α from $B \rightarrow \rho\pi$ Decays

A Working Example of Time Dependent Three Body Analysis

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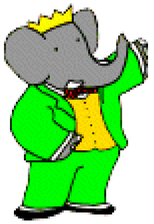
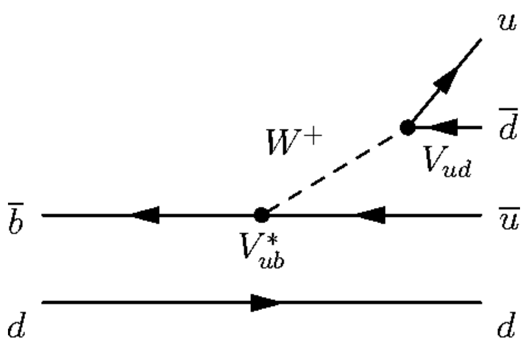
Charmless Workshop, Feb 1st, 2006

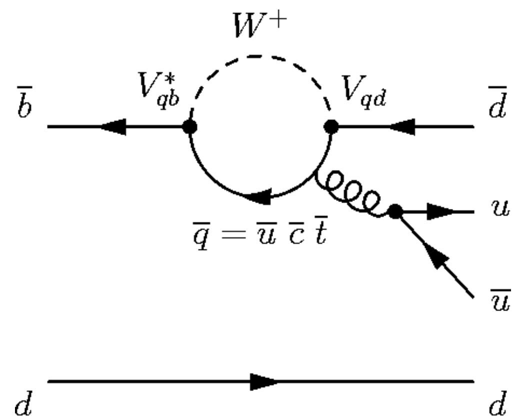


Mixing induced CPV in Charmless B Decay



$\propto V_{ub}V_{ud}^{*}$

$\propto \lambda^{3}$



$\propto V_{tb}V_{td}^{*}$

$\propto \lambda^{3}$

- *Decay-amplitude weak-phase structure for* $b \to u\bar{u}d$ *:*

$$A \cong V_{ud}V_{ub}^{*}\left(T^{u}+P^{u}-P^{c}\right)+V_{td}V_{tb}^{*}\left(P^{t}-P^{c}\right)=V_{ud}V_{ub}^{*}T+V_{td}V_{tb}^{*}P=R_{u}e^{+i\gamma}T+R_{t}e^{-i\beta}P$$

$$\lambda_{CP}=\frac{q}{p}\frac{\bar{A}}{A}\cong\frac{R_{u}e^{+i\alpha}\;T+R_{t}P}{R_{u}e^{-i\alpha}\;T+R_{t}P}=e^{2i\alpha_{\mathrm{eff}}}$$

- *Time dependent asymmetry probes* α_{eff}*:*

$$a(t)=\frac{\Gamma\left(\bar{B}^{0}_{phys}(t)\to f_{CP}\right)-\Gamma\left(B^{0}_{phys}(t)\to f_{CP}\right)}{\Gamma\left(\bar{B}^{0}_{phys}(t)\to f_{CP}\right)+\Gamma\left(B^{0}_{phys}(t)\to f_{CP}\right)}$$

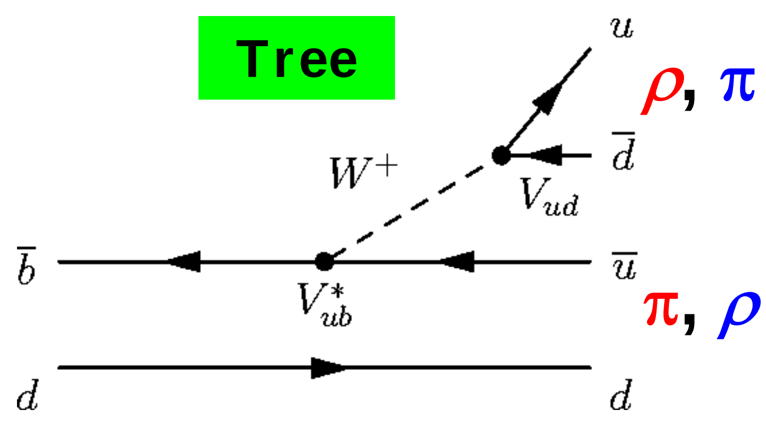
$$=\sqrt{1-C^{2}}\sin\left(2\alpha_{\mathrm{eff}}\right)\sin\left(\Delta m\Delta t\right)+C\cos\left(\Delta m\Delta t\right)$$

