are:

- The `eval` function, which lets you execute a function and specify a second function to execute if an error occurs in the first.
- The `lasterr` function, which returns a string containing the last error generated by MATLAB.

The `eval` function provides error-handling capabilities using the two-argument form

```
eval ('trystring','catchstring')
```

If the operation specified by `trystring` executes properly, `eval` simply returns. If `trystring` generates an error, the function evaluates `catchstring`. Use `catchstring` to specify a function that determines the error generated by `trystring` and takes appropriate action.

The `trystring`/`catchstring` form of `eval` is especially useful in conjunction with the `lasterr` function. `lasterr` returns a string containing the last error message generated by MATLAB. Use `lasterr` inside the `catchstring` function to "catch" the error generated by `trystring`.

For example, this function uses `lasterr` to check for a specific error message that can occur during matrix multiplication. The error message indicates that matrix multiplication is impossible because the operands have different inner dimensions. If the message occurs, the code truncates one of the matrices to perform the multiplication.

```
function C = catchfcn(A,B)
l = lasterr;
j = findstr(l,'Inner matrix dimensions')
if (~isempty(j))
    [m,n] = size(A)
    [p,q] = size(B)
    if (n>p)
        A(:,p+1:n) = []
    elseif (n<p)
        B(n+1:p,:) = []
    end
    C = A*B;
else
    C = 0;
end
```

This example uses the two-argument form of eval with the catchfcn function shown above.

```
clear
A = [1  2  3; 6  7  2; 0  1  5];
B = [9  5  6; 0  4  9];
eval('A*B','catch(A,B)')

A = 1:7;
B = randn(9,9);
eval('A*B','catchfcn(A,B)')
```

Displaying Error and Warning Messages

Use the error and fprintf functions to display error information on the screen. The error function has the syntax

```
error('error string')
```

If you call the error function from inside an M-file, error displays the text in the quoted string and causes the M-file to stop executing. For example, suppose the following appears inside the M-file myfile.m.

```
if n < 1
  error('n must be 1 or greater.')
end
```

For n equal to 0, the following text appears on the screen and the M-file stops.

```
??? Error using ==> myfile
n must be 1 or greater.
```

In MATLAB, warnings are similar to error messages, except program execution does not stop. Use the warning function to display warning messages.

```
warning('warning string')
```

The function lastwarn displays the last warning message issued by MATLAB.

Times and Dates

MATLAB provides functions for time and date handling. These functions are in a directory called `timefun` in the MATLAB Toolbox.

Category	Function	Description
Current time and date	`now`	Current date and time as serial date number.
	`date`	Current date as date string.
	`clock`	Current date and time as date vector.
Conversion	`datenum`	Convert to serial date number.
	`datestr`	Convert to string representation of date.
	`datevec`	Date components.
Utility	`calendar`	Calendar.
	`weekday`	Day of the week.
	`eomday`	End of month.
	`datetick`	Date formatted tick labels.
Timing	`cputime`	CPU time in seconds.
	`tic`, `toc`	Stopwatch timer.
	`etime`	Elapsed time.

Date Formats

MATLAB works with three different date formats: date strings, serial date numbers, and date vectors.

When dealing with dates you typically work with date strings (16-Sep-1996). MATLAB works internally with *serial date numbers* (729284). A serial date represents a calendar date as the number of days that has passed since a fixed base date. In MATLAB, serial date number 1 is January 1, 0000. MATLAB also uses serial time to represent fractions of days beginning at midnight; for example, 6 p.m. equals 0.75 serial days. So the string `'16-Sep-1996, 6:00 pm'` in MATLAB is date number 729284.75.

All functions that require dates accept either date strings or serial date numbers. If you are dealing with a few dates at the MATLAB command-line level, date strings are more convenient. If you are using functions that handle large numbers of dates or doing extensive calculations with dates, you will get better performance if you use date numbers.

Date vectors are an internal format for some MATLAB functions; you do not typically use them in calculations. A date vector contains the elements `[year month day hour minute second]`.

MATLAB provides functions that convert date strings to serial date numbers, and vice versa. Dates can also be converted to date vectors.

Here are examples of the three date formats used by MATLAB.

Date string	`02-Oct-1996`
Serial date number	`729300`
Date vector	`1996 10 2 0 0 0`

Conversions Between Date Formats

Functions that convert between date formats are:

`datenum`	Convert date string to serial date number
`datestr`	Convert serial date number to date string
`datevec`	Split date number or date string into individual date elements

Here are some examples of conversions from one date format to another.

```
d1 = datenum(‘02-Oct-1996’)

d1 =
   729300

d2 = datestr(d1+10)

d2 =
12-Oct-1996
dv1 = datevec(d1)

dv1 =
   199610  2   0   0   0

dv2 = datevec(d2)

dv2 =
   199610  12  0   0   0
```

Date String Formats

The `datenum` function is important for doing date calculations efficiently. `datenum` takes an input string in any of several formats, with `'dd-mmm-yyyy'`, `'mm/dd/yyyy'`, or `'dd-mmm-yyyy, hh:mm:ss.ss'` most common. You can form up to six fields from letters and digits separated by any other characters.

- The day field is an integer from 1 to 31.
- The month field is either an integer from 1 to 12 or an alphabetic string with at least three characters.
- The year field is a non-negative integer: if only two digits are specified, then a year 19yy is assumed; if the year is omitted, then the current year is used as a default.
- The hours, minutes, and seconds fields are optional. They are integers separated by colons or followed by ’AM’ or ’PM’.

For example, if the current year is 1996, then these are all equivalent

```
'17-May-1996'
'17-May-96'
'17-May'
'May 17, 1996'
'5/17/96'
'5/17'
```

and both of these represent the same time

```
'17-May-1996, 18:30'
'5/17/96/6:30 pm'
```

Note that the default format for numbers-only input follows the American convention. Thus `3/6` is March 6, not June 3.

If you create a vector of input date strings, use a column vector and be sure all strings are the same length. Fill in with spaces or zeros.

Output Formats

The function `datestr(D,dateform)` converts a serial date `D` to one of 19 different date string output formats showing date, time, or both. The default output for dates is a day-month-year string: `01-Mar-1996`. You select an alternative output format by using the optional integer argument `dateform`.

dateform	Format	Description
0	`01-Mar-1996 15:45:17`	day-month-year hour:minute:second
1	`01-Mar-1996`	day-month-year
2	`03/01/96`	month/day/year
3	`Mar`	month, three letters
4	`M`	month, single letter
5	`3`	month
6	`03/01`	month/day
7	`1`	day of month
8	`Wed`	day of week, three letters
9	`W`	day of week, single letter
10	`1996`	year, four digits
11	`96`	year, two digits
12	`Mar96`	month year
13	`15:45:17`	hour:minute:second
14	`03:45:17 PM`	hour:minute:second AM or PM
15	`15:45`	hour:minute
16	`03:45 PM`	hour:minute AM or PM
17	`Q1-96`	calendar quarter-year
18	`Q1`	calendar quarter

Here are some examples of converting the date March 1, 1996 to various forms using the `datestr` function.

```
d = ’01-Mar-1996’

d =

01-Mar-1996

datestr(d)

ans =
     01-Mar-1996

datestr(d, 2)

ans =
     03/01/96

datestr(d, 17)

ans =
     Q1-96
```

Current Date and Time

The function `date` returns a string for today's date.

```
date

ans =
     02-Oct-1996
```

The function `now` returns the serial date number for the current date and time.

```
now

ans =
     729300.71

datestr(now)

ans =
     02-Oct-1996 16:56:16

datestr(floor(now))

ans =
     02-Oct-1996
```

Obtaining User Input

To obtain input from a user during M-file execution, you can:

- Display a prompt and obtain keyboard input.
- Pause until the user presses a key.
- Build a complete graphical user interface.

This section covers the first two topics. The third topic is discussed in *Using MATLAB Graphics* and *Building GUIs with MATLAB*.

Prompting for Keyboard Input

The input function displays a prompt and waits for a user response. Its syntax is:

```
n = input('prompt_string')
```

The function displays the *prompt_string*, waits for keyboard input, and then returns the value from the keyboard. If the user inputs an expression, the function evaluates it and returns its value. This function is useful for implementing menu-driven applications.

input can also return user input as a string, rather than a numeric value. To obtain string input, append 's' to the function's argument list.

```
name = input('Enter address: ','s');
```

Pausing During Execution

Some M-files benefit from pauses between execution steps. For example, the petals.m script shown on page 10-5 pauses between the plots it creates, allowing the user to display a plot for as long as desired and then press a key to move to the next plot.

The pause command, with no arguments, stops execution until the user presses a key. To pause for n seconds, use:

```
pause(n)
```

Shell Escape Functions

It is sometimes useful to access your own C or Fortran programs using *shell escape functions*. Shell escape functions use the shell escape command `!` to make external stand-alone programs act like new MATLAB functions. A shell escape M-function is an M-file that:

1. Saves the appropriate variables on disk.
2. Runs an external program (which reads the data file, processes the data, and writes the results back out to disk).
3. Loads the processed file back into the workspace.

For example, look at the code for `garfield.m`, below. This function uses an external function, `gareqn`, to find the solution to Garfield's equation.

```
function y = garfield(a,b,q,r)
save gardata a b q r
!gareqn
load gardata
```

This M-file:

1. Saves the input arguments `a`, `b`, `q`, and `r` to a MAT-file in the workspace using the `save` command.
2. Uses the shell escape operator to access a C, or Fortran program called `gareqn` that uses the workspace variables to perform its computation. `gareqn` writes its results to the `gardata` MAT-file.
3. Loads the `gardata` MAT-file to obtain the results.

Reading and Writing MAT Files The MAT-file subroutine library, described in the *MATLAB Application Program Interface Guide*, provides routines for reading and writing MAT-files. Also see this document for information on MEX-files, and C or Fortran functions that you can call directly from MATLAB.

Optimizing the Performance of MATLAB Code

This section describes techniques that often improve the execution speed and memory management of MATLAB code:

- Vectorization of loops
- Vector preallocation

MATLAB is a matrix language, which means it is designed for vector and matrix operations. For best performance, you should take advantage of this where possible.

Vectorization of Loops

You can speed up your M-file code by vectorizing algorithms. *Vectorization* means converting `for` and `while` loops to equivalent vector or matrix operations.

A Simple Example

Here is one way to compute the sine of 1001 values ranging from 0 to 10.

```
i = 0;
for t = 0:.01:10
    i = i+1;
    y(i) = sin(t);
end
```

A vectorized version of the same code is:

```
t = 0:.01:10;
y = sin(t);
```

The second example executes much faster than the first and is the way MATLAB is meant to be used. Test this on your system by creating M-file scripts that contain the code shown, then using the `tic` and `toc` commands to time the M-files.

An Advanced Example

`repmat` is an example of a function that takes advantage of vectorization. It accepts three input arguments: an array `A`, a row dimension `M`, and a column dimension `N`.

repmat creates an output array that contains the elements of array A, replicated and “tiled” in an M-by-N arrangement.

```
A = [1 2 3; 4 5 6];
B = repmat(A,2,3);

B =

    1    2    3    1    2    3    1    2    3
    4    5    6    4    5    6    4    5    6
    1    2    3    1    2    3    1    2    3
    4    5    6    4    5    6    4    5    6
```

repmat uses vectorization to create the indices that place elements in the output array.

```
function B = repmat(A,M,N)
if nargin < 2
   error('Requires at least 2 inputs.')
elseif nargin == 2
   N = M;
end
[m,n] = size(A);
mind = (1:m)';
nind = (1:n)';
mind = mind(:,ones(1,M));
nind = nind(:,ones(1,N));
B = A(mind,nind);
```

1. Get row and column sizes.

2. Generate vectors of indices from 1 to row/column size.

3. Create index matrices from vectors above.

1 above obtains the row and column sizes of the input array.

2 above creates two column vectors. mind contains the integers from 1 through the row size of A. The nind variable contains the integers from 1 through the column size of A.

3 above uses a MATLAB vectorization trick to replicate a single column of data through any number of columns. The code is:

```
B = A(:,ones(1,n_cols))
```

where n_cols is the desired number of columns in the resulting matrix.

The final line of repmat uses array indexing to create the output array. Each element of the row index array, mind, is paired with each element of the column index array, nind:

1 The first element of mind, the row index, is paired with each element of nind. MATLAB moves through the nind matrix in a columnwise fashion, so mind(1,1) goes with nind(1,1), then nind(2,1), and so on. The result fills the first row of the output array.

2 Moving columnwise through mind, each element is paired with the elements of nind as above. Each complete pass through the nind matrix fills one row of the output array.

Array Preallocation

You can often improve code execution time by preallocating the arrays that store output results. Preallocation prevents MATLAB from having to resize an array each time you enlarge it. Use the appropriate preallocation function for the kind of array you are working with.

Array Type	Function	Examples
Numeric array	zeros	`y = zeros(1,100);`

Cell array	cell	`B = cell(2,3);`
		`B{1,3} = 1:3;`
		`B{2,2} = 'string';`

Structure array	struct,
	repmat
		`data = repmat(struct('x',[1 3],...`
		`       'y',[5 6]), 1, 3);`

Preallocation also helps reduce memory fragmentation if you work with large matrices. In the course of a MATLAB session, memory can become fragmented due to dynamic memory allocation and deallocation. This can result in plenty of free memory, but not enough contiguous space to hold a large variable. Preallocation helps prevent this by allowing MATLAB to “grab” sufficient space for large data constructs at the beginning of a computation.

Notes on Memory Use

This section discusses ways to conserve memory and improve memory use.

Memory Management Functions

MATLAB has five functions to improve how memory is handled:

- `clear` removes variables from memory.
- `pack` saves existing variables to disk, then reloads them contiguously. Because of time considerations, you should not use `pack` within loops or M-file functions.
- `quit` exits MATLAB and returns all allocated memory to the system.
- `save` selectively stores variables to disk.
- `load` reloads a data file saved with the `save` command.

Note `save` and `load` are faster than MATLAB low-level file I/O routines. `save` and `load` have been optimized to run faster and reduce memory fragmentation. See Chapter 2, “MATLAB Working Environment” for details on these functions.

On some systems, the `whos` command displays the amount of free memory remaining. However, be aware that:

- If you delete a variable from the workspace, the amount of free memory indicated by `whos` usually does not get larger unless the deleted variable occupied the highest memory addresses. The number actually indicates the amount of contiguous, unused memory. Clearing the highest variable makes the number larger, but clearing a variable beneath the highest variable has no effect. This means that you might have more free memory than is indicated by `whos`.
- Computers with virtual memory do not display the amount of free memory remaining because neither MATLAB nor the hardware imposes limitations.

Removing a Function From Memory

MATLAB creates a list of M- and MEX-filenames at startup for all files that reside below the `matlab/toolbox` directories. This list is stored in memory and is freed only when a new list is created during a call to the path function. Function M-file code and MEX-file relocatable code are loaded into memory when the corresponding function is called. The M-file code or relocatable code is removed from memory when:

- The function is called again and a new version now exists.
- The function is explicitly cleared with the clear command.
- All functions are explicitly cleared with the clear functions command.
- MATLAB runs out of memory.

Nested Function Calls

The amount of memory used by nested functions is the same as the amount used by calling them on consecutive lines. These two examples require the same amount of memory.

```
result = function2(function1(input99));
```

```
result = function1(input99);
result = function2(result);
```

Variables and Memory

Memory is allocated for variables whenever the left-hand side variable in an assignment does not exist. The statement

```
x = 10
```

allocates memory, but the statement

```
x(10) = 1
```

does not allocate memory if the 10th element of x exists.

To conserve memory:

- Avoid creating large temporary variables, and clear temporary variables when they are no longer needed.
- Avoid using the same variables as inputs and outputs to a function. They will be copied by reference. For example,

  ```
  y = fun(x,y)
  ```

  is not preferred because `y` is both an input and an output variable.
- Set variables equal to the empty matrix `[]` to free memory, or clear them using

  ```
  clear variable_name
  ```

- Reuse variables as much as possible.

Global Variables. Declaring variables as `global` merely puts a flag in a symbol table. It does not use any more memory than defining nonglobal variables. Consider the following example.

```
global a
a = 5;
```

Now there is one copy of `a` stored in the MATLAB workspace. Typing

```
clear a
```

removes `a` from the MATLAB workspace, but it still exists in the `global` workspace.

```
clear global a
```

removes `a` from the global workspace.

PC-Specific Topics

- There are no functions implemented to manipulate the way MATLAB handles Microsoft Windows system resources. Windows uses system resources to track fonts, windows, and screen objects. Resources can be depleted by using multiple figure windows, multiple fonts, or several Uicontrols. The best way to free up system resources is to close all inactive windows. Iconified windows still use resources.

- The performance of a permanent swap file is typically better than a temporary swap file.
- Typically a swap file twice the size of the installed RAM is sufficient.

UNIX-Specific Topics

- Memory that MATLAB requests from the operating system is not returned to the operating system until the MATLAB process in finished.
- MATLAB requests memory from the operating system when there is not enough memory available in the MATLAB heap to store the current variables. It reuses memory in the heap as long as the size of the memory segment required is available in the MATLAB heap.

  For example, on one machine these statements use approximately 15.4 MB of RAM.

  ```
  a = rand(1e6,1);
  b = rand(1e6,1);
  ```

  These statements use approximately 16.4 MB of RAM.

  ```
  c = rand(2.1e6,1);
  ```

  These statements use approximately 32.4 MB of RAM.

  ```
  a = rand(1e6,1);
  b = rand(1e6,1);
  clear
  c = rand(2.1e6,1);
  ```

  This is because MATLAB is not able to fit a 2.1 MB array in the space previously occupied by two 1 MB arrays. The simplest way to prevent overallocation of memory, is to preallocate the largest vector. This series of statements uses approximately 32.4 MB of RAM

  ```
  a = rand(1e6,1);
  b = rand(1e6,1);
  clear
  c = rand(2.1e6,1);
  ```

while these statements use only about 16.4 MB of RAM

```
c = rand(2.1e6,1);
clear
a = rand(1e6,1);
b = rand(1e6,1);
```

Allocating the largest vectors first allows for optimal use of the available memory.

What Does “Out of Memory” Mean?

Typically the `Out of Memory` message appears because MATLAB asked the operating system for a segment of memory larger than what is currently available. Use any of the techniques discussed in this section to help optimize the available memory. If the `Out of Memory` message still appears:

- Increase the size of the swap file.
- Make sure that there are no external constraints on the memory accessible to MATLAB (on UNIX systems use the limit command to check).
- Add more memory to the system.
- Reduce the size of your data.

Character Arrays (Strings)

This chapter explains MATLAB's support for string data. It describes how to create character arrays and cell arrays of strings, the two ways to represent strings. It also discusses how to perform common string operations, such as searching and replacing, and how to convert between string and numeric formats.

The string functions are located in the directory named `strfun` in the MATLAB Toolbox.

Category	Function	Description
General	`char`	Create character array (string).
	`double`	Convert string to numeric codes.
	`cellstr`	Create cell array of strings from character array.
	`blanks`	String of blanks.
	`deblank`	Remove trailing blanks.
	`eval`	Execute string with MATLAB expression.
String Tests	`ischar`	True for character array.
	`iscellstr`	True for cell array of strings.
	`isletter`	True for letters of alphabet.
	`isspace`	True for whitespace characters.
String Operations	`strcat`	Concatenate strings.
	`strvcat`	Concatenate strings vertically.
	`strcmp`	Compare strings.
	`strncmp`	Compare first N characters of strings.
	`findstr`	Find one string within another.
	`strjust`	Justify string.
	`strmatch`	Find matches for string.

Category	Function	Description
	`strrep`	Replace string with another.
	`strtok`	Find token in string.
	`upper`	Convert string to uppercase.
	`lower`	Convert string to lowercase.
String to Number Conversion	`num2str`	Convert number to string.
	`int2str`	Convert integer to string.
	`mat2str`	Convert matrix to `eval`'able string.
	`str2num`	Convert string to number.
	`sprintf`	Write formatted data to string.
	`sscanf`	Read string under format control.
Base Number Conversion	`hex2num`	Convert IEEE hexadecimal to double precision number.
	`hex2dec`	Convert hexadecimal string to decimal integer.
	`dec2hex`	Convert decimal integer to hexadecimal string.
	`bin2dec`	Convert binary string to decimal integer.
	`dec2bin`	Convert decimal integer to binary string.
	`base2dec`	Convert base B string to decimal integer.
	`dec2base`	Convert decimal integer to base B string.

Character Arrays

In MATLAB, the term *string* refers to an array of characters. MATLAB represents each character internally as its corresponding numeric value. Unless you want to access these values, however, you can simply work with the characters as they display on screen.

Specify character data by placing characters inside a pair of single quotes. For example, this line creates a 1-by-13 character array called `name`.

```
name = ’Thomas R. Lee’;
```

In the workspace, the output of `whos` shows

```
Name          Size             Bytes  Class

name          1x13                26  char array
```

You can see that a character uses two bytes of storage internally.

The `class` and `ischar` functions show `name`’s identity as a character array.

```
class(name)

ans =
char

ischar(name)

ans =
     1
```

Converting Between Characters and Numeric Values

Character arrays store each character as a 16-bit numeric value. Use the double function to convert strings to their numeric values, and char to revert to character representation.

```
name = double(name)

name =
  Columns 1 through 12

    84   104   111   109    97   115    32    82    46    32    76   101

  Column 13

   101

name = char(name)
name =

Thomas R. Lee
```

Creating Two-Dimensional Character Arrays

When creating a two-dimensional character array, be sure that each row has the same length. For example, this line is legal because both input rows have exactly 13 characters.

```
name = ['Thomas R. Lee' ; 'Sr. Developer']

name =

Thomas R. Lee
Sr. Developer
```

When creating character arrays from strings of different lengths, you can pad the shorter strings with blanks to force rows of equal length.

```
name = ['Thomas R. Lee   '; 'Senior Developer']
```

A simpler way to create string arrays is to use the char function. char automatically pads all strings to the length of the longest input string. In this example, char pads the 13-character input string ’Thomas R. Lee’ with three trailing blanks so that it will be as long as the second string.

```
name = char(’Thomas R. Lee’,’Senior Developer’)

name =

Thomas R. Lee
Senior Developer
```

When extracting strings from an array, use the deblank function to remove any trailing blanks.

```
trimname = deblank(name(1,:))

trimname =

Thomas R. Lee

size(trimname)

ans =

     1    13
```

Cell Arrays of Strings

It’s often convenient to store groups of strings in cell arrays instead of standard character arrays. This prevents you from having to pad strings with blanks to create character arrays with rows of equal length. A set of functions enables you to work with cell arrays of strings:

- You can convert between standard character arrays and cell arrays of strings.
- You can apply string comparison operations to cell arrays of strings.

For details on cell arrays see the *Structures and Cell Arrays* chapter.

Converting Between Character Arrays and Cell Arrays of Strings

The cellstr function converts a character array into a cell array of strings. Consider the character array

```
data = [’Allison Jones’;’Development  ’;’Phoenix      ’]
```

Each row of the matrix is padded so that all have equal length (in this case, 13 characters).

Now use cellstr to create a column vector of cells, each cell containing one of the strings from the data array.

```
celldata = cellstr(data)

celldata =
    ’Allison Jones’
    ’Development’
    ’Phoenix’
```

Note that the cellstr function strips off the blanks that pad the rows of the input string matrix:

```
length(celldata{3})

ans =

    7
```

Use char to convert back to a standard padded character array.

```
strings = char(celldata)

strings =

Allison Jones
Development
Phoenix
```

String Comparisons

There are several ways to compare strings and substrings:

- You can compare two strings, or parts of two strings, for equality.
- You can compare individual characters in two strings for equality.
- You can categorize every element within a string, determining whether each element is a character or whitespace.

These functions work for both character arrays and cell arrays of strings.

Comparing Strings For Equality

There are two functions that determine if two input strings are identical:

- `strcmp` determines if two strings are identical.
- `strncmp` determines if the first `n` characters of two strings are identical.