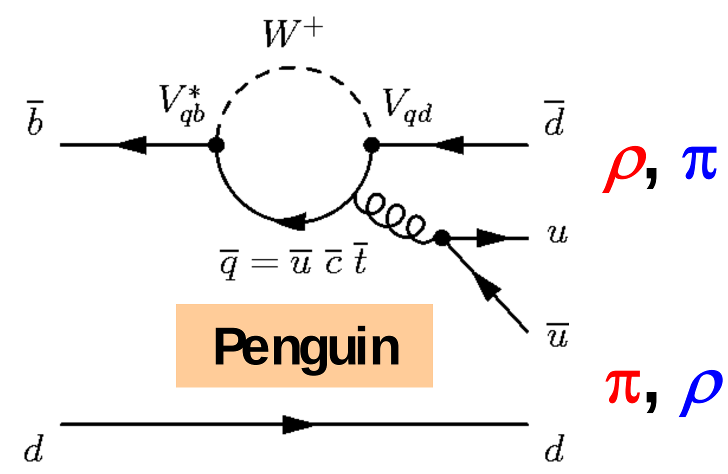
$B^0 \to \rho^{\pm}\pi^{\mp}$ Decay Amplitudes

Transition Amplitudes:

$$A\left(B^0 \to \rho^+\pi^-\right) \equiv A^{+-} = T^{+-}e^{-i\alpha} + P^{+-}$$
$$A\left(B^0 \to \rho^-\pi^+\right) \equiv A^{-+} = T^{-+}e^{-i\alpha} + P^{-+}$$

$$A\left(\overline{B}^0 \to \rho^+\pi^-\right) \equiv \overline{A}^{+-} = T^{-+}e^{+i\alpha} + P^{-+}$$
$$A\left(\overline{B}^0 \to \rho^-\pi^+\right) \equiv \overline{A}^{-+} = T^{+-}e^{+i\alpha} + P^{+-}$$





Nine unknowns:

$$T^{+-}, T^{-+}, P^{+-}, P^{-+}, \alpha$$

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* *Taking into account $\rho^0\pi^0$ adds two more unknowns, assuming SU(2)*

Quasi-two-body Analysis

- Quasi-two-body approximation, ignore interference effect
- 6 observables through a time-dependent fit:

R. Aleksan et al, 1990

$$f(\Delta t, Q_\rho, Q_{tag}) = (1 + Q_\rho A_{CP}) \frac{e^{-|\Delta t|/\tau}}{4\tau}$$

$$\left[1 + Q_{tag}\left(\left(S + Q_\rho \Delta S\right)\sin\left(\Delta m_d \Delta t\right) - \left(C + Q_\rho \Delta C\right)\cos\left(\Delta m_d \Delta t\right)\right)\right]$$

A_{CP}	*Direct CPV*
C	*Direct CPV*
ΔC	*Dilution*
S	*Mixing-induced CPV*
ΔS	*Strong phase difference*