Consider the two strings

```
str1 = 'hello';
str2 = 'help';
```

Strings `str1` and `str2` are not identical, so invoking `strcmp` returns `0` (false). For example,

```
C = strcmp(str1,str2)

C =

     0
```

Note for C programmers This is an important difference between MATLAB's `strcmp` and C's `strcmp()`, which returns 0 if the two strings are the same.

The first three characters of str1 and str2 are identical, so invoking strncmp with any value up to 3 returns 1.

```
C = strncmp(str1,str2,2)

C =
   1
```

These functions work cell-by-cell on a cell array of strings. Consider the two cell arrays of strings

```
A = {’pizza’;’chips’;’candy’};
B = {’pizza’;’chocolate’;’pretzels’};
```

Now apply the string comparison functions.

```
strcmp(A,B)


ans =
    1
    0
    0

strncmp(A,B,1)

ans =
    1
    1
    0
```

Comparing Characters for Equality with Operators

You can use MATLAB relational operators on character arrays, as long as the arrays you are comparing have equal dimensions, or one is a scalar. For example, you can use the equality operator (==) to determine which characters in two strings match.

```
A = 'fate';
B = 'cake';
A == B

ans =
   0   1   0   1
```

All of the relational operators (>, >=, <, <=, ==, !=) compare the values of corresponding characters.

Categorizing Characters Within a String

There are two functions for categorizing characters inside a string:

- isletter determines if a character is a letter
- isspace determines if a character is whitespace (blank, tab, or new line)

For example, create a string named mystring.

```
mystring = 'Room 401';
```

isletter examines each character in the string, producing an output vector of the same length as mystring.

```
A = isletter(mystring)

A =
1   1   1   1   0   0   0   0
```

The first four elements in A are 1 (true) because the first four characters of mystring are letters.

Searching and Replacing

MATLAB provides several functions for searching and replacing characters in a string. Consider a string named label.

```
label = ’Sample 1, 10/28/95’;
```

The strrep function performs the standard search-and-replace operation. Use strrep to change the date from ’10/28’ to ’10/30’.

```
newlabel = strrep(label,’28’,’30’)

newlabel =
Sample 1, 10/30/95
```

findstr returns the starting position of a substring within a longer string. To find all occurrences of the string ’amp’ inside label

```
position = findstr(’amp’,label)

position =
     2
```

The position within label where the only occurrence of ’amp’ begins is the second character.

The strtok function returns the characters before the first occurrence of a delimiting character in an input string. The default delimiting characters are the set of whitespace characters. You can use the strtok function to parse a sentence into words; for example,

```
function all_words = words(input_string)
remainder = input_string;
all_words = ’’;

while (any(remainder))
  [chopped,remainder] = strtok(remainder);
  all_words = strvcat(all_words,chopped);
end
```

String / Numeric Conversion

MATLAB's string/numeric conversion functions change numeric values into character strings. You can store numeric values as digit-by-digit string representations, or convert a value into a hexadecimal or binary string. Consider a the scalar

```
x = 5317;
```

By default, MATLAB stores the number x as a 1-by-1 double array containing the value 5317. The int2str (integer to string) function breaks this scalar into a 1-by-4 vector containing the string '5317'.

```
y = int2str(x);
size(y)

ans =
     1     4
```

A related function, num2str, provides more control over the format of the output string. An optional second argument sets the number of digits in the output string, or specifies an actual format.

```
p = num2str(pi,9)

p =
3.14159265
```

Both int2str and num2str are handy for labeling plots. For example, the following lines use num2str to prepare automated labels for the *x*-axis of a plot.

```
function plotlabel(x,y)
plot(x,y)
str1 = num2str(min(x));
str2 = num2str(max(x));
out = ['Value of f from ' str1 ' to ' str2];
xlabel(out);
```

Another class of numeric/string conversion functions changes numeric values into strings representing a decimal value in another base, such as binary or hexadecimal representation. For example, the dec2hex function converts a decimal value into the corresponding hexadecimal string.

```
dec_num = 4035
hex_num = dec2hex(dec_num)

hex_num =

FC3
```

See the `strfun` directory for a complete listing of string conversion functions.

Array/String Conversion

The MATLAB function `mat2str` changes an array to a string that MATLAB can evaluate. This string is useful input for a function such as `eval`, which evaluates input strings just as if they were typed at the MATLAB command line.

Create a 2-by-3 array `A`.

```
A = [1 2 3; 4 5 6]

A =

     1     2     3
     4     5     6
```

`mat2str` returns a string that contains the text you would enter to create `A` at the command line

```
B = mat2str(A)

B =

[1 2 3; 4 5 6]
```

12

Multidimensional Arrays

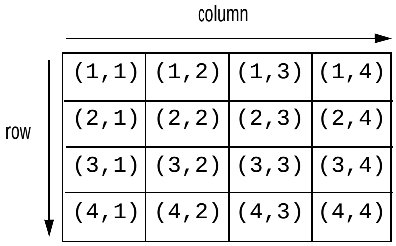
This chapter discusses *multidimensional arrays*, MATLAB arrays with more than two dimensions. Multidimensional arrays can be numeric, character, cell, or structure arrays.

Multidimensional arrays are broadly useful—for example, in the representation of multivariate data, or multiple pages of two-dimensional data. MATLAB provides a number of functions that directly support multidimensional arrays. You can extend this support by creating M-files that work with your data architecture.

Function	Description
`cat`	Concatenate arrays.
`ndgrid`	Generate arrays for N-D functions and interpolation.
`ndims`	Number of array dimensions.
`ipermute`	Inverse permute array dimensions.
`permute`	Permute array dimensions.
`shiftdim`	Shift dimensions.
`squeeze`	Remove singleton dimensions.

Multidimensional Arrays

Multidimensional arrays in MATLAB are an extension of the normal two-dimensional matrix. Matrices have two dimensions: the row dimension and the column dimension.

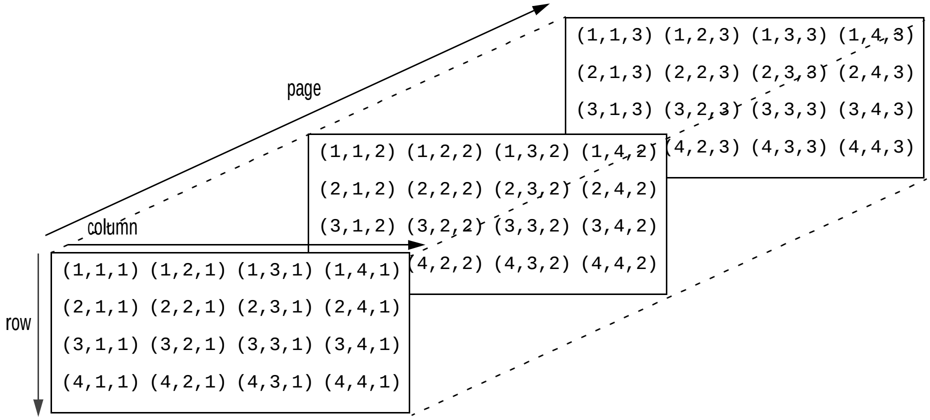


(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)
(4,1)	(4,2)	(4,3)	(4,4)

You can access a two-dimensional matrix element with two subscripts: the first representing the row index, and the second representing the column index.

Multidimensional arrays use additional subscripts for indexing. A three-dimensional array, for example, uses three subscripts:

- The first references array dimension 1, the row.
- The second references dimension 2, the column.
- The third references dimension 3. This guide uses the concept of a *page* to represent dimensions 3 and higher.

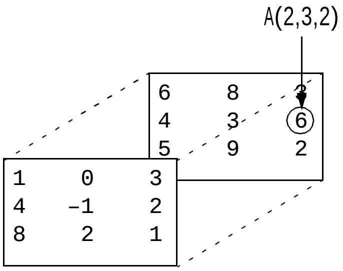


(1,1,3)	(1,2,3)	(1,3,3)	(1,4,3)
(2,1,3)	(2,2,3)	(2,3,3)	(2,4,3)
(3,1,3)	(3,2,3)	(3,3,3)	(3,4,3)
	(4,2,3)	(4,3,3)	(4,4,3)

(1,1,2)	(1,2,2)	(1,3,2)	(1,4,2)
(2,1,2)	(2,2,2)	(2,3,2)	(2,4,2)
(3,1,2)	(3,2,2)	(3,3,2)	(3,4,2)
	(4,2,2)	(4,3,2)	(4,4,2)

(1,1,1)	(1,2,1)	(1,3,1)	(1,4,1)
(2,1,1)	(2,2,1)	(2,3,1)	(2,4,1)
(3,1,1)	(3,2,1)	(3,3,1)	(3,4,1)
(4,1,1)	(4,2,1)	(4,3,1)	(4,4,1)

To access the element in the second row, third column of page 2, for example, you use the subscripts (2,3,2).



```
A(:,:,1) =

     1     0     3
     4    -1     2
     8     2     1


A(:,:,2) =

     6     8     3
     4     3     6
     5     9     2
```

As you add dimensions to an array, you also add subscripts. A four-dimensional array, for example, has four subscripts. The first two reference a row-column pair; the second two access the third and fourth dimensions of data.

Note The general multidimensional array functions reside in the datatypes directory.

Creating Multidimensional Arrays

You can use the same techniques to create multidimensional arrays that you use for two-dimensional matrices. In addition, MATLAB provides a special concatenation function that is useful for building multidimensional arrays.

This section discusses:

- Generating arrays using indexing
- Generating arrays using MATLAB functions
- Using the cat function to build multidimensional arrays

Generating Arrays Using Indexing

One way to create a multidimensional array is to create a two-dimensional array and extend it. For example, begin with a simple two-dimensional array A.

```
A = [5 7 8; 0 1 9; 4 3 6];
```

A is a 3-by-3 array, that is, its row dimension is 3 and its column dimension is 3. To add a third dimension to A,

```
A(:,:,2) = [1 0 4; 3 5 6; 9 8 7]
```

MATLAB responds with

```
A(:,:,1) =

     5     7     8
     0     1     9
     4     3     6

A(:,:,2) =

     1     0     4
     3     5     6
     9     8     7
```

You can continue to add rows, columns, or pages to the array using similar assignment statements.

Extending Multidimensional Arrays. To extend A in any dimension:

- Increment or add the appropriate subscript and assign the desired values.
- Assign the same number of elements to corresponding array dimensions. For numeric arrays, all rows must have the same number of elements, all pages must have the same number of rows and columns, and so on.

You can take advantage of MATLAB's scalar expansion capabilities, together with the colon operator, to fill an entire dimension with a single value.

```
A(:,:,3) = 5
A(:,:,3)

ans =

     5     5     5
     5     5     5
     5     5     5
```

To turn A into a 3-by-3-by-3-by-2, four-dimensional array, enter

```
A(:,:,1,2) = [1 2 3; 4 5 6; 7 8 9];
A(:,:,2,2) = [9 8 7; 6 5 4; 3 2 1];
A(:,:,3,2) = [1 0 1; 1 1 0; 0 1 1];
```

Note that after the first two assignments MATLAB pads A with zeros, as needed, to maintain the corresponding sizes of dimensions.

Generating Arrays Using MATLAB Functions

You can use MATLAB functions such as randn, ones, and zeros to generate multidimensional arrays in the same way you use them for two-dimensional arrays. Each argument you supply represents the size of the corresponding dimension in the resulting array. For example, to create a 4-by-3-by-2 array of normally distributed random numbers.

```
B = randn(4,3,2)
```

To generate an array filled with a single constant value, use the repmat function. repmat replicates an array (in this case, a 1-by-1 array) through a vector of array dimensions.

```
B = repmat(5,[3 4 2])

B(:,:,1) =

     5     5     5     5
     5     5     5     5
     5     5     5     5
```

```
B(:,:,2) =

     5     5     5     5
     5     5     5     5
     5     5     5     5
```

Note Any dimension of an array can have size zero, making it a form of empty array. For example, `10`-by-`0`-by-`20` is a valid size for a multidimensional array.

Using the cat Function to Build Multidimensional Arrays

The `cat` function is a simple way to build multidimensional arrays; it concatenates a list of arrays along a specified dimension.

```
B = cat(dim,A1,A2...)
```

where `A1`, `A2`, and so on are the arrays to concatenate, and `dim` is the dimension along which to concatenate the arrays. For example, to create a new array with `cat`

```
B = cat(3,[2 8; 0 5],[1 3; 7 9])

B(:,:,1) =

     2     8
     0     5


B(:,:,2) =

     1     3
     7     9
```

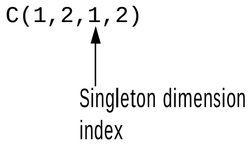
The cat function accepts any combination of existing and new data. In addition, you can nest calls to cat. The lines below, for example, create a four-dimensional array.

```
A = cat(3,[9 2; 6 5],[7 1; 8 4])
B = cat(3,[3 5; 0 1],[5 6; 2 1])
D = cat(4,A,B,cat(3,[1 2; 3 4],[4 3;2 1]))
```

cat automatically adds subscripts of 1 between dimensions, if necessary. For example, to create a 2-by-2-by-1-by-2 array, enter

```
C = cat(4,[1 2; 4 5],[7 8; 3 2])
```

In the previous case, cat inserts as many singleton dimensions as needed to create a four-dimensional array whose last dimension is not a singleton dimension. If the dim argument had been 5, the previous statement would have produced a 2-by-2-by-1-by-1-by-2 array. This adds additional 1s to indexing expressions for the array. To access the value 8 in the four-dimensional case, use

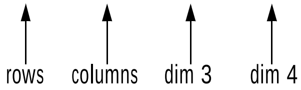


```
C(1,2,1,2)
```

Getting Information About Multidimensional Arrays

You can use MATLAB functions and commands to get information about multidimensional arrays you have created.

Information	Function	Example

Array size	`size`	`size(C)`
		
		`ans =`
		
		`     2     2     1     2`
		

Array dimensions	`ndims`	`ndims(C)`
		
		`ans =`
		
		`     4`

Array storage and format	`whos`	`whos`
		`Name     Size       Bytes   Class`
		
		`A        2x2x2         64   double array`
		`B        2x2x2         64   double array`
		`C        4-D           64   double array`
		`D        4-D          192   double array`
		
		`Grand total is 48 elements using 384 bytes`

Working with Multidimensional Arrays

Many of the concepts that apply to two-dimensional matrices extend to multidimensional arrays as well. This section describes how to apply basic indexing and reshaping techniques to multidimensional arrays.

Consider a 10-by-5-by-3 array `nddata` of random integers:

```
nddata = fix(8*randn(10,5,3));
```

Indexing

To access a single element of a multidimensional array, use integer subscripts. Each subscript indexes a dimension–the first indexes the row dimension, the second indexes the column dimension, the third indexes the first page dimension, and so on. To access element (3,2) on page 2 of nddata, for example, use nddata(3,2,2).

You can use vectors as array subscripts. In this case, each vector element must be a valid subscript, that is, within the bounds defined by the dimensions of the array. To access elements (2,1), (2,3), and (2,4) on page 3 of nddata, use

```
nddata(2,[1 3 4],3)
```

The Colon and Multidimensional Array Indexing

MATLAB's colon indexing extends to multidimensional arrays. For example, to access the entire third column on page 2 of nddata, use nddata(:,3,2).

The colon operator is also useful for accessing other subsets of data. For example, nddata(2:3,2:3,1) results in a 2-by-2 array, a subset of the data on page 1 of nddata. This matrix consists of the data in rows 2 and 3, columns 2 and 3, on the first page of the array.

The colon operator can appear as an array subscript on both sides of an assignment statement. For example, to create a 4-by-4 array of zeros

```
C = zeros(4,4)
```

Now assign a 2-by-2 subset of array nddata to the four elements in the center of C.

```
C(2:3,2:3) = nddata(2:3,1:2,2)
```

Avoiding Ambiguity in Multidimensional Indexing

Some assignment statements, such as

```
A(:,:,2) = 1:10
```

are ambiguous because they do not provide enough information about the shape of the dimension to receive the data. In the case above, the statement tries to assign a one-dimensional vector to a two-dimensional destination. MATLAB produces an error for such cases. To resolve the ambiguity, be sure you provide enough information about the destination for the assigned data, and that both data and destination have the same shape. For example,

```
A(1,:,2) = 1:10;
```

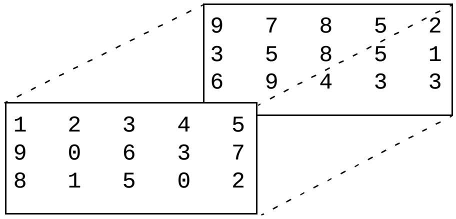
Reshaping

Unless you change its shape or size, a MATLAB array retains the dimensions specified at its creation. You change array size by adding or deleting elements. You change array shape by respecifying the array's row, column, or page dimensions while retaining the same elements. The reshape function performs the latter operation. For multidimensional arrays, its form is

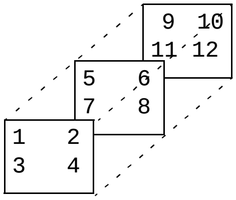
```
B = reshape(A,[s1 s2 s3 ...])
```

s1, s2, and so on represent the desired size for each dimension of the reshaped matrix. Note that a reshaped array must have the same number of elements as the original array (that is, the product of the dimension sizes is constant).

M	**reshape(M, [6 5])**


9 7 8 5 2
3 5 8 5 1
6 9 4 3 3
1 2 3 4 5
9 0 6 3 7
8 1 5 0 2
	1 3 5 7 5
	9 6 7 5 5
	8 5 2 9 3
	2 4 9 8 2
	0 3 3 8 1
	1 0 6 4 3

The `reshape` function operates in a columnwise manner. It creates the reshaped matrix by taking consecutive elements down each column of the original data construct.

<table>
<tr><th>C</th><th>reshape(C, [6 2])</th></tr>
<tr><td>


</td><td>
<pre>
1   6
3   8
2   9
4 11
5 10
7 12
</pre>
</td></tr>
</table>

Here are several new arrays from reshaping `nddata`.

```
B = reshape(nddata,[6 25])
C = reshape(nddata,[5 3 10])
D = reshape(nddata,[5 3 2 5])
```

Removing Singleton Dimensions

MATLAB creates singleton dimensions if you explicitly specify them when you create or reshape an array, or if you perform a calculation that results in an array dimension of one.

```
B = repmat(5,[2 3 1 4]);
size(B)

ans =

     2     3     1     4
```

The `squeeze` function removes singleton dimensions from an array.

```
C = squeeze(B);
size(C)

ans =

     2     3     4
```

The squeeze function does not affect two-dimensional arrays; row vectors remain rows.

Permuting Array Dimensions

The permute function reorders the dimensions of an array.

```
B = permute(A,dims);
```

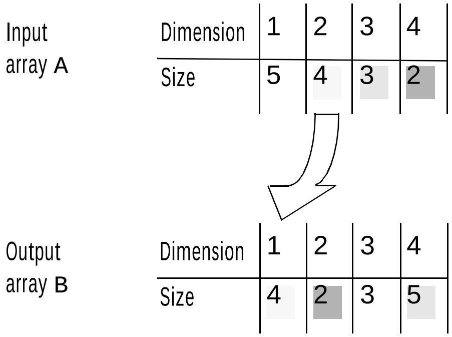
dims is a vector specifying the new order for the dimensions of A, where 1 corresponds to the first dimension (rows), 2 corresponds to the second dimension (columns), 3 corresponds to pages, and so on.

A	B= permute(A,[2 1 3])		C = permute(A,[3 2 1])	
A(:,:,1) = 1 2 3 4 5 6 7 8 9 A(:,:,2) = 0 5 4 2 7 6 9 3 1	B(:,:,1) = 1 4 7 2 5 8 3 6 9 B(:,:,2) = 0 2 9 5 7 3 4 6 1	Row and column subscripts are reversed (page-by-page transposition).	C(:,:,1) = 1 2 3 0 5 4 C(:,:,2) = 4 5 6 2 7 6 C(:,:,3) = 7 8 9 9 3 1	Row and page subscripts are reversed.

For a more detailed look at the permute function, consider a four-dimensional array A of size 5-by-4-by-3-by-2. Rearrange the dimensions, placing the column dimension first, followed by the second page dimension, the first page dimension, then the row dimension. The result is a 4 by 2 by 3 by 5 array.

```
B = permute(A,[2 4 3 1])
```

Move dimension 2 of A to first subscript position of B, dimension 4 to second subscript position, and so on.



Input array A

Dimension	1	2	3	4
Size	5	4	3	2

Output array B

Dimension	1	2	3	4
Size	4	2	3	5

The order of dimensions in permute's argument list determines the size and shape of the output array. In this example, the second dimension moves to the first position. Because the second dimension of the original array had size four, the output array's first dimension also has size four.

You can think of permute's operation as an extension of the transpose function, which switches the row and column dimensions of a matrix. For permute, the order of the input dimension list determines the reordering of the subscripts. In the example above, element (4,2,1,2) of A becomes element (2,2,1,4) of B, element (5,4,3,2) of A becomes element (4,2,3,5) of B, and so on.

Inverse Permutation

The ipermute function is the inverse of permute. Given an input array A and a vector of dimensions v, ipermute produces an array B such that permute(B,v) returns A.

For example, these statements create an array E that is equal to the input array C.

```
D = ipermute(C,[1 4 2 3]);
E = permute(D,[1 4 2 3])
```

You can obtain the original array after permuting it by calling ipermute with the same vector of dimensions.

Computation with Multidimensional Arrays

Many of MATLAB's computational and mathematical functions accept multidimensional arrays as arguments. These functions operate on specific dimensions of multidimensional arrays; that is, they operate on individual elements, on vectors, or on matrices.

Functions that Operate on Vectors

Functions that operate on vectors, like `sum`, `mean`, and so on, by default typically work on the first nonsingleton dimension of a multidimensional array. Most of these functions optionally let you specify a particular dimension on which to operate. There are exceptions, however. For example, the `cross` function, which finds the cross product of two vectors, works on the first nonsingleton dimension having length three.

Note In many cases, these functions have other restrictions on the input arguments – for example, some functions that accept multiple arrays require that the arrays be the same size. Refer to the online help for details on function arguments.

Functions that Operate Element-by-Element

MATLAB functions that operate element-by-element on two-dimensional arrays, like the trigonometric and exponential functions in the `elfun` directory, work in exactly the same way for multidimensional cases. For example, the `sin` function returns an array the same size as the function's input argument. Each element of the output array is the sine of the corresponding element of the input array.

Similarly, the arithmetic, logical, and relational operators all work with corresponding elements of multidimensional arrays that are the same size in every dimension. If one operand is a scalar and one an array, the operator applies the scalar to each element of the array.

Functions that Operate on Planes and Matrices

Functions that operate on planes or matrices, such as the linear algebra and matrix functions in the matfun directory, do not accept multidimensional arrays as arguments. That is, you cannot use the functions in the matfun directory, or the array operators *, ^, \, or /, with multidimensional arguments. Supplying multidimensional arguments or operands in these cases results in an error.

You can use indexing to apply a matrix function or operator to matrices within a multidimensional array. For example, create a three-dimensional array A:

```
A = cat(3,[1 2 3;9 8 7;4 6 5],[0 3 2;8 8 4;5 3 5],[6 4 7;6 8 5;...
5 4 3])
```

Applying the eig function to the entire multidimensional array results in an error.

```
eig(A)
??? Error using ==> eig
Input arguments must be 2-D.
```

You can, however, apply eig to planes within the array. For example, use colon notation to index just one page (in this case, the second) of the array.

```
eig(A(:,:,2))

ans =

   -2.6260
   12.9129
    2.7131
```

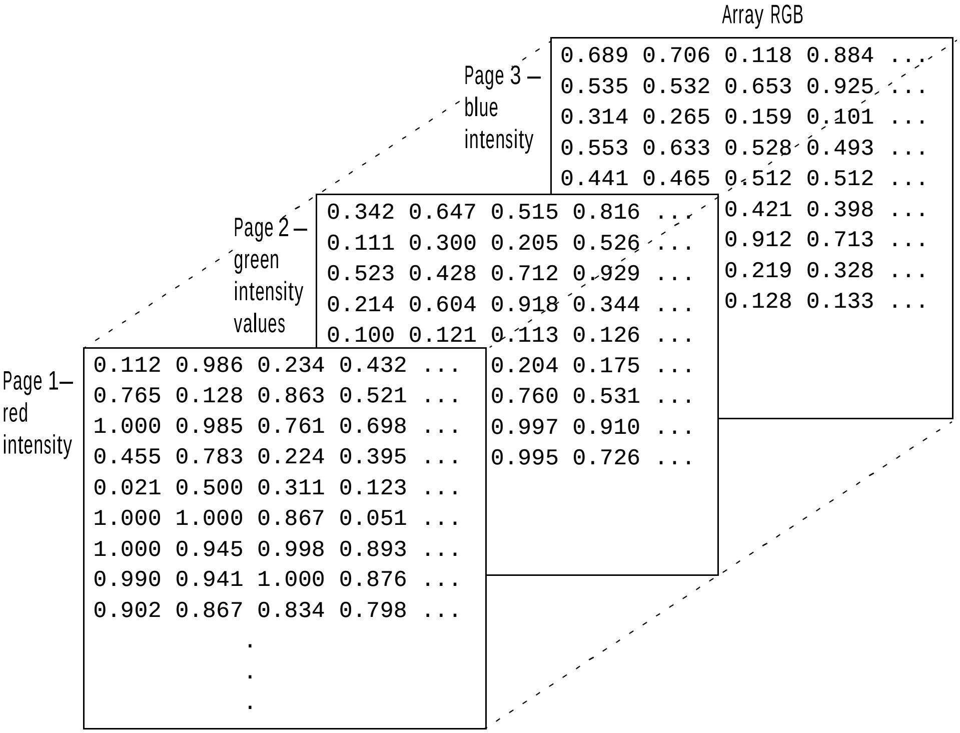
Note In the first subscripts are not colons, you must use squeeze to avoid an error. For example, eig(A(2,:,:)) results in an error because the size of the input is [1 3 3]. The expression eig(squeeze(A(2,:,:))), however, passes a valid two-dimensional matrix to eig.

Organizing Data in Multidimensional Arrays

You can use multidimensional arrays to represent data in two ways.

- As planes or pages of two-dimensional data. You can then treat these pages as matrices.
- As multivariate or multidimensional data. For example, you might have a four-dimensional array where each element corresponds to either a temperature or air pressure measurement taken at one of a set of equally spaced points in a room.

For example, consider an RGB image. For a single image, a multidimensional array is probably the easiest way to store and access data.



```
0.689 0.706 0.118 0.884 ...
0.535 0.532 0.653 0.925 ...
0.314 0.265 0.159 0.101 ...
0.553 0.633 0.528 0.493 ...
0.441 0.465 0.512 0.512 ...
            0.421 0.398 ...
            0.912 0.713 ...
            0.219 0.328 ...
            0.128 0.133 ...
```

```
0.342 0.647 0.515 0.816 ...
0.111 0.300 0.205 0.526 ...
0.523 0.428 0.712 0.929 ...
0.214 0.604 0.918 0.344 ...
0.100 0.121 0.113 0.126 ...
            0.204 0.175 ...
            0.760 0.531 ...
            0.997 0.910 ...
            0.995 0.726 ...
```

```
0.112 0.986 0.234 0.432 ...
0.765 0.128 0.863 0.521 ...
1.000 0.985 0.761 0.698 ...
0.455 0.783 0.224 0.395 ...
0.021 0.500 0.311 0.123 ...
1.000 1.000 0.867 0.051 ...
1.000 0.945 0.998 0.893 ...
0.990 0.941 1.000 0.876 ...
0.902 0.867 0.834 0.798 ...
               .
               .
               .
```

To access an entire plane of the image, use

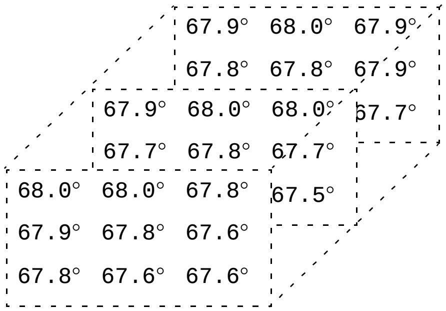
```
red_plane = RGB(:,:,1)
```

To access a subimage, use

```
subimage = RGB(20:40,50:85,:);
```

The RGB image is a good example of data that needs to be accessed in planes for operations like display or filtering. In other instances, however, the data itself might be multidimensional. For example, consider a set of temperature measurements taken at equally spaced points in a room. Here the location of each value is an integral part of the data set – the physical placement in three-space of each element is an aspect of the information. Such data also lends itself to representation as a multidimensional array.

Array TEMP



67.9° 68.0° 67.9°
67.8° 67.8° 67.9°
67.9° 68.0° 68.0° 67.7°
67.7° 67.8° 67.7°
68.0° 68.0° 67.8° 67.5°
67.9° 67.8° 67.6°
67.8° 67.6° 67.6°

Now to find the average of all the measurements, use

```
mean(mean(mean(TEMP)))
```

To obtain a vector of the “middle” values (element (2,2)) in the room on each page, use

```
B = TEMP(2,2,:);
```

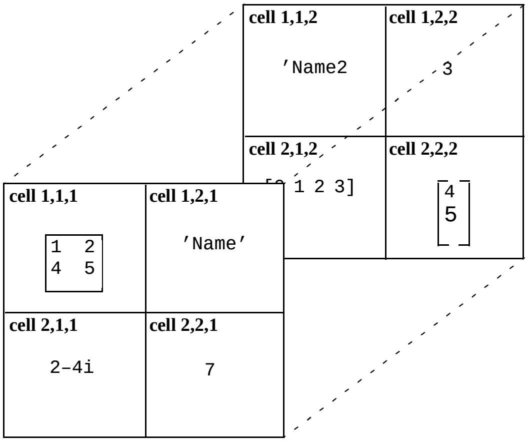
Multidimensional Cell Arrays

Like numeric arrays, the framework for multidimensional cell arrays in MATLAB is an extension of the two-dimensional cell array model. You can use the cat function to build multidimensional cell arrays, just as you use it for numeric arrays.

For example, create a simple three-dimensional cell array C.

```
A{1,1} = [1 2;4 5];
A{1,2} = ’Name’;
A{2,1} = 2–4i;
A{2,2} = 7;
B{1,1} = 'Name2';
B{1,2} = 3;
B{2,1} = 0:1:3;
B{2,2} = [4 5]';
C = cat(3,A,B);
```

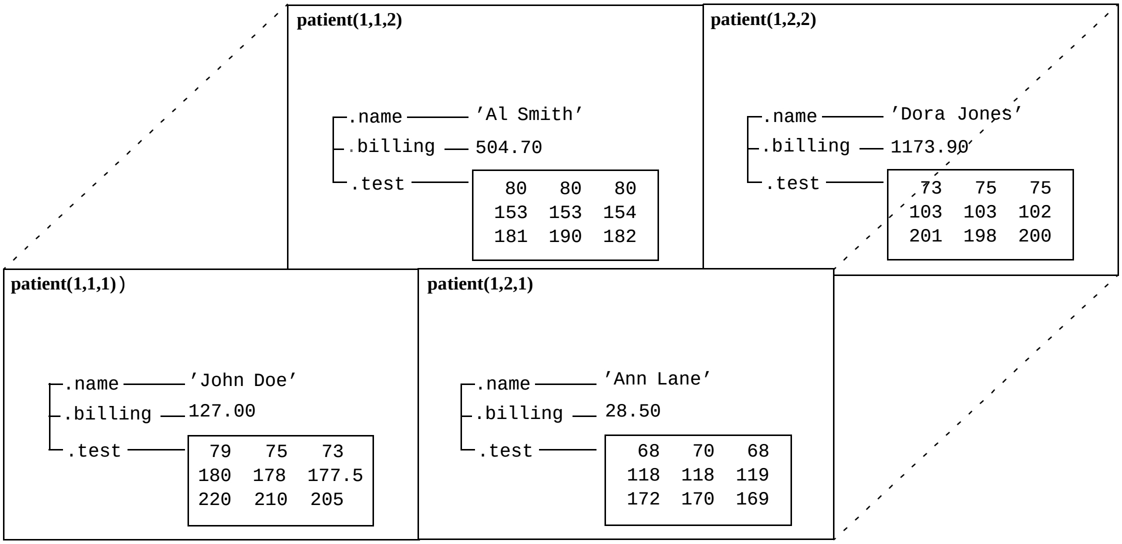
The subscripts for the cells of C look like



Multidimensional Structure Arrays

Multidimensional structure arrays are extensions of rectangular structure arrays. Like other types of multidimensional arrays, you can build them using direct assignment or the cat function.

```
patient(1,1,1).name = 'John Doe';patient(1,1,1).billing = 127.00;
patient(1,1,1).test = [79 75 73; 180 178 177.5; 220 210 205];
patient(1,2,1).name = 'Ann Lane';patient(1,2,1).billing = 28.50;
patient(1,2,1).test = [68 70 68; 118 118 119; 172 170 169];
patient(1,1,2).name = 'Al Smith';patient(1,1,2).billing = 504.70;
patient(1,1,2).test = [80 80 80; 153 153 154; 181 190 182];
patient(1,2,2).name = 'Dora Jones';patient(1,2,2).billing =
1173.90;
patient(1,2,2).test = [73 73 75; 103 103 102; 201 198 200];
```



Applying Functions to Multidimensional Structure Arrays

To apply functions to multidimensional structure arrays, operate on fields and field elements using indexing. For example, find the sum of the columns of the `test` array in `patient(1,1,2)`.

```
sum((patient(1,1,2).test))
```

Similarly, add all the `billing` fields in the `patient` array.

```
total = sum([patient.billing]);
```

Structures and Cell Arrays

Structures

Cell Arrays

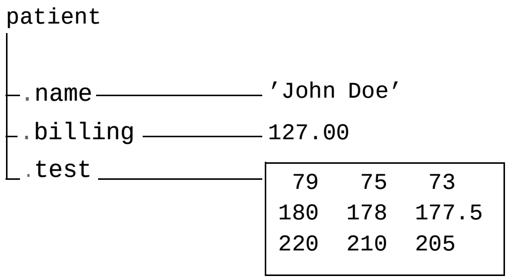
Structures are collections of different kinds of data organized by named fields. *Cell arrays* are a special class of MATLAB array whose elements consist of cells that themselves contain MATLAB arrays. Both structures and cell arrays provide a hierarchical storage mechanism for dissimilar kinds of data. They differ from each other primarily in the way they organize data. You access data in structures using named fields, while in cell arrays, data is accessed through matrix indexing operations.

This table describes the MATLAB functions for working with structures and cell arrays.

Category	**Function**	**Description**
Structure functions	`fieldnames`	Get structure field names.
	`getfield`	Get structure field contents.
	isfield	True if field is in structure array.
	isstruct	True for structures.
	`rmfield`	Remove structure field.
	`setfield`	Set structure field contents.
	`struct`	Create or convert to structure array.
	`struct2cell`	Convert structure array into cell array.
Cell array functions	`cell`	Create cell array.
	`cell2struct`	Convert cell array into structure array.
	`celldisp`	Display cell array contents.
	`cellfun`	Apply a cell function to a cell array.
	`cellplot`	Display graphical depiction of cell array.
	`deal`	Deal inputs to outputs.
	`iscell`	True for cell array.
	`num2cell`	Convert numeric array into cell array.

Structures

Structures are MATLAB arrays with named “data containers” called *fields*. The fields of a structure can contain any kind of data. For example, one field might contain a text string representing a name, another might contain a scalar representing a billing amount, a third might hold a matrix of medical test results, and so on.



Like standard arrays, structures are inherently array oriented. A single structure is a 1-by-1 structure array, just as the value 5 is a 1-by-1 numeric array. You can build structure arrays with any valid size or shape, including multidimensional structure arrays.

Note The examples in this section focus on two-dimensional structure arrays. For examples of higher-dimension structure arrays, see Chapter 12.

Building Structure Arrays

You can build structures in two ways:

- Using assignment statements
- Using the `struct` function

Building Structure Arrays Using Assignment Statements

You can build a simple 1-by-1 structure array by assigning data to individual fields. MATLAB automatically builds the structure as you go along. For example, create the 1-by-1 patient structure array shown at the beginning of this section.

```
patient.name = ’John Doe’;
patient.billing = 127.00;
patient.test = [79 75 73; 180 178 177.5; 220 210 205];
```

Now entering

```
patient
```

at the command line results in

```
name: ’John Doe’
billing: 127
test: [3x3 double]
```

patient is an array containing a structure with three fields. To expand the structure array, add subscripts after the structure name.

```
patient(2).name = ’Ann Lane’;
patient(2).billing = 28.50;
patient(2).test = [68 70 68; 118 118 119; 172 170 169];
```

The patient structure array now has size [1 2]. Note that once a structure array contains more than a single element, MATLAB does not display individual field contents when you type the array name. Instead, it shows a summary of the kind of information the structure contains.

```
patient

patient =

1x2 struct array with fields:
    name
    billing
    test
```

You can also use the fieldnames function to obtain this information. fieldnames returns a cell array of strings containing field names.

As you expand the structure, MATLAB fills in unspecified fields with empty matrices so that:

- All structures in the array have the same number of fields.
- All fields have the same field names.

For example, entering `patient(3).name = 'Alan Johnson'` expands the `patient` array to size `[1 3]`. Now both `patient(3).billing` and `patient(3).test` contain empty matrices.

Note Field sizes do not have to conform for every element in an array. In the `patient` example, the `name` fields can have different lengths, the `test` fields can be arrays of different sizes, and so on.

Building Structure Arrays Using the struct Function

You can preallocate an array of structures with the `struct` function. Its basic form is

```
str_array = struct('field1',val1,'field2',val2, ...)
```

where the arguments are field names and their corresponding values. A field value can be a single value, represented by any MATLAB data construct, or a cell array of values. All field values in the argument list must be of the same scale (single value or cell array).

You can use different methods for preallocating structure arrays. These methods differ in the way in which the structure fields are initialized. As an example, consider the allocation of a 1-by-3 structure array, `weather`, with the structure fields `temp` and `rainfall`. Three different methods for allocating such an array are shown in this table.

Method	Syntax	Initialization

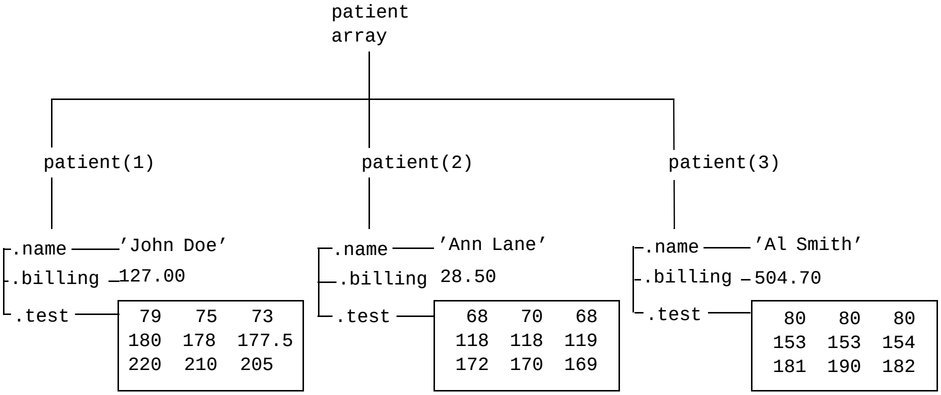
struct	`weather(3) =`
	`  struct('temp',72,'rainfall',0.0);`
		`weather(3)` is initialized with the field values shown. The fields for the other structures in the array, `weather(1)` and `weather(2)`, are initialized to the empty matrix.

struct with `repmat`	`weather =`
	`  repmat(struct('temp',72,`
	`  'rainfall',0.0),1,3);`
		All structures in the `weather` array are initialized using one set of field values.

struct with cell array syntax	`weather = struct('temp',{68,80,72},`
	`  'rainfall',{0.2,0.4,0.0});`
		The structures in the `weather` array are initialized with distinct field values specified with cell arrays.

Accessing Data in Structure Arrays

Using structure array indexing, you can access the value of any field or field element in a structure array. Likewise, you can assign a value to any field or field element. For the examples in this section, consider this structure array.



You can access subarrays by appending standard subscripts to a structure array name. For example, the line below results in a 1-by-2 structure array.

```
mypatients = patient(1:2)

1x2 struct array with fields:
    name
    billing
    test
```

The first structure in the mypatients array is the same as the first structure in the patient array.

```
mypatients(1)

ans =

       name: 'John Doe'
    billing: 127
       test: [3x3 double]
```

To access a field of a particular structure, include a period (.) after the structure name followed by the field name.

```
str = patient(2).name

str =

Ann Lane
```

To access elements within fields, append the appropriate indexing mechanism to the field name. That is, if the field contains an array, use array subscripting; if the field contains a cell array, use cell array subscripting, and so on.

```
test2b = patient(3).test(2,2)

test2b =

   153
```

Use the same notations to assign values to structure fields, for example,

```
patient(3).test(2,2) = 7;
```

You can extract field values for multiple structures at a time. For example, the line below creates a 1-by-3 vector containing all of the billing fields.

```
bills = [patient.billing]

bills =

  127.0000   28.5000  504.7000
```

Similarly, you can create a cell array containing the test data for the first two structures.

```
tests = {patient(1:2).test}

tests =

    [3x3 double]    [3x3 double]
```

Accessing Field Values Using setfield and getfield

Direct indexing is usually the most efficient way to assign or retrieve field values. If, however, you only know the field name as a string – for example, if you have used the fieldnames function to obtain the field name within an M-file – you can use the setfield and getfield functions to do the same thing.

getfield obtains a value or values from a field or field element

```
f = getfield(array,{array_index},'field',{field_index})
```

where the field_index is optional, and array_index is optional for a 1-by-1 structure array. The function syntax corresponds to

```
f = array(array_index).field(field_index);
```

For example, to access the name field in the second structure of the patient array, use:

```
str = getfield(patient,{2},'name')
```

Similarly, setfield lets you assign values to fields using the syntax

```
f = setfield(array,{array_index},'field',{field_index},value)
```

Using the size Function with Structure Arrays

Use the size function to obtain the size of a structure array, or of any structure field. Given a structure array name as an argument, size returns a vector of array dimensions. Given an argument in the form array(n).field, the size function returns a vector containing the size of the field contents.

For example, for the 1-by-3 structure array patient, size(patient) returns the vector [1 3]. The statement size(patient(1,2).name) returns the length of the name string for element (1,2) of patient.

Adding Fields to Structures

You can add a field to every structure in an array by adding the field to a single structure. For example, to add a social security number field to the patient array, use an assignment like

```
patient(2).ssn = '000-00-0000';
```

Now patient(2).ssn has the assigned value. Every other structure in the array also has the ssn field, but these fields contain the empty matrix until you explicitly assign a value to them.

Deleting Fields from Structures

You can remove a given field from every structure within a structure array using the rmfield function. Its most basic form is

```
struc2 = rmfield(array,'field')
```

where array is a structure array and 'field' is the name of a field to remove from it. To remove the name field from the patient array, for example, enter:

```
patient = rmfield(patient,'name');
```

Applying Functions and Operators

Operate on fields and field elements the same way you operate on any other MATLAB array. Use indexing to access the data on which to operate. For example, this statement finds the mean across the rows of the test array in patient(2).

```
mean((patient(2).test)')
```

There are sometimes multiple ways to apply functions or operators across fields in a structure array. One way to add all the billing fields in the patient array is:

```
total = 0;
for j = 1:length(patient)
      total = total + patient(j).billing;
end
```

To simplify operations like this, MATLAB enables you to operate on all like-named fields in a structure array. Simply enclose the array.field expression in square brackets within the function call. For example, you can sum all the billing fields in the patient array using

```
total = sum ([patient.billing]);
```

This is equivalent to using the comma-separated list.

```
total = sum ([patient(1).billing, patient(2).billing...]);
```

This syntax is most useful in cases where the operand field is a scalar field.

Writing Functions to Operate on Structures

You can write functions that work on structures with specific field architectures. Such functions can access structure fields and elements for processing.

Note When writing M-file functions to operate on structures, you must perform your own error checking. That is, you must ensure that the code checks for the expected fields.

As an example, consider a collection of data that describes measurements, at different times, of the levels of various toxins in a water source. The data consists of fifteen separate observations, where each observation contains three separate measurements.

You can organize this data into an array of 15 structures, where each structure has three fields, one for each of the three measurements taken.

The function concen, shown below, operates on an array of structures with specific characteristics. Its arguments must contain the fields lead, mercury, and chromium.

```
function [r1,r2] = concen(toxtest);
% Create two vectors. r1 contains the ratio of mercury to lead
% at each observation. r2 contains the ratio of lead to chromium.
r1 = [toxtest.mercury]./[toxtest.lead];
r2 = [toxtest.lead]./[toxtest.chromium];
% Plot the concentrations of lead, mercury, and chromium
% on the same plot, using different colors for each.
lead = [toxtest.lead];
mercury = [toxtest.mercury];
chromium = [toxtest.chromium];
plot(lead,’r’); hold on
plot(mercury,’b’)
plot(chromium,’y’); hold off
```

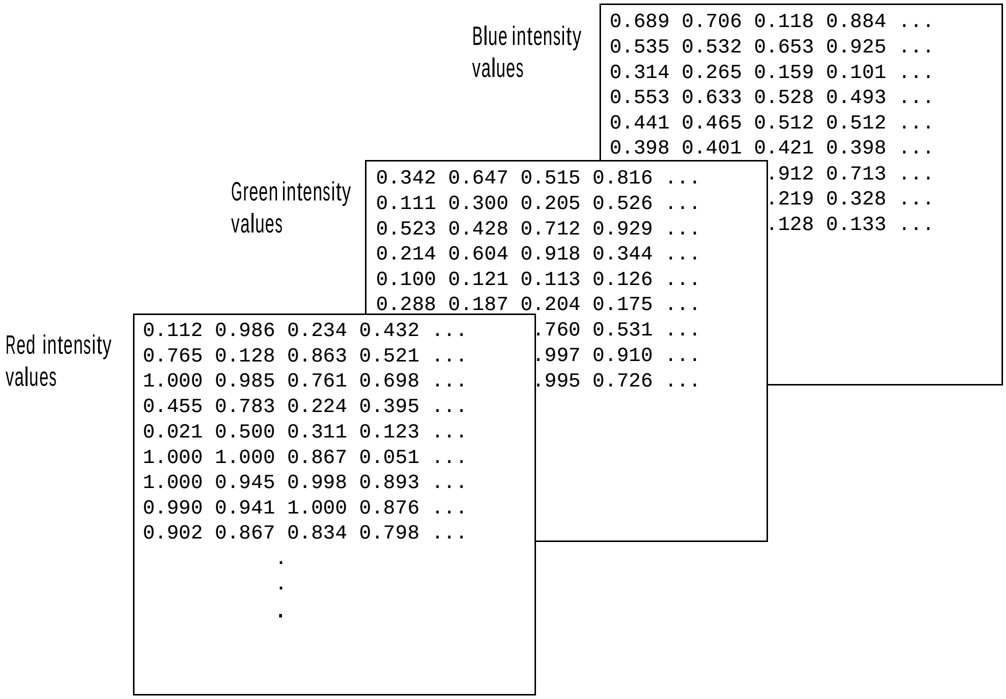
Try this function with a sample structure array like test.

```
test(1).lead = .007; test(2).lead = .031; test(3).lead = .019;
test(1).mercury = .0021; test(2).mercury = .0009;
test(3).mercury = .0013;
test(1).chromium = .025; test(2).chromium = .017;
test(3).chromium = .10;
```

Organizing Data in Structure Arrays

The key to organizing structure arrays is to decide how you want to access subsets of the information. This, in turn, determines how you build the array that holds the structures, and how you break up the structure fields.