Penguin free scenario

$$A_{CP} = 0 \qquad C = 0$$

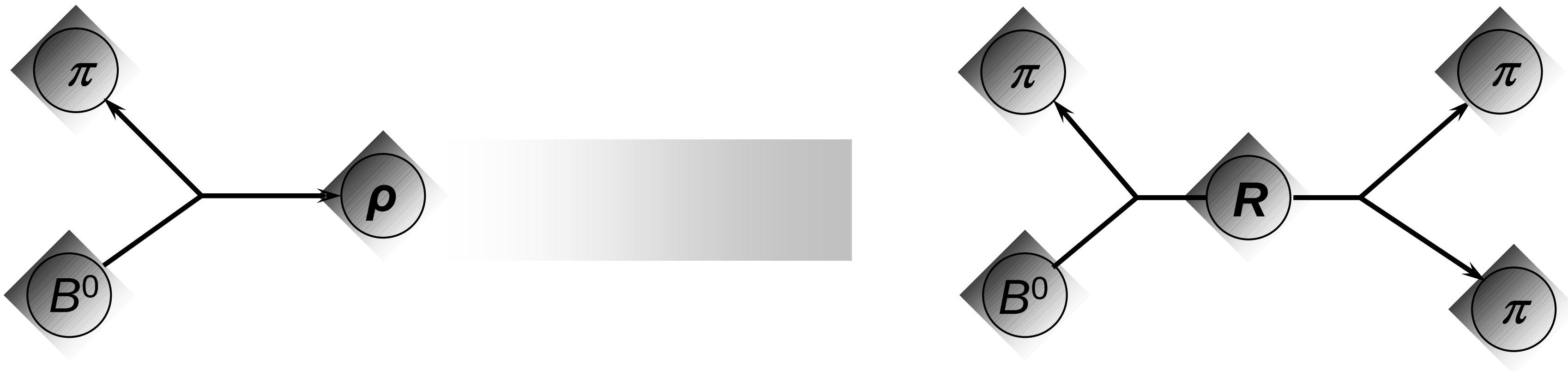
$$S_{\rho\pi} = \frac{2r_{T^{+-}}}{1 + r_{T^{+-}}^2} \sin 2\alpha \cos\delta$$

$$\Delta S_{\rho\pi} = \frac{2r_{T^{+-}}}{1 + r_{T^{+-}}^2} \cos 2\alpha \sin\delta \qquad \Delta C_{\rho\pi} = \frac{1 - r_{T^{+-}}^2}{1 + r_{T^{+-}}^2}$$

$$\delta \equiv \arg\left(A^{-+} A^{+-*}\right), r_T \equiv \left|\frac{T^{+-}}{T^{-+}}\right|$$

Alternative Approach

- Difficult to extract α with the isospin analysis
- Sensitive to the branching fractions
- Need to solve high order algebraic equations



Snyder-Quinn Method

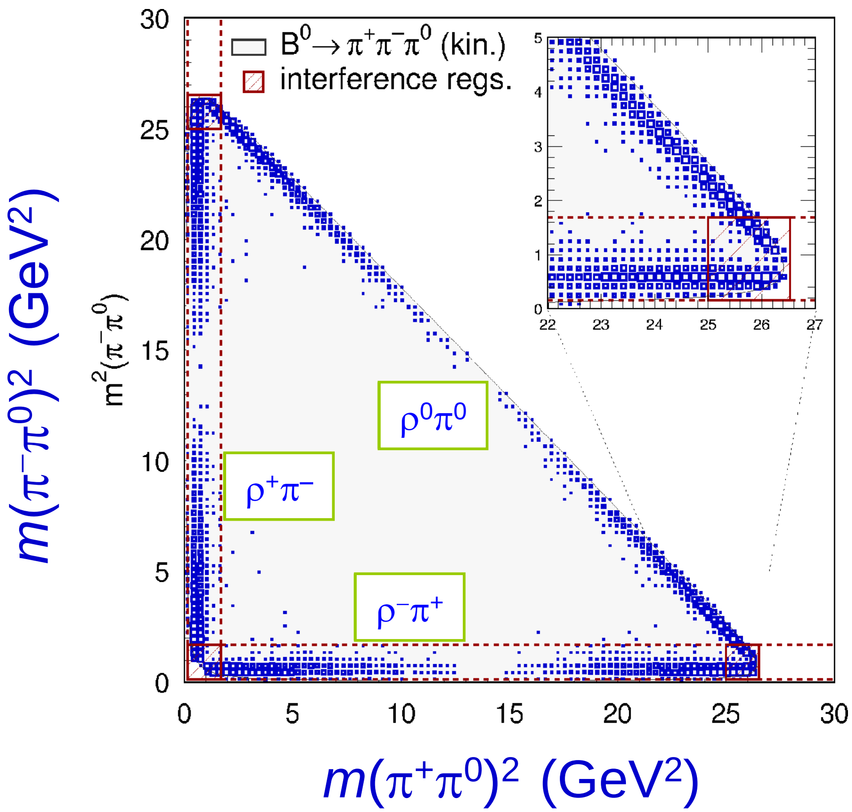
Quinn, Snyder
PRD 48, 2139, (1993)