For example, consider a 128-by-128 RGB image stored in three separate arrays; RED, GREEN, and BLUE:



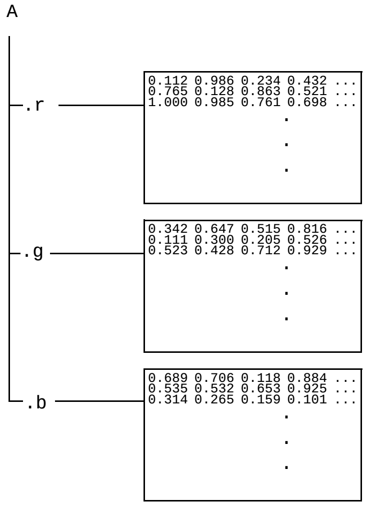
```
0.689 0.706 0.118 0.884 ...
0.535 0.532 0.653 0.925 ...
0.314 0.265 0.159 0.101 ...
0.553 0.633 0.528 0.493 ...
0.441 0.465 0.512 0.512 ...
0.398 0.401 0.421 0.398 ...
           .912 0.713 ...
           .219 0.328 ...
           .128 0.133 ...
```

```
0.342 0.647 0.515 0.816 ...
0.111 0.300 0.205 0.526 ...
0.523 0.428 0.712 0.929 ...
0.214 0.604 0.918 0.344 ...
0.100 0.121 0.113 0.126 ...
0.288 0.187 0.204 0.175 ...
           .760 0.531 ...
           .997 0.910 ...
           .995 0.726 ...
```

```
0.112 0.986 0.234 0.432 ...
0.765 0.128 0.863 0.521 ...
1.000 0.985 0.761 0.698 ...
0.455 0.783 0.224 0.395 ...
0.021 0.500 0.311 0.123 ...
1.000 1.000 0.867 0.051 ...
1.000 0.945 0.998 0.893 ...
0.990 0.941 1.000 0.876 ...
0.902 0.867 0.834 0.798 ...
           .
           .
           .
```

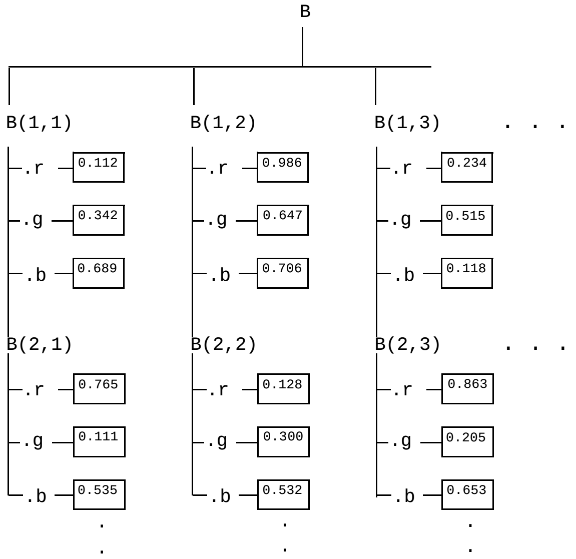
There are at least two ways you can organize such data into a structure array:

Plane organization



1-by-1 structure array where each field is a 128-by-128 array

Element-by-element organization



128-by-128 structure array where each field is a single data element

Plane Organization

In case 1 above, each field of the structure is an entire plane of the image. You can create this structure using

```
A.r = RED;
A.g = GREEN;
A.b = BLUE;
```

This approach allows you to easily extract entire image planes for display, filtering, or other tasks that work on the entire image at once. To access the entire red plane, for example, use:

```
red_plane = A.red;
```

Plane organization has the additional advantage of being extensible to multiple images in this case. If you have a number of images, you can store them as A(2), A(3), and so on, each containing an entire image.

The disadvantage of plane organization is evident when you need to access subsets of the planes. To access a subimage, for example, you need to access each field separately.

```
red_sub = A.r(2:12,13:30);
grn_sub = A.g(2:12,13:30);
blue_sub = A.b(2:12,13:30);
```

Element-by-Element Organization

Case 2 has the advantage of allowing easy access to subsets of data. To set up the data in this organization, use:

```
for i = 1:size(RED,1)
   for j = 1:size(RED,2)
      B(i,j).r = RED(i,j);
      B(i,j).g = GREEN(i,j);
      B(i,j).b = BLUE(i,j);
   end
end
```

With element-by-element organization, you can access a subset of data with a single statement.

```
Bsub = B(1:10,1:10);
```

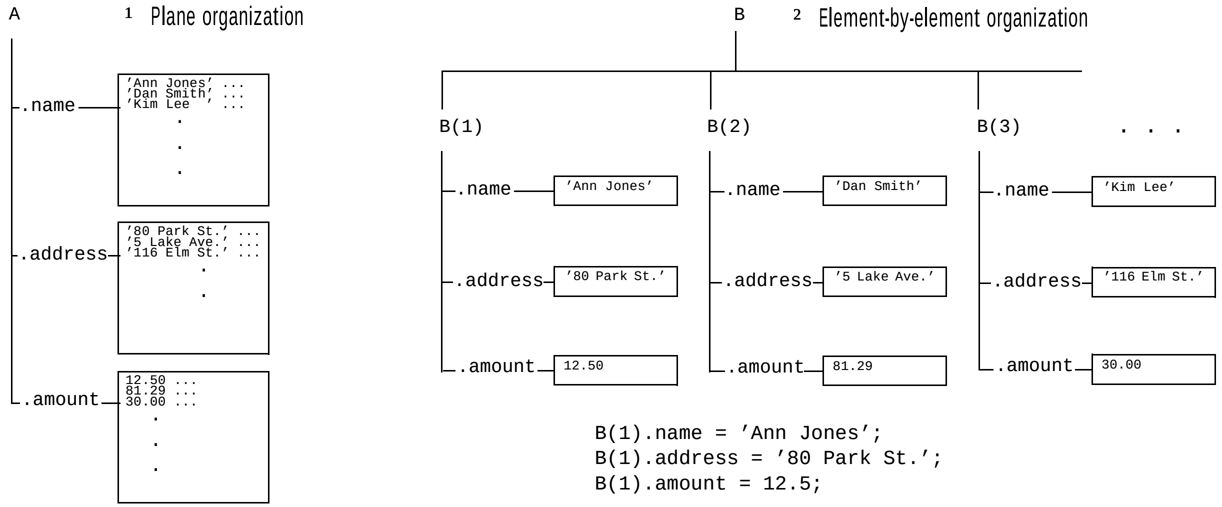
To access an entire plane of the image using the element-by-element method, however, requires a loop.

```
red_plane = zeros(128,128);
for i = 1:(128*128)
    red_plane(i) = B(i).r;
end
```

Element-by-element organization is not the best structure array choice for most image processing applications; however, it can be the best for other applications wherein you will routinely need to access corresponding subsets of structure fields. The example in the following section demonstrates this type of application.

Example: A Simple Database

Consider organizing a simple database:



```
B(1).name = ’Ann Jones’;
B(1).address = ’80 Park St.’;
B(1).amount = 12.5;

B(2).name = ’Dan Smith’;
B(2).address = ’5 Lake Ave.’;
B(2).amount = 81.29;
    .
    .
    .
```

```
A.name = strvcat(’Ann Jones’,’Dan Smith’,...);
A.address = strvcat(’80 Park St.’,’5 Lake Ave.’,..
A.amount = [12.5;81.29;30; ...];
```

Each of the possible organizations has advantages depending on how you want to access the data:

- Plane organization makes it easier to operate on all field values at once. For example, to find the average of all the values in the `amount` field,

  Using plane organization

  ```
  avg = mean(A.amount);
  ```

  Using element-by-element organization

  ```
  avg = mean([B.amount]);
  ```

- Element-by-element organization makes it easier to access all the information related to a single client. Consider an M-file, `client.m`, which displays the name and address of a given client on screen.

  Using plane organization, pass individual fields

  ```
  function client(name,address)
  disp(name)
  disp(address)
  ```

  Using element-by-element organization, pass an entire structure

  ```
  function client(B)
  disp(B)
  ```

  To call the `client` function,

  Using plane organization

  ```
  client(A.name(2,:),A.address(2,:))
  ```

  Using element-by-element organization

  ```
  client(B(2))
  ```

- Element-by-element organization makes it easier to expand the string array fields. If you do not know the maximum string length ahead of time for plane organization, you may need to frequently recreate the `name` or `address` field to accommodate longer strings.

Typically, your data does not dictate the organization scheme you choose. Rather, you must consider how you want to access and operate on the data.

Nesting Structures

A structure field can contain another structure, or even an array of structures. Once you have created a structure, you can use the `struct` function or direct assignment statements to nest structures within existing structure fields.

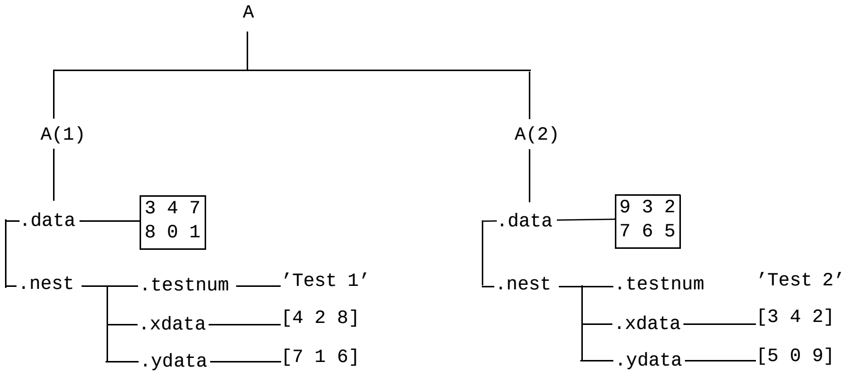
Building Nested Structures with the struct Function

To build nested structures, you can nest calls to the `struct` function. For example, create a 1-by-1 structure array.

```
A = struct('data',[3 4 7; 8 0 1],'nest',...
    struct('testnum','Test 1', 'xdata',[4 2 8],...
    'ydata',[7 1 6]))
```

You can build nested structure arrays using direct assignment statements. These statements add a second element to the array.

```
A(2).data = [9 3 2; 7 6 5];
A(2).nest.testnum = 'Test 2';
A(2).nest.xdata = [3 4 2];
A(2).nest.ydata = [5 0 9];
```



Indexing Nested Structures

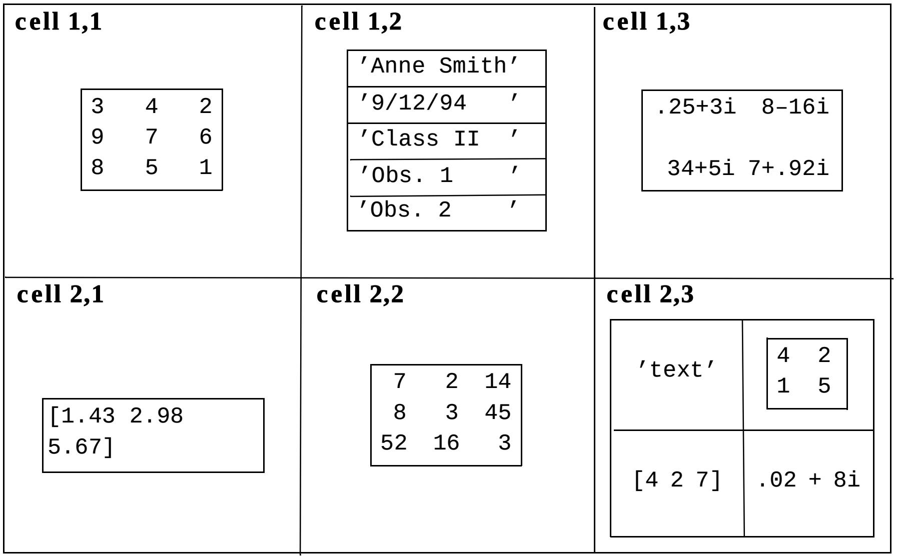
To index nested structures, append nested field names using dot notation. The first text string in the indexing expression identifies the structure array, and subsequent expressions access field names that contain other structures.

For example, the array A created earlier has three levels of nesting:

- To access the nested structure inside A(1), use A(1).nest.
- To access the xdata field in the nested structure in A(2), use A(2).nest.xdata.
- To access element 2 of the ydata field in A(1), use A(1).nest.ydata(2).

Cell Arrays

A cell array is a MATLAB array for which the elements are *cells*, containers that can hold other MATLAB arrays. For example, one cell of a cell array might contain a real matrix, another an array of text strings, and another a vector of complex values.



You can build cell arrays of any valid size or shape, including multidimensional structure arrays.

Note The examples in this section focus on two-dimensional cell arrays. For examples of higher-dimension cell arrays, see Chapter 12.

Creating Cell Arrays

You can create cell arrays by:

- Using assignment statements
- Preallocating the array using the `cells` function, then assigning data to cells

Using Assignment Statements

You can build a cell array by assigning data to individual cells, one cell at a time. MATLAB automatically builds the array as you go along. There are two ways to assign data to cells:

- Cell indexing

  Enclose the cell subscripts in parentheses using standard array notation. Enclose the cell contents on the right side of the assignment statement in curly braces, "{}." For example, create a 2-by-2 cell array A.

  ```
  A(1,1) = {[1 4 3; 0 5 8; 7 2 9]};
  A(1,2) = {'Anne Smith'};
  A(2,1) = {3+7i};
  A(2,2) = {–pi:pi/10:pi}
  ```

Note The notation "{}" denotes the empty cell array, just as "[]" denotes the empty matrix for numeric arrays. You can use the empty cell array in any cell array assignments.

- Content indexing

  Enclose the cell subscripts in curly braces using standard array notation. Specify the cell contents on the right side of the assignment statement.

  ```
  A{1,1} = [1 4 3; 0 5 8; 7 2 9];
  A{1,2} = 'Anne Smith';
  A{2,1} = 3+7i;
  A{2,2} = –pi:pi/10:pi
  ```

The various examples in this guide do not use one syntax throughout, but attempt to show representative usage of cell and content addressing. You can use the two forms interchangeably.

Note If you already have a numeric array of a given name, don't try to create a cell array of the same name by assignment without first clearing the numeric array. If you do not clear the numeric array, MATLAB assumes that you are trying to "mix" cell and numeric syntaxes, and generates an error. Similarly, MATLAB does not clear a cell array when you make a single assignment to it. If any of the examples in this section give unexpected results, clear the cell array from the workspace and try again.

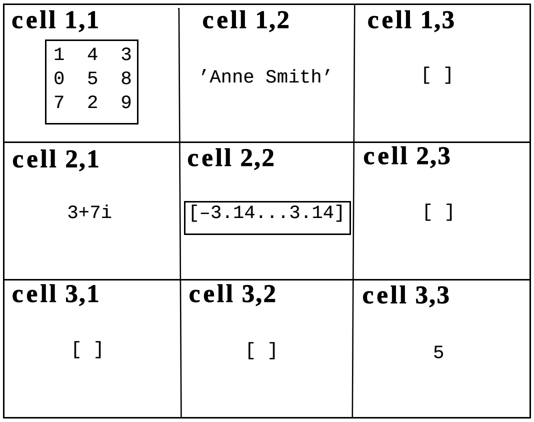
MATLAB displays the cell array `A` in a condensed form.

```
A = 

          [3x3 double]    'Anne Smith'
     [3.0000+ 7.0000i]    [1x21 double]
```

To display the full cell contents, use the `celldisp` function. For a high-level graphical display of cell architecture, use `cellplot`.

If you assign data to a cell that is outside the dimensions of the current array, MATLAB automatically expands the array to include the subscripts you specify. It fills any intervening cells with empty matrices. For example, the assignment below turns the 2-by-2 cell array A into a 3-by-3 cell array.

```
A(3,3) = {5};
```



cell 1,1	cell 1,2	cell 1,3

1 4 3
0 5 8
7 2 9
	'Anne Smith'	[ ]

cell 2,1	**cell 2,2**	**cell 2,3**
3+7i	[–3.14...3.14]	[ ]
cell 3,1	**cell 3,2**	**cell 3,3**
[ ]	[ ]	5

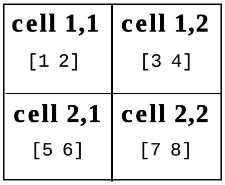
Cell Array Syntax: Using Braces

The curly braces, "{}", are cell array constructors, just as square brackets are numeric array constructors. Curly braces behave similarly to square brackets, except that you can nest curly braces to denote nesting of cells (see page 13-29 for details).

Curly braces use commas or spaces to indicate column breaks and semicolons to indicate row breaks between cells. For example,

```
C = {[1 2], [3 4]; [5 6], [7 8]}
```

results in



cell 1,1	cell 1,2
[1 2]	[3 4]
cell 2,1	**cell 2,2**
[5 6]	[7 8]

Use square brackets to concatenate cell arrays, just as you do for numeric arrays.

Preallocating Cell Arrays with the cell Function

The `cell` function allows you to preallocate empty cell arrays of the specified size. For example, this statement creates an empty 2-by-3 cell array.

```
B = cell(2,3)
```

Use assignment statements to fill the cells of `B`.

```
B(1,3) = {1:3};
```

Obtaining Data from Cell Arrays

You can obtain data from cell arrays and store the result as either a standard array or a new cell array. This section discusses:

- Accessing cell contents using content indexing
- Accessing a subset of cells using cell indexing

Accessing Cell Contents Using Content Indexing

You can use content indexing on the right side of an assignment to access some or all of the data in a single cell. Specify the variable to receive the cell contents on the left side of the assignment. Enclose the cell index expression on the right side of the assignment in curly braces. This indicates that you are assigning cell contents, not the cells themselves.

Consider the 2-by-2 cell array `N`.

```
N{1,1} = [1 2; 4 5];
N{1,2} = 'Name';
N{2,1} = 2-4i;
N{2,2} = 7;
```

You can obtain the string in N{1,2} using

```
c = N{1,2}

c =

Name
```

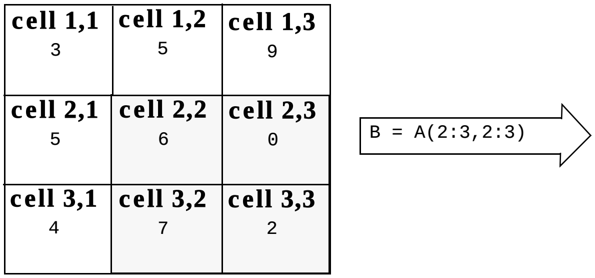
Note In assignments, you can use content indexing to access only a single cell, not a subset of cells. For example, the statements A{1,:} = *value* and B = A{1,:} are both invalid. However, you can use a subset of cells any place you would normally use a comma-separated list of variables (for example, as function inputs or when building an array). See ““Replacing Lists of Variables with Cell Arrays”” on page 13-25 for details.

To obtain subsets of a cell’s contents, concatenate indexing expressions. For example, to obtain element (2,2) of the array in cell N{1,1}, use:

```
d = N{1,1}(2,2)

d =

     5
```

Accessing a Subset of Cells Using Cell Indexing

Use cell indexing to assign any set of cells to another variable, creating a new cell array. Use the colon operator to access subsets of cells within a cell array.



cell 1,1 3	**cell 1,2** 5	**cell 1,3** 9
cell 2,1 5	**cell 2,2** 6	**cell 2,3** 0
cell 3,1 4	**cell 3,2** 7	**cell 3,3** 2

```
B = A(2:3,2:3)
```

cell 1,1 6	**cell 1,2** 0
cell 2,1 7	**cell 2,2** 2

Deleting Cells

You can delete an entire dimension of cells using a single statement. Like standard array deletion, use vector subscripting when deleting a row or column of cells and assign the empty matrix to the dimension.

```
A(cell_subscripts) = []
```

When deleting cells, curly braces do not appear in the assignment statement at all.

Reshaping Cell Arrays

Like other arrays, you can reshape cell arrays using the `reshape` function. The number of cells must remain the same after reshaping; you cannot use `reshape` to add or remove cells.

```
A = cell(3,4);
size(A)

ans =

     3     4

B = reshape(A,6,2);
size(B)

ans =


     6     2
```

Replacing Lists of Variables with Cell Arrays

Cell arrays can replace comma-separated lists of MATLAB variables in:

- Function input lists
- Function output lists
- Display operations
- Array constructions (square brackets and curly braces)

If you use the colon to index multiple cells in conjunction with the curly brace notation, MATLAB treats the contents of each cell as a separate variable. For example, assume you have a cell array T where each cell contains a separate vector. The expression T{1:5} is equivalent to a comma-separated list of the vectors in the first five cells of T.

Consider the cell array C.

```
C(1) = {[1 2 3]};
C(2) = {[1 0 1]};
C(3) = {1:10};
C(4) = {[9 8 7]};
C(5) = {3};
```

To convolve the vectors in C(1) and C(2) using conv,

```
d = conv(C{1:2})

d =

     1     2     4     2     3
```

Display vectors two, three, and four with

```
C{2:4}

ans =

    1   0   1

ans =

    1   2   3   4   5   6   7   8   9   10

ans =

    9   8   7
```

Similarly, you can create a new numeric array using the statement

```
B = [C{1}; C{2}; C{4}]

B =
    1   2   3
    1   0   1
    9   8   7
```

You can also use content indexing on the left side of an assignment to create a new cell array where each cell represents a separate output argument.

```
[D{1:2}] = eig(B)

D =

    [3x3 double]    [3x3 double]
```

You can display the actual eigenvalues and eigenvectors using `D{1}` and `D{2}`.

Note The `varargin` and `varargout` arguments allow you to specify variable numbers of input and output arguments for MATLAB functions that you create. Both `varargin` and `varargout` are cell arrays, allowing them to hold various sizes and kinds of MATLAB data. See Chapter 10 for details.

Applying Functions and Operators

Use indexing to apply functions and operators to the contents of cells. For example, use content indexing to call a function with the contents of a single cell as an argument.

```
A{1,1} = [1 2; 3 4];
A{1,2} = randn(3,3);
A{1,3} = 1:5;
B = sum(A{1,1})

B =

     4     6
```

To apply a function to several cells of a non-nested cell array, use a loop.

```
for i = 1:length(A)
    M{i} = sum(A{1,i});
end
```

Organizing Data in Cell Arrays

Cell arrays are useful for organizing data that consists of different sizes or kinds of data. Cell arrays are better than structures for applications where:

- You need to access multiple fields of data with one statement.
- You want to access subsets of the data as comma-separated variable lists.
- You don't have a fixed set of field names.
- You routinely remove fields from the structure.

As an example of accessing multiple fields with one statement, assume that your data consists of:

- A 3-by-4 array consisting of measurements taken for an experiment
- A 15-character string containing a technician's name
- A 3-by-4-by-5 array containing a record of measurements taken for the past five experiments

For many applications, the best data construct for this data is a structure. However, if you routinely access only the first two fields of information, then a cell array might be more convenient for indexing purposes.

This example shows how to access the first and second elements of the cell array `TEST`.

```
[newdata,name] = deal(TEST{1:2})
```

This example shows how to access the first and second elements of the structure `TEST`.

```
newdata = TEST.measure
name = TEST.name
```

The varargin and varargout arguments are examples of the utility of cell arrays as substitutes for comma-separated lists. Create a 3-by-3 numeric array A.

```
A = [0 1 2;4 0 7;3 1 2];
```

Now apply the normest (2-norm estimate) function to A, and assign the function output to individual cells of B.

```
[B{1:2}] = normest(A)

B =

    [8.8826]    [4]
```

All of the output values from the function are stored in separate cells of B. B(1) contains the norm estimate; B(2) contains the iteration count.

Nesting Cell Arrays

A cell can contain another cell array, or even an array of cell arrays. (Cells that contain noncell data are called *leaf cells*.) You can use nested curly braces, the cell function, or direct assignment statements to create nested cell arrays. You can then access and manipulate individual cells, subarrays of cells, or cell elements.

Building Nested Arrays with Nested Curly Braces

You can nest pairs of curly braces to create a nested cell array. For example,

```
clear A
A(1,1) = {magic(5)};
A(1,2) = {{[5 2 8; 7 3 0; 6 7 3] 'Test 1'; [2–4i 5+7i] {17 []}}}

A =
    [5x5 double]    {2x2 cell}
```

Note that the right side of the assignment is enclosed in two sets of curly braces. The first set represents cell (1,2) of cell array A. The second "packages" the 2-by-2 cell array inside the outer cell.