Idea: Extract α and the strong phases using the interference between $B^0 \to \pi^+\pi^-\pi^0$ amplitudes

$\pi^+\pi^-\pi^0$ amplitude parameterization:

$$A_{3\pi} = f_+ A^{+-} + f_- A^{-+} + f_0 A^{00}$$
$$\overline{A}_{3\pi} = f_+ \overline{A}^{+-} + f_- \overline{A}^{-+} + f_0 \overline{A}^{00}$$

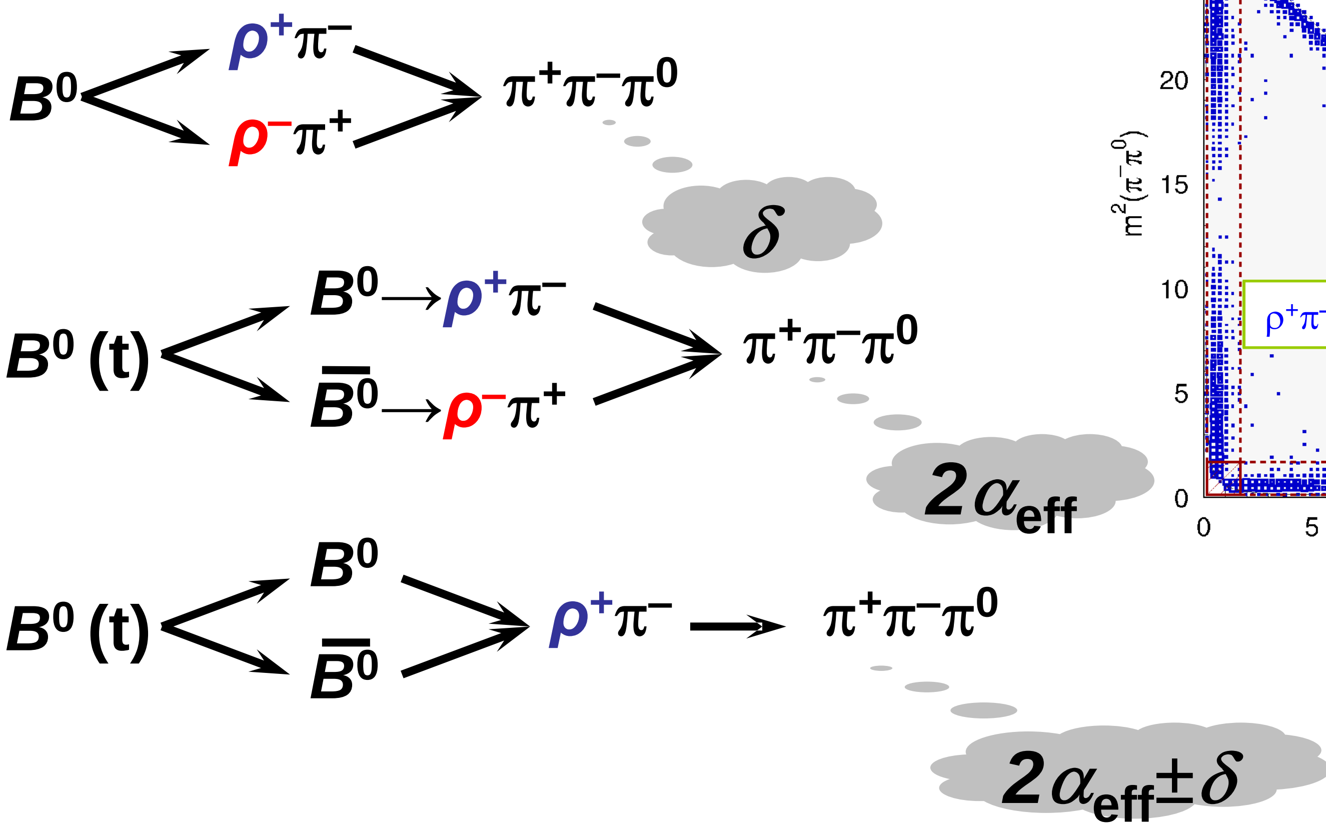
- The $f_{+,-,0}$ are relativistic Breit-Wigner form factors