Building Nested Arrays with the cell Function

To nest cell arrays with the `cell` function, assign the output of `cell` to an existing cell:

1 Create an empty 1-by-2 cell array.

```
A = cell(1,2)
```

2 Create a 2-by-2 cell array inside `A(1,2)`.

```
A(1,2) = {cell(2,2)}
```

3 Fill `A`, including the nested array, using assignments.

```
A(1,1) = {magic(5)};
A{1,2}(1,1) = {[5 2 8; 7 3 0; 6 7 3]};
A{1,2}(1,2) = {’Test 1’};
A{1,2}(2,1) = {[2–4i 5+7i]};
A{1,2}(2,2) = {cell(1,2)}
A{1,2}{2,2}(1) = {17};
```

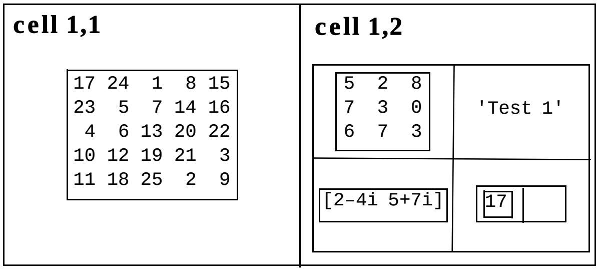
Note the use of curly braces until the final level of nested subscripts. This is required because you need to access cell contents to access cells within cells.

You can also build nested cell arrays with direct assignments using the statements shown in step 3 above.

Indexing Nested Cell Arrays

To index nested cells, concatenate indexing expressions. The first set of subscripts accesses the top layer of cells, and subsequent sets of parentheses access successively deeper layers.

For example, array `A` has three levels of nesting:



- To access the 5-by-5 array in cell `(1,1)`, use `A{1,1}`.
- To access the 3-by-3 array in position `(1,1)` of cell `(1,2)`, use `A{1,2}{1,1}`.
- To access the 2-by-2 cell array in cell `(1,2)`, use `A{1,2}`.
- To access the empty cell in position `(2,2)` of cell `(1,2)`, use `A{1,2}{2,2}{1,2}`.

Converting Between Cell and Numeric Arrays

Use for loops to convert between cell and numeric formats. For example, create a cell array F,

```
F{1,1} = [1 2; 3 4];
F{1,2} = [–1 0; 0 1];
F{2,1} = [7 8; 4 1];
F{2,2} = [4i 3+2i; 1–8i 5];
```

Now use three for loops to copy the contents of F into a numeric array NUM.

```
for k = 1:4
   for i = 1:2
      for j = 1:2
         NUM(i,j,k) = F{k}(i,j);
      end
   end
end
```

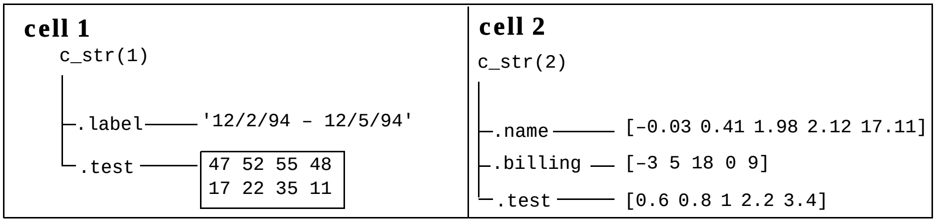
Similarly, you must use for loops to assign each value of a numeric array to a single cell of a cell array.

```
G = cell(1,16);
for m = 1:16
   G{m} = NUM(m);
end
```

Cell Arrays of Structures

Use cell arrays to store groups of structures with different field architectures.

```
c_str = cell(1,2)
c_str{1}.label = '12/2/94 - 12/5/94';
c_str{1}.obs = [47 52 55 48; 17 22 35 11];
c_str{2}.xdata = [-0.03 0.41 1.98 2.12 17.11];
c_str{2}.ydata = [-3 5 18 0 9];
c_str{2}.zdata = [0.6 0.8 1 2.2 3.4];
```



Cell 1 of the c_str array contains a structure with two fields, one a string and the other a vector. Cell 2 contains a structure with three vector fields.

When building cell arrays of structures, you must use content indexing. Similarly, you must use content indexing to obtain the contents of structures within cells. The syntax for content indexing is:

```
cell_array{index}.field
```

For example, to access the label field of the structure in cell 1, use c_str{1}.label.

14

MATLAB Classes and Objects

Classes and Objects: An Overview

This chapter describes how to define classes in MATLAB. Classes and objects enable you to add new data types and new operations to MATLAB. The *class* of a variable describes the structure of the variable and indicates the kinds of operations and functions that can apply to the variable. An *object* is an instance of a particular class. The phrase “object-oriented programming” describes an approach to writing programs that emphasizes the use of classes and objects.

You can view classes as new data types having specific behaviors defined for the class. For example, a polynomial class might redefine the addition operator (+) so that it correctly performs the operation of addition on polynomials. Operations defined to work with objects of a particular class are know as *methods* of that class.

You can also view classes as new items that you can treat as single entities. An example is an arrow object that MATLAB can display on graphs (perhaps composed of MATLAB line and patch objects) and that has properties like a Handle Graphics object. You can create an arrow simply by instantiating the arrow class.

You can add classes to your MATLAB environment by specifying a MATLAB structure that provides data storage for the object and creating a class directory containing M-files that operate on the object. These M-files contain the methods for the class. The class directory can also include functions that define the way various MATLAB operators, including arithmetic operations, subscript referencing, and concatenation, apply to the objects. Redefining how a built-in operator works for your class is known as *overloading* the operator.

Features of Object-Oriented Programming

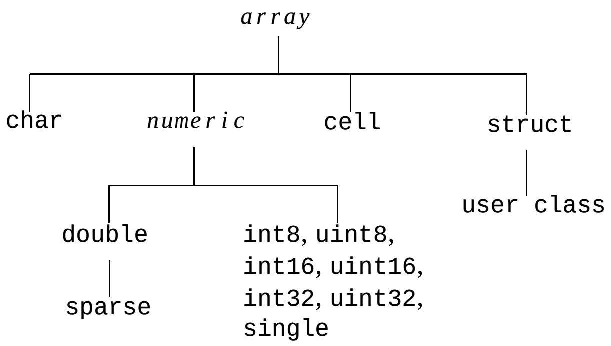
When using well-designed classes, object-oriented programming can significantly increase code reuse and make your programs easier to maintain and extend. Programming with classes and objects differs from ordinary structured programming in these important ways:

- **Function and operator overloading**. You can create methods that override existing MATLAB functions. When you call a function with a user-defined object as an argument, MATLAB first checks to see if there is a method defined for the object's class. If there is, MATLAB calls it, rather than the normal MATLAB function.

- **Encapsulation of data and methods**. Object properties are not visible from the command line; you can access them only with class methods. This protects the object properties from operations that are not intended for the object's class.
- **Inheritance**. You can create class hierarchies of parent and child classes in which the child class inherits data fields and methods from the parent. A child class can inherit from one parent (*single inheritance*) or many parents (*multiple inheritance*). Inheritance can span one or more generations. Inheritance enables sharing common parent functions and enforcing common behavior amongst all child classes.
- **Aggregation**. You can create classes using *aggregation*, in which an object contains other objects. This is appropriate when an object type is part of another object type. For example, a savings account object might be a part of a financial portfolio object.

MATLAB Data Class Hierarchy

You can add new data types to MATLAB by extending the MATLAB data class hierarchy. The MATLAB data class hierarchy is shown in this diagram.



The *array* class, shown at the root of the diagram, is the fundamental data class in MATLAB upon which all other data classes are based. The *array* class is a *virtual* class that you cannot directly instantiate. This means that you cannot create a variable with the type *array*. The *numeric* class is also a virtual class. The remaining classes, `char`, `double`, `sparse`, `cell`, `struct`, and the individual storage types, such as uint8, are the classes that you use to work with data in MATLAB.

The diagram shows a user class that inherits from the struct class. All classes that you create are structure based since this is the point in the class hierarchy where you can insert you own classes.

Creating Objects

You create an object by calling the class constructor and passing it the appropriate input arguments. In MATLAB, constructors have the same name as the class name. For example, the statement,

```
p = polynom([1 0 –2 –5]);
```

creates an object named p belonging to the class polynom. Once you have created a polynom object, you can operate on the object using methods that are defined for the polynom class. See “Example: A Polynomial Class” on page 14-23 for a description of the polynom class.

Invoking Methods on Objects

Class methods are M-file functions that take an object as one of the input arguments. The methods for a specific class must be placed in the class directory for that class (the @*class_name* directory). This is the first place that MATLAB looks to find a class method.

The syntax for invoking a method on an object is similar to a function call. Generally, it looks like:

```
[out1,out2,...] = method_name(object,arg1,arg2, ...);
```

For example, suppose a user-defined class called polynom has a char method defined for the class. This method converts a polynom object to a character string and returns the string. This statement calls the char method on the polynom object p.

```
s = char(p);
```

Using the class function, you can confirm that the returned value s is a character string:

```
class(s)
ans =
   char
s
s =
   x^3–2*x–5
```

You can use the methods command to produce a list of all of the methods that are defined for a class.

Private Methods

Private methods can be called only by other methods of their class. You define private methods by placing the associated M-files in a private subdirectory of the @*class_name* directory. In the example,

```
@class_name/private/update_obj.m
```

the method update_obj has scope only within the *class_name* class. This means that update_obj can be called by any method that is defined in the @*class_name* directory, but it cannot be called from the MATLAB command line or by methods outside of the class directory, including parent methods.

Private methods and private functions differ in that private methods (in fact all methods) have an object as one of their input arguments and private functions do not. You can use private functions as helper functions, such as described in the next section.

Helper Functions

In designing a class, you may discover the need for functions that perform support tasks for the class, but do not directly operate on an object. These functions are called *helper functions*. A helper function can be a subfunction in a class method file or a private function. When determining which version of a particular function to call, MATLAB looks for these functions in the order listed above. For more information about the order in which MATLAB calls functions and methods, see “How MATLAB Determines Which Method to Call” on page 14-68.

Debugging Class Methods

You can use the MATLAB debugging commands with object methods in the same way that you use them with other M-files. The only difference is that you need to include the class directory name before the method name in the command call, as shown in this example using dbstop.

```
dbstop @polynom/char
```

While debugging a class method, you have access to all methods defined for the class, including inherited methods, private methods, and private functions.

For more information about debugging MATLAB functions, see Chapter 3, “Debugger and Profiler.”

Changing Class Definition

If you change the class definition, such as the number or names of fields in a class, you must issue a

```
clear classes
```

command to propagate the changes to your MATLAB session. This command also clears all objects from the workspace. See the clear command help entry for more information.

Setting Up Class Directories

The M-files defining the methods for a class are collected together in a directory referred to as the class directory. The directory name is formed with the class name preceded by the character @. For example, one of the examples used in this chapter is a class involving polynomials in a single variable. The name of the class, and the name of the class constructor, is polynom. The M-files defining a polynomial class would be located in directory with the name @polynom.

The class directories are subdirectories of directories on the MATLAB search path, but are not themselves on the path. For instance, the new @polynom directory could be a subdirectory of MATLAB’s working directory or your own personal directory that has been added to the search path.

Adding the Class Directory to the MATLAB Path

After creating the class directory, you need to update the MATLAB path so that MATLAB can locate the class source files. The class directory should not be directly on the MATLAB path. Instead, you should add the parent directory to the MATLAB path. For example, if the `@polynom` class directory is located at

```
c:\my_classes\@polynom
```

you add the class directory to the MATLAB path with the `addpath` command

```
addpath c:\my_classes;
```

If you create a class directory with the same name as another class, MATLAB treats the two class directories as a single directory when locating class methods. For more information, see “How MATLAB Determines Which Method to Call” on page 14-68.

Data Structure

One of the first steps in the design of a new class is the choice of the data structure to be used by the class. Objects are stored in MATLAB structures. The fields of the structure, and the details of operations on the fields, are visible only within the methods for the class. The design of the appropriate data structure can affect the performance of the code.

Tips for C++ and Java Programmers

If you are accustomed to programming in other object-oriented languages, such as C++ or Java, you will find that the MATLAB programming language differs from these languages in some important ways:

- In MATLAB, method dispatching is not syntax based, as it is in C++ and Java. When the argument list contains objects of equal precedence, MATLAB uses the left-most object to select the method to call.
- In MATLAB, there is no equivalent to a destructor method. To remove an object from the workspace, use the `clear` function.
- Construction of MATLAB data types occurs at runtime rather than compile time. You register an object as belonging to a class by calling the `class` function.
- When using inheritance in MATLAB, the inheritance relationship is established in the child class by creating the parent object, and then calling

the class function. For more information on writing constructors for inheritance relationships, see “Building on Other Classes” on page 14-34.

- When using inheritance in MATLAB, the child object contains a parent object in a property with the name of the parent class.
- In MATLAB, there is no passing of variables by reference. When writing methods that update an object, you must pass back the updated object and use an assignment statement. For instance, this call to the set method updates the name field of the object A and returns the updated object.

```
A = set(A,'name','John Smith');
```

- In MATLAB, there is no equivalent to an abstract class.
- In MATLAB, there is no equivalent to a Java interface.
- In MATLAB, there is no equivalent to the C++ scoping operator.
- In MATLAB, there is no virtual inheritance or virtual base classes.
- In MATLAB, there is no equivalent to C++ templates.

References for Object-Oriented Design

For more detailed information about object-oriented design, we recommend these references:

- *Object Oriented Software Construction* - Bertrand Meyer
- *Object Oriented Analysis and Design with Applications* - Grady Booch

Designing User Classes in MATLAB

This section discusses how to approach the design of a class and describes the basic set of methods that should be included in a class.

The MATLAB Canonical Class

When you design a MATLAB class, you should include a standard set of methods that enable the class to behave in a consistent and logical way within the MATLAB environment. Depending on the nature of the class you are defining, you may not need to include all of these methods and you may include a number of other methods to realize the class's design goals.

This table lists the basic methods included in MATLAB classes.

Class Method	Description
class constructor	Creates an object of the class
`display`	Called whenever MATLAB displays the contents of an object (e.g., when an expression is entered without terminating with a semicolon)
`set` and `get`	Accesses class properties
`subsref` and `subsasgn`	Enables indexed reference and assignment for user objects
`end`	Supports `end` syntax in indexing expressions using an object; e.g., `A(1:end)`
`subsindex`	Supports using an object in indexing expressions
converters like `double` and `char`	Methods that convert an object to a MATLAB data type

The following sections discuss the implementation of each type of method, as well as providing references to examples used in this chapter.

The Class Constructor Method

The @ directory for a particular class must contain an M-file known as the *constructor* for that class. The name of the constructor is the same as the name of the directory (excluding the @ prefix and `.m` extension) that defines the name of the class. The constructor creates the object by initializing the data structure and instantiating an object of the class.

Guidelines for Writing a Constructor

Class constructors must perform certain functions so that objects behave correctly in the MATLAB environment. In general, a class constructor must handle three possible combinations of input arguments:

- No input arguments
- An object of the same class as an input argument
- The input arguments used to create an object of the class (typically data of some kind)

No Input Arguments. If there are no input arguments, the constructor should create a default object. Since there are no inputs, you have no data from which to create the object, so you simply initialize the object's data structures with empty or default values, call the `class` function to instantiate the object, and return the object as the output argument. Support for this syntax is required for two reasons:

- When loading objects into the workspace, the `load` function calls the class constructor with no arguments.
- When creating arrays of objects, MATLAB calls the class constructor to add objects to the array.

Object Input Argument. If the first input argument in the argument list is an object of the same class, the constructor should simply return the object. Use the `isa` function to determine if an argument is a member of a class. See "Overloading the + Operator" on page 14-29 for an example of a method that uses this constructor syntax.

Data Input Arguments. If the input arguments exist and are not objects of the same class, then the constructor creates the object using the input data. Of course, as in any function, you should perform proper argument checking in your constructor function. A typical approach is to use a `varargin` input argument and a switch statement to control program flow. This provides an easy way to accommodate the three cases: no inputs, object input, or the data inputs used to create an object.

It is in this part of the constructor that you assign values to the object’s data structure, call the class function to instantiate the object, and return the object as the output argument. If necessary, place the object in an object hierarchy using the superiorto and inferiorto functions.

Using the class Function in Constructors

Within a constructor method, you use the class function to associate an object structure with a particular class. This is done using an internal class tag that is only accessible using the class and isa functions. For example, this call to the class function identifies the object p to be of type polynom.

```
p = class(p,'polynom');
```

Examples of Constructor Methods

See the following sections for examples of constructor methods:

- “The Polynom Constructor Method” on page 14-24
- “The Asset Constructor Method” on page 14-38
- “The Portfolio Constructor Method” on page 14-55

Identifying Objects Outside the Class Directory

The class and isa functions used in constructor methods can also be used outside of the class directory. The expression

```
isa(a,'class_name')
```

checks whether a is an object of the specified class. For example, if p is a polynom object, each of the following expressions is true.

```
isa(pi,'double')
isa('hello','char')
isa(p,'polynom')
```

Outside of the class directory, the class function takes only one argument (it is only within the constructor that class can have more than one argument). The expression

```
class(a)
```

returns a string containing the class name of a. For example

```
class(pi),
class('hello'),
class(p)
```

return

```
'double',
'char',
'polynom'
```

Use the whos command to see what objects are in the MATLAB workspace.

```
whos

  Name      Size         Bytes  Class
  p         1x1          156 polynom object
```

The display Method

MATLAB calls a method named display whenever an object is the result of a statement that is not terminated by a semicolon. For example, creating the variable a, which is a double, calls MATLAB's display method for doubles:

```
>>a = 5

a =

   5
```

You should define a display method so MATLAB can display values on the command line when referencing objects from your class. In many classes, display can simply print the variable name, and then use the char converter method to print the contents or value of the variable, since MATLAB displays output as strings. You must define the char method to convert the object's data to a character string.

Examples of display Methods

See the following sections for examples of `display` methods:

- "The Polynom display Method" on page 14-27
- "The Asset display Method" on page 14-44
- "The Stock display Method" on page 14-52
- "The Portfolio display Method" on page 14-57

Accessing Object Data

You need to write methods for your class that provide access to an object's data. Accessor methods can use a variety of approaches, but all methods that change object data always accept an object as an input argument and return a new object with the data changed. This is necessary because MATLAB does not support passing arguments by reference (i.e., pointers). Functions can change only their private, temporary copy of an object. Therefore, to change an existing object, you must create a new one, and then replace the old one.

The following sections provide more detail about implementation techniques for the `set`, `get`, `subsasgn`, and `subsref` methods.

The set and get Methods

The `set` and `get` methods provide a convenient way to access object data in certain cases. For example, suppose you have created a class that defines an arrow object that MATLAB can display on graphs (perhaps composed of existing MATLAB line and patch objects).

To produce a consistent interface, you could define `set` and `get` methods that operate on arrow objects the way the MATLAB `set` and `get` functions operate on built-in graphics objects. The `set` and `get` verbs convey what operations they perform, but insulate the user from the internals of the object.

Examples of set and get Methods. See the following sections for examples of `set` and `get` methods:

- "The Asset get Method" on page 14-40 and the "The Asset set Method" on page 14-41
- "The Stock get Method" on page 14-47 and the "The Stock set Method" on page 14-50

Property Name Methods

As an alternative to a general `set` method, you can write a method to handle the assignment of an individual property. The method should have the same name as the property name.

For example, if you defined a class that creates objects representing employee data, you might have a field in an employee object called `salary`. You could then define a method called `salary.m` that takes an employee object and a value as input arguments and returns the object with the specified value set.

Indexed Reference Using subsref and subsasgn

User classes implement new data types in MATLAB. It is useful to be able to access object data via an indexed reference, as is possible with MATLAB's built-in data types. For example, if `A` is an array of class `double`, `A(i)` returns the `i`th element of `A`.

As the class designer, you can decide what an index reference to an object means. For example, suppose you define a class that creates polynomial objects and these objects contain the coefficients of the polynomial.

An indexed reference to a polynomial object,

```
p(3)
```

could return the value of the coefficient of x^3, the value of the polynomial at $x = 3$, or something different depending on the intended design.

You define the behavior of indexing for a particular class by creating two class methods – `subsref` and `subsasgn`. MATLAB calls these methods whenever an subscripted reference or assignment is made on an object from the class. If you do not define these methods for a class, indexing is undefined for objects of this class.

In general, the rules for indexing objects are the same as the rules for indexing structure arrays. For details, see Chapter 13, "Structures and Cell Arrays."

Handling Subscripted Reference

The use of a subscript or field designator with an object on the right-hand side of an assignment statement is known as a *subscripted reference*. MATLAB calls a method named `subsref` in these situations. Object subscripted references can be of three forms – an array index, a cell array index, and a structure field name:

```
A(I)
A{I}
A.field
```

Each of these results in a call by MATLAB to the subsref method in the class directory. MATLAB passes two arguments to subsref:

```
B = subsref(A,S)
```

The first argument is the object being referenced. The second argument, S, is a structure array with two fields:

- S.type is a string containing '()', '{}', or '.' specifying the subscript type. The parentheses represent a numeric array; the curly braces, a cell array; and the dot, a structure array.
- S.subs is a cell array or string containing the actual subscripts. A colon used as a subscript is passed as the string ':'.

For instance, the expression

```
A(1:2,:)
```

causes MATLAB to call subsref(A,S), where S is a 1-by-1 structure with

```
S.type = '()'
S.subs = {1:2,':'}
```

Similarly, the expression

```
A{1:2}
```

uses

```
S.type ='{}'
S.subs = {1:2}
```

The expression

```
A.field
```

calls subsref(A,S) where

```
S.type = '.'
S.subs = 'field'
```

These simple calls are combined for more complicated subscripting expressions. In such cases, length(S) is the number of subscripting levels. For example,

```
A(1,2).name(3:4)
```

calls subsref(A,S), where S is a 3-by-1 structure array with the values:

```
S(1).type = '()'    S(2).type = '.'      S(3).type = '()'
S(1).subs = '{1,2}' S(2).subs = 'name'   S(3).subs = '{3:4}'
```

How to Write subsref

The subsref method must interpret the subscripting expressions passed in by MATLAB. A typical approach is to use the switch statement to determine the type of indexing used and to obtain the actual indices. The following three code fragments illustrate how to interpret the input arguments. In each case, the function must return the value B.

For an array index:

```
switch S.type
case '()'
   B = A(S.subs{:});
end
```

For a cell array:

```
switch S.type
case '{}'
   B = A(S.subs{:}); % B is a cell array
end
```

For a structure array:

```
switch S.type
case '.'
   switch S.subs
   case 'field1'
       B = A.field1;
   case 'field2'
       B = A.field2;
   end
end
```

Examples of the subsref Method

See the following sections for examples of the subsref method:

- "The Polynom subsref Method" on page 14-28
- "The Asset subsref Method" on page 14-40
- "The Stock subsref Method" on page 14-48
- "The Portfolio subsref Method" on page 14-64

Subscripted Assignment

The use of a subscript or field designator with an object on the left-hand side of an assignment statement is known as a *subscripted assignment*. MATLAB calls a method named subsasgn in these situations. Object subscripted assignment can be of three forms – an array index, a cell array index, and a structure field name:

```
A(I) = B
A{I} = B
A.field = B
```

Each of these results in a call to subsasgn of the form:

```
A = subsasgn(A,S,B)
```

The first argument, A, is the object being referenced. The second argument, S, has the same fields as those used with subsref. The third argument, B, is the new value.

Examples of the subsasgn Method

See the following sections for examples of the subsasgn method:

- “The Asset subsasgn Method” on page 14-42
- “The Stock subsasgn Method” on page 14-51

Defining end Indexing for an Object

When you use end in an object indexing expression, MATLAB calls the object’s end class method. If you want to be able to use end in indexing expressions involving objects of your class, you must define an end method for your class.