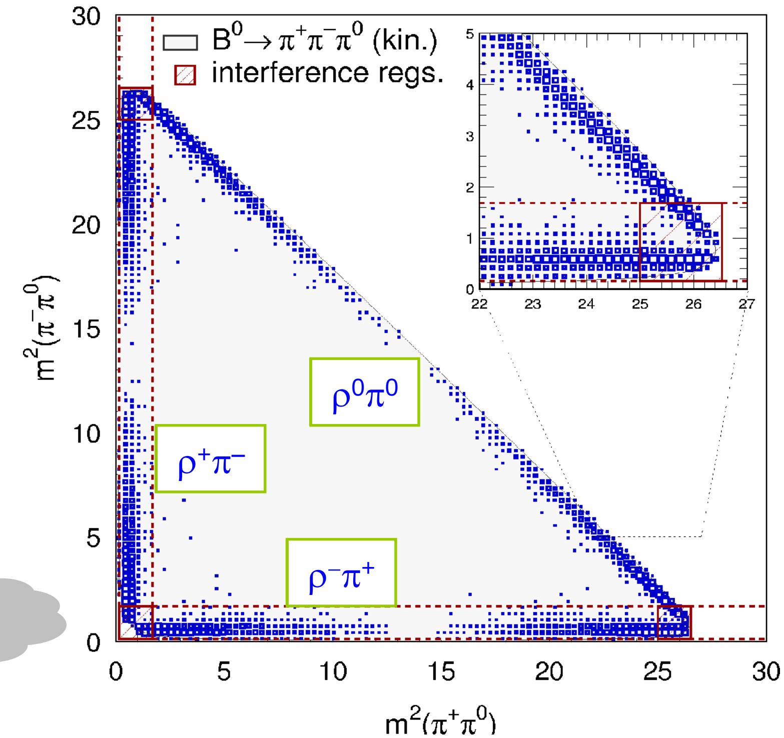


$$f(\Delta t, Q_{tag}) \propto \left(|A_{3\pi}|^2 + |\overline{A}_{3\pi}|^2\right)\frac{e^{-|\Delta t|/\tau}}{4\tau}$$
$$\left(1 + 2Q_{tag}\frac{\mathrm{Im}\left[\overline{A}_{3\pi}A_{3\pi}^*\right]}{|A_{3\pi}|^2 + |\overline{A}_{3\pi}|^2}\sin(\Delta m_d \Delta t) - Q_{tag}\frac{|A_{3\pi}|^2 - |\overline{A}_{3\pi}|^2}{|A_{3\pi}|^2 + |\overline{A}_{3\pi}|^2}\cos(\Delta m_d \Delta t)\right)$$

$B^0 \to \pi^+\pi^-\pi^0$: *Snyder-Quinn Method*

Solving the problem with only neural B decay!





Conceptually, it's pretty simple, one measure 11 amplitudes and phases, then solve for 11 known including α