The end method has the calling sequence

```
end(a,k,n)
```

where a is the user object, k is the index in the expression where the end syntax is used, and n is the total number of indices in the expression.

For example, consider the expression

```
A(end–1,:)
```

MATLAB calls the end method defined for the object A using the arguments

```
end(A,1,2)
```

That is, the end statement occurs in the first index element and there are two index elements. The class method for end must then return the index value for the last element of the first dimension. When you implement the end method for your class, you must ensure it returns a value appropriate for the object.

Indexing an Object with Another Object

When MATLAB encounters an object as an index, it calls the subsindex method defined for the object. For example, suppose you have an object a and you want to use this object to index into another object b.

```
c = b(a);
```

A subsindex method might do something as simple as convert the object to double format to be used as an index, as shown in this sample code.

```
function d = subsindex(a)
%SUBSINDEX
% convert the object a to double format to be used
% as an index in an indexing expression
d = double(a);
```

subsindex values are 0-based, not 1-based.

Converter Methods

A converter method is a class method that has the same name as another class, such as char or double. Converter methods accept an object of one class as input and return an object of another class. Converters enable you to:

- Use methods defined for another class
- Ensure that expressions involving objects of mixed class types execute properly

A converter function call is of the form

```
b = class_name(a)
```

where a is an object of a class other than *class_name*. In this case, MATLAB looks for a method called *class_name* in the class directory for object a. If the input object is already of type *class_name*, then MATLAB calls the constructor, which just returns the input argument.

Examples of Converter Methods

See the following sections for examples of converter methods:

- "The Polynom to Double Converter" on page 14-25
- "The Polynom to Char Converter" on page 14-25

Overloading Operators and Functions

In many cases, you may want to change the behavior of MATLAB's operators and functions for cases when the arguments are objects. You can accomplish this by *overloading* the relevant functions. Overloading enables a function to handle different types and numbers of input arguments and perform whatever operation is appropriate for the highest-precedence object. See "Object Precedence" on page 14-66 for more information on object precedence.

Overloading Operators

Each built-in MATLAB operator has an associated function name (e.g., the + operator has an associated `plus.m` function). You can overload any operator by creating an M-file with the appropriate name in the class directory. For example, if either `p` or `q` is an object of type *class_name*, the expression

```
p + q
```

generates a call to a function `@`*class_name*`/plus.m`, if it exists. If `p` and `q` are both objects of different classes, then MATLAB applies the rules of precedence to determine which method to use.

Examples of Overloaded Operators

See the following sections for examples of overloaded operators:

- "Overloading the + Operator" on page 14-29
- "Overloading the - Operator" on page 14-30
- "Overloading the * Operator" on page 14-30

The following table lists the function names for most of MATLAB's operators.

Operation	M-File	Description
`a + b`	`plus(a,b)`	Binary addition
`a – b`	`minus(a,b)`	Binary subtraction
`–a`	`uminus(a)`	Unary minus
`+a`	`uplus(a)`	Unary plus
`a.*b`	`times(a,b)`	Element-wise multiplication
`a*b`	`mtimes(a,b)`	Matrix multiplication
`a./b`	`rdivide(a,b)`	Right element-wise division
`a.\b`	`ldivide(a,b)`	Left element-wise division
`a/b`	`mrdivide(a,b)`	Matrix right division
`a\b`	`mldivide(a,b)`	Matrix left division
`a.^b`	`power(a,b)`	Element-wise power
`a^b`	`mpower(a,b)`	Matrix power
`a < b`	`lt(a,b)`	Less than
`a > b`	`gt(a,b)`	Greater than
`a <= b`	`le(a,b)`	Less than or equal to
`a >= b`	`ge(a,b)`	Greater than or equal to
`a ~= b`	`ne(a,b)`	Not equal to
`a == b`	`eq(a,b)`	Equality
`a & b`	`and(a,b)`	Logical AND
`a | b`	`or(a,b)`	Logical OR
`~a`	`not(a)`	Logical NOT

Operation	M-File	Description

`a:d:b`
`a:b`
	`colon(a,d,b)`
	`colon(a,b)`
		Colon operator

`a'`	`ctranspose(a)`	Complex conjugate transpose
`a.'`	`transpose(a)`	Matrix transpose
`command window output`	`display(a)`	Display method
`[a b]`	`horzcat(a,b,...)`	Horizontal concatenation
`[a; b]`	`vertcat(a,b,...)`	Vertical concatenation
`a(s1,s2,...sn)`	`subsref(a,s)`	Subscripted reference
`a(s1,...,sn) = b`	`subsasgn(a,s,b)`	Subscripted assignment
`b(a)`	`subsindex(a)`	Subscript index

Overloading Functions

You can overload any function by creating a function of the same name in the class directory. When a function is invoked on an object, MATLAB always looks in the class directory before any other location on the search path. To overload the `plot` function for a class of objects, for example, simply place your version of `plot.m` in the appropriate class directory.

Examples of Overloaded Functions

See the following sections for examples of overloaded functions:

- "Overloading Functions for the Polynom Class" on page 14-31
- "The Portfolio pie3 Method" on page 14-57

Example: A Polynomial Class

This example implements a MATLAB data type for polynomials by defining a new class called polynom. The class definition specifies a structure for data storage and defines a directory (`@polynom`) of methods that operate on polynom objects.

Polynom Data Structure

The polynom class represents a polynomial with a row vector containing the coefficients of powers of the variable, in decreasing order. Therefore, a polynom object `p` is a structure with a single field, `p.c`, containing the coefficients. This field is accessible only within the methods in the `@polynom` directory.

Polynom Methods

To create a class that is well behaved within the MATLAB environment and provides useful functionality for a polynomial data type, the polynom class implements the following methods:

- A constructor method `polynom.m`
- A polynom to double converter
- A polynom to char converter
- A `display` method
- A `subsref` method
- Overloaded `+`, –, and `*` operators
- Overloaded `roots`, `polyval`, `plot`, and `diff` functions

The following sections describe the implementation of these methods.

The Polynom Constructor Method

Here is the polynom class constructor, `@polynom/polynom.m`.

```
function p = polynom(a)
%POLYNOM Polynomial class constructor.
%   p = POLYNOM(v) creates a polynomial object from the vector v,
%   containing the coefficients of descending powers of x.
if nargin == 0
   p.c = [];
   p = class(p,'polynom');
elseif isa(a,'polynom')
   p = a;
else
   p.c = a(:).';
   p = class(p,'polynom');
end
```

Constructor Calling Syntax

You can call the polynom constructor method with one of three different arguments:

- No Input Argument – If you call the constructor function with no arguments, it returns a polynom object with empty fields.
- Input Argument is an Object – If you call the constructor function with an input argument that is already a polynom object, MATLAB returns the input argument. The `isa` function (pronounced "is a") checks for this situation.
- Input Argument is a coefficient vector – If the input argument is a variable that is not a polynom object, reshape it to be a row vector and assign it to the `.c` field of the object's structure. The `class` function creates the `polynom` object, which is then returned by the constructor.

An example use of the `polynom` constructor is the statement

```
p = polynom([1 0 –2 –5])
```

This creates a polynomial with the specified coefficients.

Converter Methods for the Polynom Class

A converter method converts an object of one class to an object of another class. Two of the most important converter methods contained in MATLAB classes are double and char. Conversion to double produces MATLAB's traditional matrix, although this may not be appropriate for some classes. Conversion to char is useful for producing printed output.

The Polynom to Double Converter

The double converter method for the polynom class is a very simple M-file, @polynom/double.m, which merely retrieves the coefficient vector.

```
function c = double(p)
% POLYNOM/DOUBLE  Convert polynom object to coefficient vector.
%   c = DOUBLE(p) converts a polynomial object to the vector c
%   containing the coefficients of descending powers of x.
c = p.c;
```

On the object p,

```
p = polynom([1 0 –2 –5])
```

the statement

```
double(p)
```

returns

```
ans =
     1     0    –2    –5
```

The Polynom to Char Converter

The converter to char is a key method because it produces a character string involving the powers of an independent variable, x. Therefore, once you have specified x, the string returned is a syntactically correct MATLAB expression, which you can then evaluate.

Here is @polynom/char.m.

```
function s = char(p)
% POLYNOM/CHAR
% CHAR(p) is the string representation of p.c
if all(p.c == 0)
   s = ’0’;
else
   d = length(p.c) – 1;
   s = [];
   for a = p.c;
       if a ~= 0;
           if ~isempty(s)
               if a > 0
                   s = [s ' + '];
               else
                   s = [s ' – '];
                   a = –a;
               end
           end
           if a ~= 1 | d == 0
               s = [s num2str(a)];
               if d > 0
                   s = [s '*'];
               end
           end
           if d >= 2
               s = [s 'x^' int2str(d)];
           elseif d == 1
               s = [s 'x'];
           end
       end
       d = d – 1;
   end
end
```

Evaluating the Output

If you create the polynom object p

```
p = polynom([1 0 –2 –5]);
```

and then call the char method on p

```
char(p)
```

MATLAB produces the result

```
ans =
   x^3 – 2*x – 5
```

The value returned by char is a string that you can pass to eval once you have defined a scalar value for x. For example,

```
x = 3;
eval(char(p))
ans =
   16
```

See “The Polynom subsref Method” on page 14-28 for a better method to evaluate the polynomial.

The Polynom display Method

Here is @polynom/display.m. This method relies on the char method to produce a string representation of the polynomial, which is then displayed on the screen. This method produces output that is the same as standard MATLAB output. That is, the variable name is displayed followed by an equal sign, then a blank line, then a new line with the value.

```
function display(p)
% POLYNOM/DISPLAY Command window display of a polynom
disp(’ ’);
disp([inputname(1),’ = ’])
disp(’ ’);
disp([’   ’ char(p)])
disp(’ ’);
```

The statement

```
p = polynom([1 0 –2 –5])
```

creates a polynom object. Since the statement is not terminated with a semicolon, the resulting output is:

```
p =
   x^3 - 2*x - 5
```

The Polynom subsref Method

Suppose the design of the polynom class specifies that a subscripted reference to a polynom object causes the polynomial to be evaluated with the value of the independent variable equal to the subscript. That is, for a polynom object p,

```
p = polynom([1 0 –2 –5]);
```

the following subscripted expression returns the value of the polynomial at x = 3 and x = 4:

```
p([3 4])
ans =

   16  51
```

subsref Implementation Details

This implementation takes advantage of the char method already defined in the polynom class to produce an expression that can then be evaluated.

```
function b = subsref(a,s)
% SUBSREF
switch s.type
case '()'
   ind = s.subs{:};
   for i = 1:length(ind)
       b(i) = eval(strrep(char(a),'x',num2str(ind(i))));
   end
otherwise
   error('Specify value for x as p(x)')
end
```

Once the polynomial expression has been generated by the char method, the strrep function is used to swap the passed in value for the character x. The eval function then evaluates the expression and returns the value in the output argument.

Overloading Arithmetic Operators

Several arithmetic operations are meaningful on polynomials and should be implemented for the polynom class. When overloading arithmetic operators, keep in mind what data types you want to operate on. In this section, the `plus`, `minus`, and `mtimes` methods are defined for the polynom class to handle addition, subtraction, and multiplication on polynom/polynom and polynom/double combinations of operands.

Overloading the + Operator

If either `p` or `q` is a polynom, the expression

```
p + q
```

generates a call to a function `@polynom/plus.m`, if it exists (unless `p` or `q` is an object of a higher precedence, as described in “Object Precedence” on page 14-66).

The following M-file redefines the + operator for the polynom class:

```
function r = plus(p,q)
% POLYNOM/PLUS  Implement p + q for polynoms.
p = polynom(p);
q = polynom(q);
k = length(q.c) – length(p.c);
r = polynom([zeros(1,k) p.c] + [zeros(1,–k) q.c]);
```

The function first makes sure that both input arguments are polynomials. This ensures that expressions such as

```
p + 1
```

that involve both a polynom and a double, work correctly. The function then accesses the two coefficient vectors and, if necessary, pads one of them with zeros to make them the same length. The actual addition is simply the vector sum of the two coefficient vectors. Finally, the function calls the `polynom` constructor a third time to create the properly typed result.

Overloading the – Operator

You can implement the overloaded minus operator (–) using the same approach as the plus (+) operator. MATLAB calls @polynom/minus.m to compute p–q.

```
function r = minus(p,q)
% POLYNOM/MINUS Implement p – q for polynoms.
p = polynom(p);
q = polynom(q);
k = length(q.c) – length(p.c);
r = polynom([zeros(1,k) p.c] – [zeros(1,–k) q.c]);
```

Overloading the ∗ Operator

MATLAB calls the method @polynom/mtimes.m to compute the product p*q. The letter m at the beginning of the function name comes from the fact that it is overloading MATLAB's *matrix* multiplication. Multiplication of two polynomials is simply the convolution of their coefficient vectors.

```
function r = mtimes(p,q)
% POLYNOM/MTIMES   Implement p * q for polynoms.
p = polynom(p);
q = polynom(q);
r = polynom(conv(p.c,q.c));
```

Using the Overloaded Operators

Given the polynom object

```
p = polynom([1 0 –2 –5])
```

MATLAB calls these two functions @polynom/plus.m and @polynom/mtimes.m when you issue the statements

```
q = p+1
r = p*q
```

to produce

```
q =
   x^3 – 2*x – 4
r =
   x^6 – 4*x^4 – 9*x^3 + 4*x^2 + 18*x + 20
```

Overloading Functions for the Polynom Class

MATLAB already has several functions for working with polynomials represented by coefficient vectors. They should be overloaded to also work with the new polynom object. In many cases, the overloading methods can simply apply the original function to the coefficient field.

Overloading roots for the Polynom Class

The method @polynom/roots.m finds the roots of polynom objects.

```
function r = roots(p)
% POLYNOM/ROOTS.  ROOTS(p) is a vector containing the roots of p.
r = roots(p.c);
```

The statement

```
roots(p)
```

results in

```
ans =
   2.0946
   –1.0473+ 1.1359i
   –1.0473– 1.1359i
```

Overloading polyval for the Polynom Class

The function polyval evaluates a polynomial at a given set of points. @polynom/polyval.m uses nested multiplication, or Horner's method to reduce the number of multiplication operations used to compute the various powers of x.

```
function y = polyval(p,x)
% POLYNOM/POLYVAL  POLYVAL(p,x) evaluates p at the points x.
y = 0;
for a = p.c
   y = y.*x + a;
end
```

Overloading plot for the Polynom Class

The overloaded plot function uses both root and polyval. The function selects the domain of the independent variable to be slightly larger than an interval containing all real roots. Then polyval is used to evaluate the polynomial at a few hundred points in the domain.

```
function plot(p)
% POLYNOM/PLOT  PLOT(p) plots the polynom p.
r = max(abs(roots(p)));
x = (–1.1:0.01:1.1)*r;
y = polyval(p,x);
plot(x,y);
title(char(p))
grid on
```

Overloading diff for the Polynom Class

The method @polynom/diff.m differentiates a polynomial by reducing the degree by 1 and multiplying each coefficient by its original degree.

```
function q = diff(p)
% POLYNOM/DIFF  DIFF(p) is the derivative of the polynom p.
c = p.c;
d = length(c) – 1;  % degree
q = polynom(p.c(1:d).*(d:–1:1));
```

Listing Class Methods

The function call

```
methods('class_name')
```

or its command form

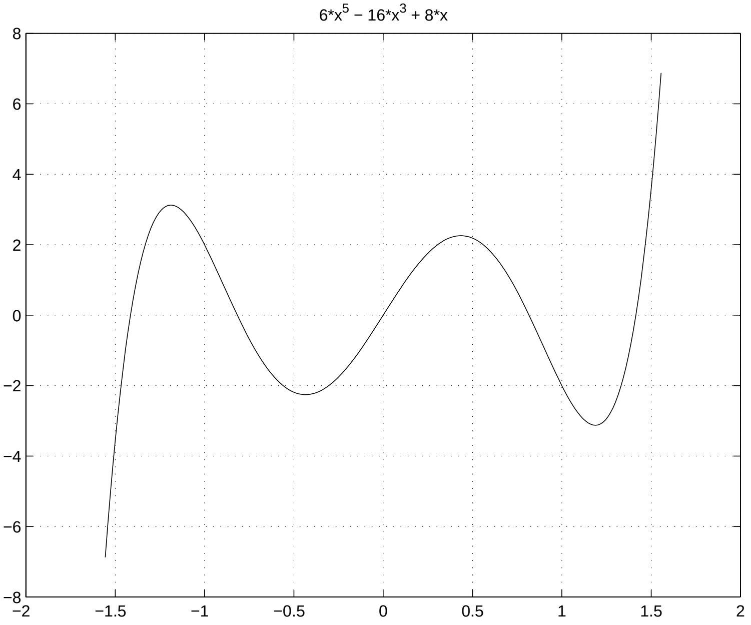
```
methods class_name
```

shows all the methods available for a particular class. For the polynom example, the output is:

```
methods polynom

Methods for class polynom:

char   display   minus    plot    polynom   roots
diff   double    mtimes   plus    polyval   subsref
```

Plotting the two polynom objects x and p calls most of these methods.

```
x = polynom([1 0]);
p = polynom([1 0 -2 -5]);
plot(diff(p*p + 10*p + 20*x) - 20)
```



Building on Other Classes

A MATLAB object can *inherit* properties and behavior from another MATLAB object. When one object (the child) inherits from another (the parent), the child object includes all the fields of the parent object and can call the parent's methods. The parent methods can access those fields that a child object inherited from the parent class, but not fields new to the child class.

Inheritance is a key feature of object-oriented programming. It makes it easy to reuse code by allowing child objects to take advantage of code that exists for parent objects. Inheritance enables a child object to behave exactly like a parent object, which facilitates the development of related classes that behave similarly, but are implemented differently.

There are two kinds of inheritance:

- Simple inheritance, in which a child object inherits characteristics from one parent class.
- Multiple inheritance, in which a child object inherits characteristics from more than one parent class.

This section also discusses a related topic, *aggregation*. Aggregation allows one object to contain another object as one of its fields.

Simple Inheritance

A class that inherits attributes from a single parent class, and adds new attributes of its own, uses simple inheritance. Inheritance implies that objects belonging to the child class have the same fields as the parent class, as well as additional fields. Therefore, methods associated with the parent class can operate on objects belonging to the child class. The methods associated with the child class, however, cannot operate on objects belonging to the parent class. You cannot access the parent's fields directly from the child class; you must use access methods defined for the parent.

The constructor function for a class that inherits the behavior of another has two special characteristics:

- It calls the constructor function for the parent class to create the inherited fields.
- The calling syntax for the `class` function is slightly different, reflecting both the child class and the parent class.

The general syntax for establishing a simple inheritance relationship using the `class` function is

```
child_obj = class(child_obj,'child_class',parent_obj);
```

Simple inheritance can span more than one generation. If a parent class is itself an inherited class, the child object will automatically inherit from the grandparent class.

Visibility of Class Properties and Methods

The parent class does not have knowledge of the child properties or methods. The child class cannot access the parent properties directly, but must use parent access methods (e.g., `get` or `subsref` method) to access the parent properties. From the child class methods, this access is accomplished via the parent field in the child structure. For example, when a constructor creates a child object `c`,

```
c = class(c,'child_class_name',parent_object);
```

MATLAB automatically creates a field, `c.`*parent_class_name*, in the object's structure that contains the parent object. You could then have a statement in the child's display method that calls the parent's display method:

```
display(c.parent_class_name)
```

See "Designing the Stock Class" on page 14-45 for examples that use simple inheritance.

Multiple Inheritance

In the multiple inheritance case, a class of objects inherits attributes from more than one parent class. The child object gets fields from all the parent classes, as well as fields of its own.

Multiple inheritance can encompass more than one generation. For example, each of the parent objects could have inherited fields from multiple grandparent objects, and so on. Multiple inheritance is implemented in the constructors by calling `class` with more than three arguments.

```
obj = class(structure,'class_name',parent1,parent2,...)
```

You can append as many parent arguments as desired to the class input list.

Multiple parent classes can have associated methods of the same name. In this case, MATLAB calls the method associated with the parent that appears first in the `class` function call in the constructor function. There is no way to access subsequent parent function of this name.

Aggregation

In addition to standard inheritance, MATLAB objects support *containment* or *aggregation*. That is, one object can contain (embed) another object as one of its fields. For example, a rational object might use two polynom objects, one for the numerator and one for the denominator.

You can call a method for the contained object only from within a method for the outer object. When determining which version of a function to call, MATLAB considers only the outermost containing class of the objects passed as arguments; the classes of any contained objects are ignored.

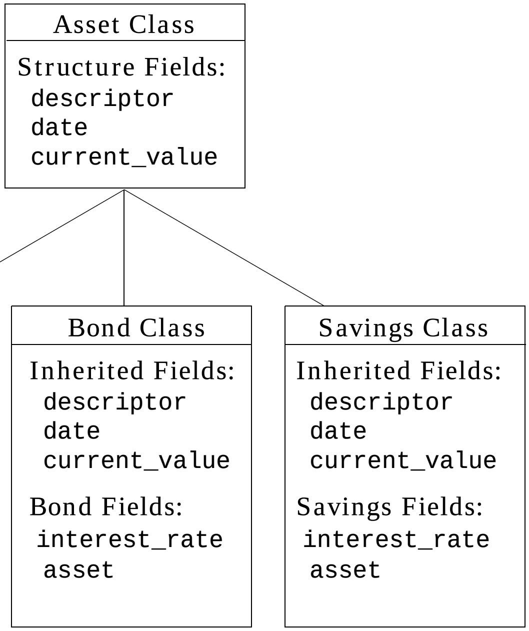
See “Example: The Portfolio Container” on page 14-54 for an example of aggregation.

Example: Assets and Asset Subclasses

As an example of simple inheritance, consider a general asset class that can be used to represent any item that has monetary value. Some examples of an asset are: stocks, bonds, savings accounts, and any other piece of property. In designing this collection of classes, the `asset` class holds the data that is common to all of the specialized asset subclasses. The individual asset subclasses, such as the `stock` class, inherit the `asset` properties and contribute additional properties. The subclasses are “kinds of” assets.

Simple Inheritance

An example of a simple inheritance relationship using an asset parent class is shown in this diagram.



As shown in the diagram, the `stock`, `bond`, and `savings` classes inherit structure fields from the `asset` class. In this example, the `asset` class is used to provide storage for data common to all subclasses and to share asset methods with these subclasses. This example shows how to implement the asset and stock classes. The bond and savings classes can be implemented in a way that is very similar to the stock class, as would other types of asset subclasses.

Designing the Asset Class

The asset class provides storage and access for information common to all asset children. It is not intended to be instantiated directly, so it does not require an extensive set of methods. To serve its purpose, the class needs to contain the following methods:

- Constructor
- get and set
- subsref and subsasgn
- display

The Asset Constructor Method

The asset class is based on a structure array with four fields:

- descriptor – Identifier of the particular asset (e.g., stock name, savings account number, etc.)
- date – The date the object was created (calculated by the date command)
- current_value – The current value of the asset (calculated from subclass data)

This information is common to asset child objects (stock, bond, and savings), so it is handled from the parent object to avoid having to define the same fields in each child class. This is particularly helpful as the number of child classes increases.

```
function a = asset(varargin)
% ASSET Constructor function for asset object
% a = asset(descirptor, current_value)
switch nargin
case 0
% if no input arguments, create a default object
   a.descriptor = 'none';
   a.date = date;
   a.current_value = 0;
   a = class(a,'asset');
case 1
% if single argument of class asset, return it
   if (isa(varargin{1},'asset'))
       a = varargin{1};
   else
       error('Wrong argument type')
   end
case 2
% create object using specified values
   a.descriptor = varargin{1};
   a.date = date;
   a.current_value = varargin{2};
   a = class(a,'asset');
otherwise
   error('Wrong number of input arguments')
end
```

The function uses a switch statement to accommodate three possible scenarios:

- Called with no arguments, the constructor returns a default asset object.
- Called with one argument that is an asset object, the object is simply returned.
- Called with two arguments (subclass descriptor, and current value), the constructor returns a new asset object.

The asset constructor method is not intended to be called directly; it is called from the child constructors since its purpose is to provide storage for common data.

The Asset get Method

The asset class needs methods to access the data contained in asset objects. The following function implements a get method for the class. It uses capitalized property names rather than literal field names to provide an interface similar to other MATLAB objects.

```
function val = get(a,prop_name)
% GET Get asset properties from the specified object
% and return the value
switch prop_name
case 'Descriptor'
   val = a.descriptor;
case 'Date'
   val = a.date;
case 'CurrentValue'
   val = a.current_value;
otherwise
   error([prop_name,' Is not a valid asset property'])
end
```

This function accepts an object and a property name and uses a switch statement to determine which field to access. This method is called by the subclass get methods when accessing the data in the inherited properties.

The Asset subsref Method

The subsref method provides access to the data contained in an asset object using one-based numeric indexing and structure field name indexing. The outer switch statement determines if the index is a numeric or field name syntax. The inner switch statements map the index to the appropriate value.

MATLAB calls subsref whenever you make a subscripted reference to an object (e.g., A(i), A{i}, or A.*fieldname*). See the subsref help entry for more information on object subscripted reference.

```
function b = subsref(a,index)
%SUBSREF Define field name indexing for asset objects
switch index.type
case '()'
   switch index.subs{:}
   case 1
       b = a.descriptor;
   case 2
       b = a.date;
   case 3
       b = a.current_value;
   otherwise
       error('Index out of range')
   end
case '.'
   switch index.subs
   case 'descriptor'
       b = a.descriptor;
   case 'date'
       b = a.date;
   case 'current_value'
       b = a.current_value;
   otherwise
       error('Invalid field name')
   end
case '{}'
   error('Cell array indexing not supported by asset objects')
end
```

The Asset set Method

The asset class set method is called by subclass set methods. This method accepts an asset object and variable length argument list of property name/ property value pairs and returns the modified object. Subclass set methods call the asset set method and require the capability to return the modified object since MATLAB does not support passing arguments by reference. See “The Stock set Method” on page 14-50 for an example.

```
function a = set(a,varargin)
% SET Set asset properties and return the updated object
property_argin = varargin;
while length(property_argin) >= 2,
   prop = property_argin{1};
   val = property_argin{2};
   property_argin = property_argin(3:end);
   switch prop
   case ’Descriptor’
       a.descriptor = val;
   case ’Date’
       a.date = val;
   case ’CurrentValue’
       a.current_value = val;
   otherwise
       error(’Asset properties: Descriptor, Date, CurrentValue’)
   end
end
```

The Asset subsasgn Method

The subsasgn method is the assignment equivalent of the subsref method. This version enables you to change the data contained in an object using one-based numeric indexing and structure field name indexing. The outer switch statement determines if the index is a numeric or field name syntax. The inner switch statements map the index value to the appropriate value in the stock structure.

MATLAB calls subsasgn whenever you execute an assignment statement (e.g., A(i) = val, A{i} = val, or A.*fieldname* = val). See the subsasgn help entry for more information on assignment statements in MATLAB.

```
function a = subsasgn(a,index,val)
% SUBSASGN Define index assignment for asset objects
switch index.type
case ’()’
   switch index.subs{:}
   case 1
       a.descriptor = val;
   case 2
       a.date = val;
   case 3
       a.current_value = val;
   otherwise
       error(’Index out of range’)
   end
case ’.’
   switch index.subs
   case ’descriptor’
       a.descriptor = val;
   case ’date’
       a.date = val;
   case ’current_value’
       a.current_value = val;
   otherwise
       error(’Invalid field name’)
   end
end
```

The subsasgn method enables you to assign values to the asset object data structure using two techniques. For example, suppose you have a child stock object `s`.

```
s = stock(’XYZ’,100,25);
```

Within stock class methods, you could change the `descriptor` field with either of the following statements

```
s.asset(1) = ’ABC’;
```

or

```
s.asset.descriptor = ’ABC’;
```

See the “The Stock subsasgn Method” on page 14-51 for an example of how the child subsasgn method calls the parent subsasgn method.

The Asset display Method

The asset display method is designed to be called from child-class display methods. Its purpose is to display the data it stores for the child object. The method simply formats the data for display in a way that is consistent with the formatting of the child’s display method.

```
function display(a)
% DISPLAY(a) Display an asset object
stg = sprintf('Descriptor: %s\nDate: %s\nCurrent Value:%9.2f',...
   a.descriptor,a.date,a.current_value);
disp(stg)
```

The stock class display method can now call this method to display the data stored in the parent class. This approach isolates the stock display method from changes to the asset class.

The Asset fieldcount Method

The asset fieldcount method returns the number of fields in the asset object data structure. fieldcount enables asset child methods to determine the number of fields in the asset object during execution, rather than requiring the child methods to have knowledge of the asset class. This allows you to make changes to the number of fields in the asset class data structure without having to change child-class methods.

```
function num_fields = fieldcount(asset_obj)
% Determines the number of fields in an asset object
% Used by asset child class methods
num_fields = length(fieldnames(struct(asset_obj)));
```

The struct function converts an object to its equivalent data structure, enabling access to the structure’s contents.

Other Asset Methods

The asset class provides inherited data storage for its child classes, but is not instanced directly. The set, get, and display methods provide access to the stored data. It is not necessary to implement the full complement of methods for asset objects (such as converters, `end`, and `subsindex`) since only the child classes access the data.

Designing the Stock Class

A stock object is designed to represent one particular asset in a person's investment portfolio. This object contains two properties of its own and inherits three properties from its parent asset object.

Stock properties:

- `NumberShares` – The number of shares for the particular stock object.
- `SharePrice` – The value of each share.

Asset properties:

- `Descriptor` – The identifier of the particular asset (e.g., stock name, savings account number, etc.).
- `Date` – The date the object was created (calculated by the `date` command).
- `CurrentValue` – The current value of the asset.

Note that the property names are not actually the same as the field names of the structure array used internally by stock and asset objects. The property name interface is controlled by the stock and asset `set` and `get` methods and is designed to resemble the interface of other MATLAB object properties.

The `asset` field in the stock object structure contains the parent asset object and is used to access the inherited fields in the parent structure.

Stock Class Methods

The stock class implements the following methods:

- Constructor
- `get` and `set`
- `subsref` and `subsasgn`
- `display`

The Stock Class Constructor

The stock constructor creates a stock object from three input arguments:

- The stock name
- The number of shares
- The share price

The constructor must create an asset object from within the stock constructor to be able to specify it as a parent to the stock object. The stock constructor must, therefore, call the asset constructor. The class function, which is called to create the stock object, defines the asset object as the parent.

Keep in mind that the asset object is created in the temporary workspace of the stock constructor function and is stored as a field (.asset) in the stock structure. The stock object inherits the asset fields, but the asset object is not returned to the base workspace.

```
function s = stock(varargin)
% STOCK Stock class constructor.
% s = stock(descriptor, num_shares, share_price)
switch nargin
case 0
% if no input arguments, create a default object
   s.num_shares = 0;
   s.share_price = 0;
   a = asset('none',0);
   s = class(s, 'stock',a);
case 1
% if single argument of class stock, return it
   if (isa(varargin{1},'stock'))
       s = varargin{1};
   else
       error('Input argument is not a stock object')
   end
case 3
% create object using specified values
   s.num_shares = varargin{2};
   s.share_price = varargin{3};
   a = asset(varargin{1},varargin{2} * varargin{3});
   s = class(s,'stock',a);
otherwise
   error('Wrong number of input arguments')
end
```

Constructor Calling Syntax

The stock constructor method can be called in one of three ways:

- No Input Argument – If called with no arguments, the constructor returns a default object with empty fields.
- Input Argument is a Stock Object – If called with a single input argument that is a stock object, the constructor returns the input argument. A single argument that is not a stock object generates an error.
- Three Input Arguments – If there are three input arguments, the constructor uses them to define the stock object.
- Otherwise – If none of the above three conditions are met, return an error.

For example, this statement creates a stock object to record the ownership of 100 shares of XYZ corporation stocks with a price per share of 25 dollars.

```
XYZ_stock = stock('XYZ',100,25);
```

The Stock get Method

The get method provides a way to access the data in the stock object using a “property name” style interface, similar to Handle Graphics. While in this example the property names are similar to the structure field name, they can be quite different. You could also choose to exclude certain fields from access via the get method or return the data from the same field for a variety of property names, if such behavior suits your design.

```
function val = get(s,prop_name)
% GET Get stock property from the specified object
% and return the value. Property names are: NumberShares
% SharePrice, Descriptor, Date, Type, CurrentValue
switch prop_name
case 'NumberShares'
   val = s.num_shares;
case 'SharePrice'
   val = s.share_price;
case 'Descriptor'
   val = get(s.asset,'Descriptor'); % call asset get method
case 'Date'
   val = get(s.asset,'Date');
case 'Type'
   val = get(s.asset,'Type');
case 'CurrentValue'
   val = get(s.asset,'CurrentValue');
otherwise
   error([prop_name ,'Is not a valid stock property'])
end
```

Note that the asset object is accessed via the stock object's asset field (`s.asset`). MATLAB automatically creates this field when the `class` function is called with the parent argument.

The Stock subsref Method

The `subsref` method defines subscripted indexing for the stock class. In this example, `subsref` is implemented to enable numeric and structure field name indexing of stock objects.

```
function b = subsref(s,index)
% SUBSREF Define field name indexing for stock objects
fc = fieldcount(s.asset);
switch index.type
case ’()’
   if (index.subs{:} <= fc)
       b = subsref(s.asset,index);
   else
       switch index.subs{:} – fc
       case 1
           b = s.num_shares;
       case 2
           b = s.share_price;
       otherwise
       error([’Index must be in the range 1 to ’,num2str(fc + 2)])
       end
   end
case ’.’
   switch index.subs
   case ‘num_shares’
       b = s.num_shares;
   case ‘share_price’
       b = s.share_price;
   otherwise
       b = subsref(s.asset,index);
   end
end
```

The outer `switch` statement determines if the index is a numeric or field name syntax.

The `fieldcount` asset method determines how many fields there are in the asset structure, and the `if` statement calls the asset `subsref` method for indices `1` to `fieldcount`. See “The Asset fieldcount Method” on page 14-44 and the “The Asset subsref Method” on page 14-40 for a description of these methods.

Numeric indices greater than the number returned by `fieldcount` are handled by the inner `switch` statement, which maps the index value to the appropriate field in the stock structure.

Field-name indexing assumes field names other than num_shares and share_price are asset fields, which eliminates the need for knowledge of asset fields by child methods. The asset subsref method performs field-name error checking.

See the subsref help entry for general information on implementing this method.

The Stock set Method

The set method provides a “property name” interface like the get method. It is designed to update the number of shares, the share value, and the descriptor. The current value and the date are automatically updated.

```
function s = set(s,varargin)
% SET Set stock properties to the specified values
% and return the updated object
property_argin = varargin;
while length(property_argin) >= 2,
   prop = property_argin{1};
   val = property_argin{2};
   property_argin = property_argin(3:end);
   switch prop
   case ’NumberShares’
       s.num_shares = val;
   case ’SharePrice’
       s.share_price = val;
   case ’Descriptor’
       s.asset = set(s.asset,’Descriptor’,val);
   otherwise
       error(’Invalid property’)
   end
end
s.asset = set(s.asset,’CurrentValue’,...
           s.num_shares * s.share_price,’Date’,date);
```

Note that this function creates and returns a new stock object with the new values, which you then copy over the old value. For example, given the stock object,

```
s = stock(’XYZ’,100,25);
```

the following `set` command updates the share price:

```
s = set(s,’SharePrice’,36);
```

It is necessary to copy over the original stock object (i.e., assign the output to `s`) because MATLAB does not support passing arguments by reference. Hence the `set` method actually operates on a copy of the object.

The Stock subsasgn Method

The `subsasgn` method enables you to change the data contained in a stock object using numeric indexing and structure field name indexing. MATLAB calls `subsasgn` whenever you execute an assignment statement (e.g., `A(i) = val`, `A{i} = val`, or `A.fieldname = val`).

```
function s = subsasgn(s,index,val)
% SUBSASGN Define index assignment for stock objects
fc = fieldcount(s.asset);
switch index.type
case ’()’
   if (index.subs{:} <= fc)
       s.asset = subsasgn(s.asset,index,val);
   else
       switch index.subs{:}–fc
       case 1
           s.num_shares = val;
       case 2
           s.share_price = val;
       otherwise
       error([’Index must be in the range 1 to ’,num2str(fc + 2)])
       end
   end
case ’.’
   switch index.subs
   case ’num_shares’
       s.num_shares = val;
   case ’share_price’
       s.share_price = val;
   otherwise
       s.asset = subsasgn(s.asset,index,val);
   end
end
```

The outer switch statement determines if the index is a numeric or field name syntax.

The fieldcount asset method determines how many fields there are in the asset structure and the if statement calls the asset subsasgn method for indices 1 to fieldcount. See “The Asset fieldcount Method” on page 14-44 and the “The Asset subsasgn Method” on page 14-42 for a description of these methods.

Numeric indices greater than the number returned by fieldcount are handled by the inner switch statement, which maps the index value to the appropriate field in the stock structure.

Field-name indexing assumes field names other than num_shares and share_price are asset fields, which eliminates the need for knowledge of asset fields by child methods. The asset subsasgn method performs field-name error checking.

The subsasgn method enables you to assign values to stock object data structure using two techniques. For example, suppose you have a stock object

```
s = stock('XYZ',100,25)
```

You could change the descriptor field with either of the following statements

```
s(1) = 'ABC';
```

or

```
s.descriptor = 'ABC';
```

See the subsasgn help entry for general information on assignment statements in MATLAB.

The Stock display Method

When you issue the statement (without terminating with a semicolon)

```
XYZStock = stock('XYZ',100,25)
```

MATLAB looks for a method in the @stock directory called display. The display method for the stock class produces this output.

```
Descriptor: XYZ
Date: 17-Nov-1998
Type: stock
Current Value:  2500.00
Number of shares: 100
Share price: 25.00
```

Here is the stock display method.

```
function display(s)
% DISPLAY(s) Display a stock object
display(s.asset)
stg = sprintf('Number of shares: %g\nShare price: %3.2f\n',...
   s.num_shares,s.share_price);
disp(stg)
```

First, the parent asset object is passed to the asset display method to display its fields (MATLAB calls the asset display method because the input argument is an asset object). The stock object's fields are displayed in a similar way using a formatted text string.

Note that if you did not implement a stock class display method, MATLAB would call the asset display method. This would work, but would display only the descriptor, date, type, and current value.

Example: The Portfolio Container

Aggregation is the containment of one class by another class. The basic relationship is: each contained class “is a part of” the container class.

For example, consider a financial portfolio class as a container for a set of assets (stocks, bonds, savings, etc.). Once the individual assets are grouped, they can be analyzed, and useful information can be returned. The contained objects are not accessible directly, but only via the portfolio class methods.

See “Example: Assets and Asset Subclasses” on page 14-37 for information about the assets collected by this portfolio class.

Designing the Portfolio Class

The portfolio class is designed to contain the various assets owned by a given individual and provide information about the status of his or her investment portfolio. This example implements a somewhat over-simplified portfolio class that:

- Contains an individual’s assets
- Displays information about the portfolio contents
- Displays a 3-D pie chart showing the relative mix of asset types in the portfolio

Required Portfolio Methods

The portfolio class implements only three methods:

- `portfolio` – The portfolio constructor.
- `display` – Displays information about the portfolio contents.
- `pie3` – Overloaded version of `pie3` function designed to take a single portfolio object as an argument.

Since a portfolio object contains other objects, the portfolio class methods can use the methods of the contained objects. For example, the portfolio `display` method calls the stock class `display` method, and so on.

The Portfolio Constructor Method

The portfolio constructor method takes as input arguments a client's name and a variable length list of asset subclass objects (stock, bond, and savings objects in this example). The portfolio object uses a structure array with the following fields:

- name – The client's name.
- ind_assets – The array of asset subclass objects (stock, bond, savings).
- total_value – The total value of all assets. The constructor calculates this value from the objects passed in as arguments.
- account_number – The account number. This field is assigned a value only when you save a portfolio object (see "Saving and Loading Objects" on page 14-61).

```
function p = portfolio(name,varargin)
% PORTFOLIO Create a portfolio object containing the
% client’s name and a list of assets
switch nargin
case 0
   % if no input arguments, create a default object
   p.name = ’none’;
   p.total_value = 0;
   p.ind_assets = {};
   p.account_number = ’’;
   p = class(p,’portfolio’);
case 1
   % if single argument of class portfolio, return it
   if isa(name,’portfolio’)
       p = name;
   else
       disp([inputname(1) ’ is not a portfolio object’])
       return
   end
otherwise
   % create object using specified arguments
   p.name = name;
   p.total_value = 0;
   for i = 1:length(varargin)
       p.ind_assets(i) = {varargin{i}};
       asset_value = get(p.ind_assets{i},’CurrentValue’);
       p.total_value = p.total_value + asset_value;
   end
   p.account_number = ’’;
   p = class(p,’portfolio’);
end
```

Constructor Calling Syntax

The portfolio constructor method can be called in one of three different ways:

- No input arguments – If called with no arguments, it returns an object with empty fields.
- Input argument is an object – If the input argument is already a portfolio object, MATLAB returns the input argument. The isa function checks for this case.
- More than two input arguments – If there are more than two input arguments, the constructor assumes the first is the client's name and the rest are asset subclass objects. A more thorough implementation would perform more careful input argument checking, for example, using the isa function to determine if the arguments are the correct class of objects.

The Portfolio display Method

The portfolio display method lists the contents of each contained object by calling the object's display method. It then lists the client name and total asset value.

```
function display(p)
% DISPLAY Display a portfolio object
for i=1:length(p.ind_assets)
   display(p.ind_assets{i})
end
stg = sprintf('\nAssets for Client: %s\nTotal Value: %9.2f\n',...
p.name,p.total_value);
disp(stg)
```

The Portfolio pie3 Method

The portfolio class overloads the MATLAB pie3 function to accept a portfolio object and display a 3-D pie chart illustrating the relative asset mix of the client's portfolio. MATLAB calls the @portfolio/pie3.m version of pie3 whenever the input argument is a single portfolio object.

```
function pie3(p)
% PIE3 Create a 3-D pie chart of a portfolio
stock_amt = 0; bond_amt = 0; savings_amt = 0;
for i=1:length(p.ind_assets)
   if isa(p.ind_assets{i},'stock')
       stock_amt = stock_amt + ...
           get(p.ind_assets{i},'CurrentValue');
   elseif isa(p.ind_assets{i},'bond')
       bond_amt = bond_amt + ...
           get(p.ind_assets{i},'CurrentValue');
   elseif isa(p.ind_assets{i},'savings')
       savings_amt = savings_amt + ...
           get(p.ind_assets{i},'CurrentValue');
   end
end
i = 1;
if stock_amt ~= 0
   label(i) = {'Stocks'};
   pie_vector(i) = stock_amt;
   i = i +1;
end
if bond_amt ~= 0
   label(i) = {'Bonds'};
   pie_vector(i) = bond_amt;
   i = i +1;
end
if savings_amt ~= 0
   label(i) = {'Savings'};
   pie_vector(i) = savings_amt;
end
pie3(pie_vector,label)
set(gcf,'Renderer','zbuffer')
set(findobj(gca,'Type','Text'),'FontSize',14)
cm = gray(64);
colormap(cm(48:end,:))
stg(1) = {['Portfolio Composition for ',p.name]};
stg(2) = {['Total Value of Assets: $',num2str(p.total_value)]};
title(stg,'FontSize',12)
```

There are three parts in the overloaded pie3 method.

- The first uses the asset subclass get methods to access the CurrentValue property of each contained object. The total value of each class is summed.
- The second part creates the pie chart labels and builds a vector of graph data, depending on which objects are present.
- The third part calls the MATLAB pie3 function, makes some font and colormap adjustments, and adds a title.

Creating a Portfolio

Suppose you have implemented a collection of asset subclasses in a manner similar to the stock class. You can then use a portfolio object to present the individual's financial portfolio. For example, given the following assets:

```
XYZStock = stock('XYZ',200,12);
SaveAccount = savings('Acc # 1234',2000,3.2);
Bonds = bond('U.S. Treasury',1600,12);
```

Now create a portfolio object.

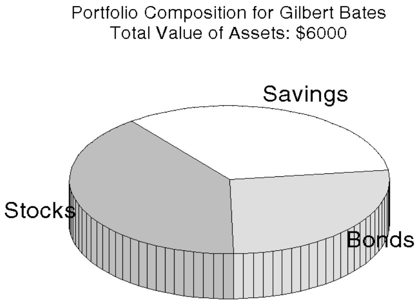
```
p = portfolio('Gilbert Bates',XYZStock,SaveAccount,Bonds)
```

The portfolio display method summarizes the portfolio contents (because this statement is not terminated by a semicolon):

```
Descriptor: XYZ
Date: 24-Nov-1998
Type: stock
Current Value:   2400.00
Number of shares: 200
Share price: 12.00

Descriptor: Acc # 1234
Date: 24-Nov-1998
Type: savings
Current Value:   2000.00
Interest Rate: 3.2%

Descriptor: U.S. Treasury
Date: 24-Nov-1998
Type: bond
Current Value:   1600.00
Interest Rate: 12%

Assets for Client: Gilbert Bates
Total Value:   6000.00
```

The portfolio pie3 method displays the relative mix of assets using a pie chart:

```
pie3(p)
```



Saving and Loading Objects

You can use the MATLAB `save` and `load` commands to save and retrieve user-defined objects to and from `.mat` files, just like any other variables.

When you load objects, MATLAB calls the object's class constructor to register the object in the workspace. The constructor function for the object class you are loading must be able to be called with no input arguments and return a default object. See "Guidelines for Writing a Constructor" on page 14-10 for more information.

Modifying Objects During Save or Load

When you issue a `save` or `load` command on objects, MATLAB looks for class methods called `saveobj` and `loadobj` in the class directory. You can overload these methods to modify the object before the save or load operation. For example, you could define a `saveobj` method that saves related data along with the object or you could write a `loadobj` method that updates objects to a newer version when this type of object is loaded into the MATLAB workspace.

Example – Defining saveobj and loadobj

In the section "Example: The Portfolio Container" on page 14-54, portfolio objects are used to collect information about a client's investment portfolio. Now suppose you decide to add an account number to each portfolio object that is saved. You can define a portfolio `saveobj` method to carry out this task automatically during the save operation.

Suppose further that you have already saved a number of portfolio objects without the account number. You want to update these objects during the load operation so that they are still valid portfolio objects. You can do this by defining a `loadobj` method for the portfolio class.

Summary of Code Changes

To implement the account number scenario, you need to add or change the following functions:

- portfolio – The portfolio constructor method needs to be modified to create a new field, account_number, which is initialized to the empty string when an object is created.
- saveobj – A new portfolio method designed to add an account number to a portfolio object during the save operation, only if the object does not already have one.
- loadobj – A new portfolio method designed to update older versions of portfolio objects that were saved before the account number structure field was added.
- subsref – A new portfolio method that enables subscripted reference to portfolio objects outside of a portfolio method.
- getAccountNumber – a MATLAB function that returns an account number that consists of the first three letters of the client's name.

The saveobj Method

MATLAB looks for the portfolio saveobj method whenever the save command is passed a portfolio object. If @portfolio/saveobj exists, MATLAB passes the portfolio object to saveobj, which must then return the modified object as an output argument. The following implementation of saveobj determines if the object has already been assigned an account number from a previous save operation. If not, saveobj calls getAccountNumber to obtain the number and assigns it to the account_number field.

```
function b = saveobj(a)
if isempty(a.account_number)
   a.account_number = getAccountNumber(a);
end
b = a;
```

The loadobj Method

MATLAB looks for the portfolio loadobj method whenever the load command detects portfolio objects in the .mat file being loaded. If loadobj exists, MATLAB passes the portfolio object to loadobj, which must then return the modified object as an output argument. The output argument is then loaded into the workspace.

If the input object does not match the current definition as specified by the constructor function, then MATLAB converts it to a structure containing the same fields and the object's structure with all the values intact (that is, you now have a structure, not an object).

The following implementation of loadobj first uses isa to determine whether the input argument is a portfolio object or a structure. If the input is an object, it is simply returned since no modifications are necessary. If the input argument has been converted to a structure by MATLAB, then the new account_number field is added to the structure and is used to create an updated portfolio object.

```
function b = loadobj(a)
% loadobj for portfolio class
if isa(a,'portfolio')
   b = a;
else % a is an old version
   a.account_number = getAccountNumber(a);
   b = class(a,'portfolio');
end
```

Changing the Portfolio Constructor

The portfolio structure array needs an additional field to accommodate the account number. To create this field, add the line

```
p.account_number = '';
```

to @portfolio/portfolio.m in both the zero argument and variable argument sections.

The getAccountNumber Function

In this example, getAccountNumber is a MATLAB function that returns an account number composed of the first three letters of the client name prepended to a series of digits. To illustrate implementation techniques, getAccountNumber is not a portfolio method so it cannot access the portfolio object data directly. Therefore, it is necessary to define a portfolio subsref method that enables access to the name field in a portfolio object's structure.

For this example, getAccountNumber simply generates a random number, which is formatted and concatenated with elements 1 to 3 from the portfolio name field.

```
function n = getAccountNumber(p)
% provides a account number for object p
n = [upper(p.name(1:3)) strcat(num2str(round(rand(1,7)*10))')'];
```

Note that the portfolio object is indexed by field name, and then by numerical subscript to extract the first three letters. The subsref method must be written to support this form of subscripted reference.

The Portfolio subsref Method

When MATLAB encounters a subscripted reference, such as that made in the getAccountNumber function,

```
p.name(1:3)
```

MATLAB calls the portfolio subsref method to interpret the reference. If you do not define a subsref method, the above statement is undefined for portfolio objects (recall that here p is an object, not just a structure).

The portfolio subsref method must support field-name and numeric indexing for the getAccountNumber function to access the portfolio name field.

```
function b = subsref(p,index)
% SUBSREF Define field name indexing for portfolio objects
switch index(1).type
case '.'
   switch index(1).subs
   case 'name'
      if length(index)== 1
         b = p.name;
      else
         switch index(2).type
         case '()'
            b = p.name(index(2).subs{:});
         end
      end
   end
end
```

Note that the portfolio implementation of `subsref` is designed to provide access to specific elements of the `name` field; it is not a general implementation that provides access to all structure data, such as the stock class implementation of `subref`.

See the `subsref` help entry for more information about indexing and objects.

New Portfolio Class Behavior

With the additions and changes made in this example, the portfolio class now:

- Includes a field for an account number
- Adds the account number when a portfolio object is saved for the first time
- Automatically updates the older version of portfolio objects when you load them into the MATLAB workspace

Object Precedence

Object precedence is a means to resolve the question of which of possibly many versions of an operator or function to call in a given situation. Object precedence enables you to control the behavior of expressions containing different classes of objects. For example, consider the expression:

objectA + objectB

Ordinarily, MATLAB assumes that the objects have equal precedence and calls the method associated with the leftmost object. However, there are two exceptions:

- User-defined classes have precedence over MATLAB built-in classes.
- User-defined classes can specify their relative precedence with respect to other user-defined classes using the `inferiorto` and `superiorto` functions.

For example, in the section "Example: A Polynomial Class" on page 14-23 the polynom class defines a `plus` method that enables addition of polynom objects. Given the polynom object `p`:

```
p = polynom([1 0 –2 –5])
p =
   x^3–2*x–5
```

The expression,

```
1 + p
ans =
   x^3–2*x–4
```

calls the polynom `plus` method (which converts the double, 1, to a polynom object, and then adds it to `p`). The user-defined polynom class has precedence over the MATLAB double class.

Specifying Precedence of User-Defined Classes

You can specify the relative precedence of user-defined classes by calling the `inferiorto` or `superiorto` function in the class constructor.

The `inferiorto` function places a class below other classes in the precedence hierarchy. The calling syntax for the `inferiorto` function is

```
inferiorto('class1','class2',...)
```

You can specify multiple classes in the argument list, placing the class below many other classes in the hierarchy.

Similarly, the `superiorto` function places a class above other classes in the precedence hierarchy. The calling syntax for the `superiorto` function is

```
superiorto('class1','class2',...)
```

Location in Hierarchy

If *objectA* is above *objectB* in the precedence hierarchy, then the expression,

```
objectA + objectB
```

calls `@`*classA*`/plus.m`. Conversely, if *objectB* is above *objectA* in the precedence hierarchy, then MATLAB calls `@`*classB*`/plus.m`.

See "How MATLAB Determines Which Method to Call" on page 14-68 for related information.

How MATLAB Determines Which Method to Call

In MATLAB, functions exist in directories in the computer's file system. A directory may contain many functions (M-files). Function names are unique only within a single directory (e.g., more than one directory many contain a function called `pie3`). When you type a function name on the command line, MATLAB must search all the directories it is aware of to determine which function to call. This list of directories is called the *MATLAB path*.

When looking for a function, MATLAB searches the directories in the order they are listed in the path, and calls the first function whose name matches the name of the specified function.

If you write an M-file called `pie3.m` and put it in a directory that is searched before the `specgraph` directory that contains MATLAB's `pie3` function, then MATLAB uses your `pie3` function instead (note that this is not true for built-in functions like `plot`, which are always found first).

Object oriented-programming allows you to have many methods (MATLAB functions located in class directories) with the same name and enables MATLAB to determine which method to use based on the type or class of the variables passed to the function. For example, if `p` is a portfolio object, then,

```
pie3(p)
```

calls `@portfolio/pie3.m` because the argument is a portfolio object.

The Process of Selecting a Method

When you call a method for which there are multiple versions with the same name, MATLAB determines the method to call by:

- Looking at the classes of the objects in the argument list to determine which argument has the highest object precedence; the class of this object controls the method selection and is called the *dispatch type*.
- Applying the *function precedence order* to determine which of possibly several implementations of a method to call. This order is determined by the location and type of function.

Determining the Dispatch Type

MATLAB first determines which argument controls the method selection. The class type of this argument then determines the class in which MATLAB searches for the method. The controlling argument is either:

- The argument with the highest precedence, or
- The leftmost of arguments having equal precedence

User-defined objects take precedence over MATLAB's built-in classes such as `double` or `char`. You can set the relative precedence of user-defined objects with the `inferiorto` and `superiorto` functions, as described in "Object Precedence" on page 14-66.

MATLAB searches for functions by name. When you call a function, MATLAB knows the name, number of arguments, and the type of each argument. MATLAB uses the dispatch type to choose among multiple functions of the same name, but does not consider the number of arguments.

Function Precedence Order

The function precedence order determines the precedence of one function over another based on the type of function and its location on the MATLAB path. From the perspective of method selection, MATLAB contains two types of functions: those built into MATLAB, and those written as M-files. MATLAB treats these types differently when determining the function precedence order.

MATLAB selects the correct function for a given context by applying the following function precedence rules, in the order given:

1. **Overloaded Built-in Function**. If there is a method in the class directory of the dispatching argument that has the same name as a MATLAB function, then this method is called instead of the MATLAB function.

2. **Nonoverloaded MATLAB Functions**. If there is no overloaded method, then the MATLAB function is called. Note that MATLAB built-in functions take precedence over both subfunctions and private functions, but MATLAB M-file functions do not.

3. **Subfunctions**. Subfunctions take precedence over other functions on the path with the same name. Note that subfunctions with the same name as MATLAB built-in functions (or overloaded built-in functions) can never be

called, because of rules 1 and 2. However, subfunctions always take precedence over M-file and overloaded M-file functions.

4. **Private Functions**. A function in a private directory (i.e., a directory named `private`) that is below the directory containing the calling function takes precedence over other functions on the path having the same name. Note that private functions with the same name as MATLAB built-in functions (or overloaded built-in functions) can never be called, because of rules 1 and 2. However, private functions always take precedence over M-file and overloaded M-file functions.

5. **Class Constructor Functions**. Constructor functions (functions having names that are the same as the @ directory, for example `@polynom/polynom.m`) take precedence over other MATLAB functions. Therefore, if you create an M-file called `polynom.m` and put it on your path before the constructor `@polynom/polynom.m` version, MATLAB will always call the constructor version.

6. **Overloaded Methods**. MATLAB calls an overloaded method if it is not masked by a subfunction or private function.

7. **Current Directory**. A function in the current working directory is selected before one elsewhere on the path.

8. **Elsewhere On Path**. Finally, a function anywhere else on the path is selected.

Selection from Multiple Directories

There may be a number of directories on the path that contain methods with the same name. MATLAB stops searching when it finds the first implementation of the method on the path, regardless of the implementation type (MEX-file, P-code, M-file).

Selection from Multiple Implementation Types

There are four file precedence types. MATLAB uses file precedence to select between identically named functions in the same directory. The order of precedence for file types is:

1. MEX-files

2. MDL-file (Simulink model)

3. P-code
4. M-file

For example, if MATLAB finds a P-code and an M-file version of a method in a class directory, then the P-code version is used. It is, therefore, important to regenerate the P-code version whenever you edit the M-file.

Querying Which Method MATLAB Will Call

You can determine which method MATLAB will call using the `which` command. For example,

```
which pie3
your_matlab_path/toolbox/matlab/specgraph/pie3.m
```

However, if `p` is a portfolio object,

```
which pie3(p)
dir_on_your_path/@portfolio/pie3.m % portfolio method
```

The `which` command determines which version of `pie3` MATLAB will call if you passed a portfolio object as the input argument. To see a list of all versions of a particular function that are on your MATLAB path, use the `–all` option. See the `which` reference page for more information on this command.

File I/O

MATLAB's file input and output (I/O) functions read and write arbitrary binary and formatted text files. They allow you to read data collected in other formats and to write out data for other programs or devices.

The low-level file I/O functions live in a directory called `iofun` in the MATLAB Toolbox.

Category	Function	Description
File Opening and Closing	`fopen`	Open file.
	`fclose`	Close file.
Binary I/O	`fread`	Read binary data from file.
	`fwrite`	Write binary data to file.
Formatted I/O	`fscanf`	Read formatted data from file.
	`fprintf`	Write formatted data to file.
	`fgetl`	Read line from file, discard newline character.
	`fgets`	Read line from file, keep newline character.
String Conversion	`sprintf`	Write formatted data to string.
	`sscanf`	Read string under format control.
File Positioning	`ferror`	Inquire file I/O error status.
	`feof`	Test for end-of-file.
	`fseek`	Set file position indicator.
	`ftell`	Get file position indicator.
	`frewind`	Rewind file.
Temporary Files	`tempdir`	Get temporary directory name.
	`tempname`	Get temporary filename.

Opening and Closing Files

Files are opened with the fopen command and closed with the fclose command.

Function	Purpose
fopen	Open file.
fclose	Close file.

Before reading or writing a text or binary file you must open it with the fopen command.

```
fid = fopen(’filename’,’permission’)
```

The permission string specifies the kind of access you require. Possible permission strings include:

- r for reading only
- w for writing only
- a for appending only
- r+ for both reading and writing

Note Systems such as Microsoft Windows that distinguish between text and binary files may require additional characters in the permission string, such as ’rb’ to open a binary file for reading.

fopen returns a *file identifier* (fid), the value you use to access the open file.

This fopen statement opens the data file named penny.dat for reading:

```
fid = fopen(’penny.dat’,’r’)
```

MATLAB File I/O Functions and ANSI Standard C

Many of the MATLAB file I/O functions are based on the I/O functions of the ANSI Standard C Library. If you know C, therefore, you are probably familiar with these routines. However, not all MATLAB file I/O commands work the same way as their C language counterparts. Check MATLAB command syntax and functionality using the online help facility or the online MATLAB Function Reference.

Using the File Identifier (fid)

The file identifier that `fopen` returns (if successful) is a nonnegative integer. This integer acts as a handle to the file and is an argument to MATLAB file I/O functions.

There are times when `fopen` might fail. For example, `fopen` fails if you try to open a file that does not exist. If `fopen` fails, it does the following:

- It assigns `-1` to the file identifier.
- It assigns an error message to an optional second output argument. Note that the error messages are system dependent and are not provided for all errors on all systems. The function `ferror` may also provide information about errors.

It's good practice to test the file identifier each time you open a file. For example, this code loops until the user enters the name of a readable file:

```
fid=0;
while fid < 1
   filename=input('Open file: ', 's');
   [fid,message] = fopen(filename, 'r');
   if fid == –1
     disp(message)
   end
end
```

Now assume that `nofile.mat` does not exist but that `goodfile.mat` does exist. On one system, the results are:

```
Open file: nofile.mat
Cannot open file. Existence? Permissions? Memory? . . .

Open file: goodfile.mat
```

Closing a File

When you finish reading or writing, use `fclose` to close the file. For example, this line closes the file associated with file identifier `fid`:

```
status = fclose(fid);
```

This line closes all open files:

```
status = fclose('all');
```

Both forms return `0` if the file or files were successfully closed or `-1` if the attempt was unsuccessful.

MATLAB automatically closes all open files when you exit from MATLAB. It is still good practice, however, to close a file explicitly with `fclose` when you are finished using it. Not doing so can unnecessarily drain system resources.

Note Closing a file does not clear the file identifier variable `fid`. However, subsequent attempts to access a file through this file identifier variable will not work.

Temporary Files and Directories

The tempdir and tempname commands assist in locating temporary data on your system.

Function	Purpose
tempdir	Get temporary directory name.
tempname	Get temporary filename.

You can create temporary files. Some systems delete temporary files every time you reboot the system. On other systems, designating a file as temporary may mean only that the file is not backed up.

A function named tempdir returns the name of the directory or folder that has been designated to hold temporary files on your system. For example, issuing tempdir on a UNIX system returns the /tmp directory.

MATLAB also provides a tempname function that returns a filename in the temporary directory. The returned filename is a suitable destination for temporary data. For example, if you need to store some data in a temporary file, then you might issue the following command first:

```
fid = fopen(tempname, 'w');
```

Note The filename that tempname generates is not guaranteed to be unique; however, it is likely to be so.

Binary Files

This section explains how to read from or write to binary files.

Function	Purpose
`fread`	Read binary data from file.
`fwrite`	Write binary data to file.

Reading Binary Files

The `fread` function reads all or part of a binary file (as specified by a file identifier) and stores it in a matrix. In its simplest form, it reads an entire file and interprets each byte of input as the next element of the matrix. For example, the following code reads the data from a file named `nickel.dat` into matrix `A`.

```
fid = fopen(’nickel.dat’,’r’);
A = fread(fid);
```

To echo the data to the screen after reading it, use `char` to display the contents of `A` as characters, transposing the data so it displays horizontally:

```
disp(char(A’))
```

The `char` function causes MATLAB to interpret the contents of `A` as characters instead of as numbers. Transposing `A` displays it in its more natural horizontal format.

Controlling the Number of Values Read

`fread` accepts an optional second argument that controls the number of values read (if unspecified, the default is the entire file). For example, this statement reads the first 100 data values of the file specified by `fid` into the column vector `A`:

```
A = fread(fid,100);
```

Replacing the number `100` with the matrix dimensions `[10 10]` reads the same 100 elements into a 10-by-10 array.

Controlling the Data Type of Each Value

An optional third argument to fread controls the data type of the input. The data type argument controls both the number of bits read for each value and the interpretation of those bits as character, integer, or floating-point values. MATLAB supports a wide range of precisions, which you can specify with MATLAB-specific strings or their C or Fortran equivalents.

Some common precisions include:

- ’char’ and ’uchar’ for signed and unsigned characters (usually 8 bits)
- ’short’ and ’long’ for short and long integers (usually 16 and 32 bits, respectively)
- ’float’ and ’double’ for single and double precision floating-point values (usually 32 and 64 bits, respectively)

Note The meaning of a given precision can vary across different hardware platforms. For example, a ’uchar’ is not always 8 bits. fread also provides a number of more specific precisions, such as ’int8’ and ’float32’. If in doubt, use these precisions, which are not platform dependent. Look up fread in online help for a complete list of precisions.

For example, if fid refers to an open file containing single-precision floating-point values, then the following command reads the next 10 floating-point values into a column vector A:

```
A = fread(fid,10,’float’);
```

Writing Binary Files

The fwrite function writes the elements of a matrix to a file in a specified numeric precision, returning the number of values written. For instance, these lines create a 100-byte binary file containing the 25 elements of the 5-by-5 magic square, each stored as 4-byte integers:

```
fwriteid = fopen(’magic5.bin’,’w’);
count = fwrite(fwriteid,magic(5),’int32’);
status = fclose(fwriteid);
```

In this case, `fwrite` sets the `count` variable to `25` unless an error occurs, in which case the value is less.

Controlling Position in a File

Once you open a file with `fopen`, MATLAB maintains a file position indicator that specifies a particular location within a file. MATLAB uses the file position indicator to determine where in the file the next read or write operation will begin. The table below summarizes MATLAB functions for controlling the file position indicator:

Function	Purpose
`feof`	Determine if file position indicator is at end-of-file.
`fseek`	Set file position indicator.
`ftell`	Get file position indicator.
`frewind`	Reset file position indicator to beginning of file.

The `fseek` and `ftell` functions let you set and query the position in the file at which the next input or output operation takes place:

- The `fseek` function repositions the file position indicator, letting you skip over data or back up to an earlier part of the file.
- The `ftell` function gives the offset in bytes of the file position indicator for a specified file.

The syntax for `fseek` is

```
status = fseek(fid,offset,origin)
```

`fid` is the file identifier for the file. `offset` is a positive or negative offset value, specified in bytes. `origin` is an origin from which to calculate the move, specified as a string.

`’cof’`	Current position in file
`’bof’`	Beginning of file
`’eof’`	End of file

Understanding File Position

To see how fseek and ftell work, consider this short M-file:

```
A = 1:5;
fid = fopen('five.bin','w');
fwrite(fid, A,'short');
status = fclose(fid);
```

This code writes out the numbers 1 through 5 to a binary file named five.bin. The call to fwrite specifies that each numerical element be stored as a short. Consequently, each number uses two storage bytes.

Now reopen five.bin for reading:

```
fid = fopen('five.bin','r');
```

This call to fseek moves the file position indicator forward six bytes from the beginning of the file:

```
status = fseek(fid,6,'bof');
```

File Position	bof	1	2	3	4	5	6	7	8	9	10	eof
File Contents		0	1	0	2	0	3	0	4	0	5	
File Position Indicator								↑				

This call to fread reads whatever is at file positions 7 and 8 and stores it in variable four:

```
four = fread(fid,1,'short');
```

The act of reading advances the file position indicator. To determine the current file position indicator, call ftell:

```
position = ftell(fid)

position =

     8
```

File Position	bof	1	2	3	4	5	6	7	8	9	10	eof
File Contents		0	1	0	2	0	3	0	4	0	5	
File Position Indicator										↑		

This call to `fseek` moves the file position indicator back four bytes:

```
status = fseek(fid,–4,'cof');
```

File Position	bof	1	2	3	4	5	6	7	8	9	10	eof
File Contents		0	1	0	2	0	3	0	4	0	5	
File Position Indicator						↑						

Calling `fread` again reads in the next value (3).

```
three = fread(fid,1,'short');
```

Formatted Files

This section explains how to read from and write to formatted text files.

Function	Purpose
`fgetl`	Read line from file, discard newline character.
`fgets`	Read line from file, keep newline character.
`fscanf`	Read formatted data from file.
`fprintf`	Write formatted data to file.

Reading Strings Line-By-Line from Text Files

MATLAB provides two functions, `fgetl` and `fgets`, that read lines from formatted text files and store them in string vectors. The two functions are almost identical; the only difference is that `fgets` copies the newline character to the string vector but `fgetl` does not.

The following M-file function demonstrates a possible use of `fgetl`. This function uses `fgetl` to read an entire file one line at a time. For each line, the function determines whether an input literal string (`literal`) appears in the line.

If it does, the function prints the entire line preceded by the number of times the literal string appears on the line.

```
function y = litcount(filename, literal)
% Search for number of string matches per line.

fid = fopen(filename, ’rt’);
y = 0;
while feof(fid) == 0
   line = fgetl(fid);
   matches = findstr(line, literal);
   num = length(matches);
   if num > 0
      y = y + num;
      fprintf(1,’%d:%s\n’,num,line);
   end
end
fclose(fid);
```

Given the following input datafile called `badpoem`:

```
Oranges and lemons,
Pineapples and tea.
Orangutans and monkeys,
Dragonflys or fleas.
```

Calling the `litcount` function with the string `’an’` produces the output:

```
litcount(’badpoem’,’an’)
2: Oranges and lemons,
1: Pineapples and tea.
3: Orangutans and monkeys,
```

Reading Formatted Text

The `fscanf` function is like the `fscanf` function in standard C. Both functions operate in a similar manner, reading a line of data from a file and assigning it to one or more variables. Both functions use the same set of conversion specifiers to control the interpretation of the input data.

The conversion specifiers for `fscanf` begin with a `%` character; common conversion specifiers include:

- %s to match a string
- %d to match an integer in base 10 format
- %g to match a double-precision floating-point value

Despite all the similarities between the MATLAB and C versions of fscanf, there are some significant differences. For example, consider a file named moon.dat for which the contents are as follows:

```
3.654234533
2.71343142314
5.34134135678
```

The following code reads all three elements of this file into a matrix named MyData:

```
fid = fopen('moon.dat','r');
MyData = fscanf(fid,'%g');
status = fclose(fid);
```

Notice that this code does not use any loops. Instead, the fscanf function continues to read in text as long as the input format is compatible with the format specifier.

An optional size argument controls the number of matrix elements read. For example, if fid refers to an open file containing strings of integers, then this line reads 100 integer values into the column vector A:

```
A = fscanf(fid,'%5d',100);
```

This line reads 100 integer values into the 10-by-10 matrix A.

```
A = fscanf(fid,'%5d',[10 10]);
```

A related function, sscanf, takes its input from a string instead of a file. For example, this line returns a column vector containing 2 and its square root.

```
root2 = num2str([2, sqrt(2)]);
rootvalues = sscanf(root2,'%f');
```

Writing Text Files

The fprintf function converts data to character strings and outputs them to the screen or a file. A format control string containing conversion specifiers and any optional text specify the output format. The conversion specifiers control the output of array elements; `fprintf` copies text directly.

Common conversion specifiers include:

- `%e` for exponential notation
- `%f` for fixed point notation
- `%g` to automatically select the shorter of `%e` and `%f`

Optional fields in the format specifier control the minimum field width and precision. For example, this code creates a text file containing a short table of the exponential function:

```
x = 0:0.1:1;
y = [x; exp(x)];
```

The code below writes `x` and `y` into a newly created file named `exptable.txt`:

```
fid = fopen('exptable.txt','w');
fprintf(fid,'Exponential Function\n\n');
fprintf(fid,'%6.2f  %12.8f\n',y);
status = fclose(fid);
```

The first call to `fprintf` outputs a title, followed by two carriage returns. The second call to `fprintf` outputs the table of numbers. The format control string specifies the format for each line of the table:

- A fixed-point value of six characters with two decimal places
- Two spaces
- A fixed-point value of twelve characters with eight decimal places

`fprintf` converts the elements of array `y` in column order. The function uses the format string repeatedly until it converts all the array elements.

Now use `fscanf` to read the exponential data file:

```
fid = fopen('exptable.txt','r');
title = fgetl(fid);
[table,count] = fscanf(fid,'%f %f',[2 11]);
table = table';
status = fclose(fid);
```

The second line reads the file title. The third line reads the table of values, two floating-point values on each line until it reaches end of file. count returns the number of values matched.

A function related to fprintf, sprintf, outputs its results to a string instead of a file or the screen. For example:

```
root2 = sprintf('The square root of %f is %10.8e.\n',2,sqrt(2));
```

Index