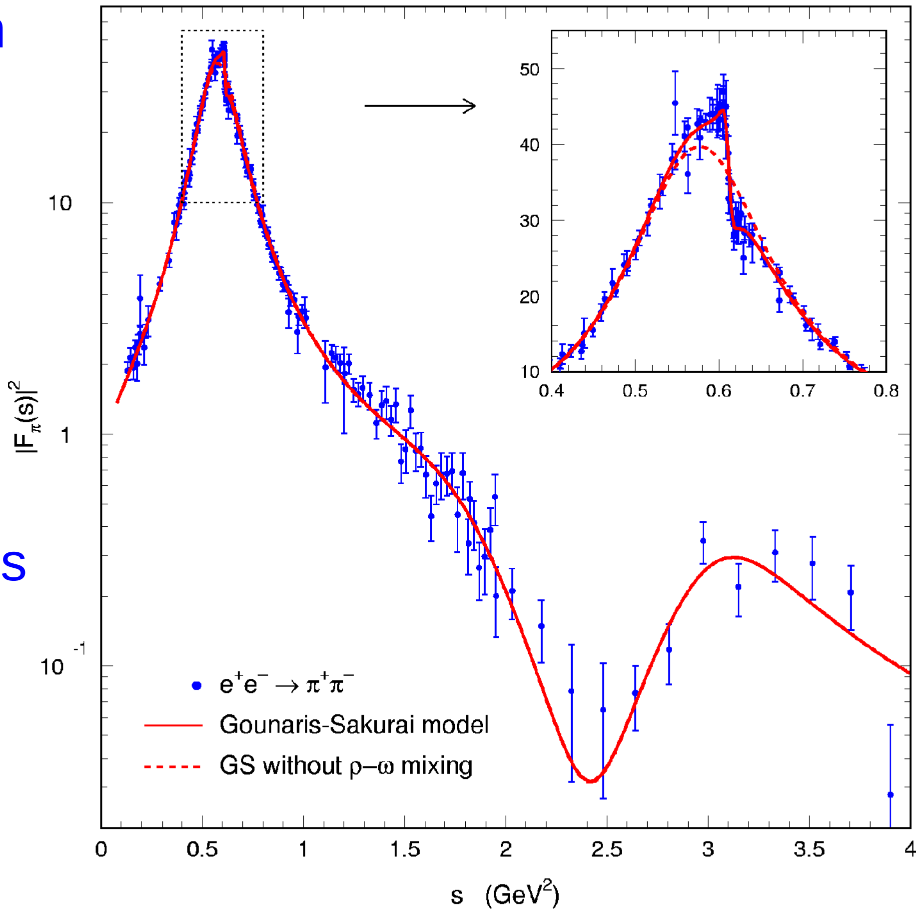
Main Model Assumptions

- The strong phase difference between the ρ(770) and its radial excitations are independent of the charge of the resonances

  Tested to very good accuracy in $\tau \to \pi^+\pi^0\nu$ *and* $e^+e^- \to \pi^+\pi^-$ *data*

- The ratio P/T is the same for the ground state and the radial excitations of the ρ

  True in naive factorization. Same assumptions go into isospin analysis



Assumptions theoretically motivated and necessary to limit the fit parameters

- *Hypothesis tested and validated in data (+ systematics study)*

Fitting Strategy

- Directly fitting for amplitudes and phases suffers from mirror solutions and local minima with limited statistics.
- Alternative fit approach:
  $\Rightarrow$ expand $A_{3\pi}$ as sum of Breit-Wigner bilinears
  $\Rightarrow$ fit the coefficients of Breit-Wigner bilinears

Quinn, Silva
PRD 62, 054002, (2000)

$$\left|A_{3\pi}\right|^2 \pm \left|\bar{A}_{3\pi}\right|^2 = \sum_{\kappa\in\{+,0,-\}} \left|f_\kappa\right|^2 U_\kappa^\pm + 2\sum_{\sigma<\kappa\in\{+,0,-\}} \left(\mathrm{Re}\left[f_\kappa f_\sigma^*\right] U_\kappa^\pm - \mathrm{Im}\left[f_\kappa f_\sigma^*\right] U_{\kappa\sigma}^{\pm,\mathrm{Im}}\right)$$

$$\mathrm{Im}\left(\bar{A}_{3\pi} A_{3\pi}^*\right) = \sum_{\kappa\in\{+,0,-\}} \left|f_\kappa\right|^2 I_\kappa + \sum_{\sigma<\kappa\in\{+,0,-\}} \left(\mathrm{Re}\left[f_\kappa f_\sigma^*\right] I_{\kappa\sigma}^{\mathrm{Im}} + \mathrm{Im}\left[f_\kappa f_\sigma^*\right] I_{\kappa\sigma}^{\mathrm{Re}}\right)$$

- Instead of 11 unknowns, one now gets 27 *interdependent* observables. we can safely fit 16 of them if $\rho^0\pi^0$ is small.
- Extract physics parameters using *U*s and *I*s fit results, such as quasi-two-body *CP* parameters, $\rho^0\pi^0$ branching fraction, α scan, …

Hypothesis: only the dominant ρ^+, ρ^- and ρ^0 resonances are taken into account

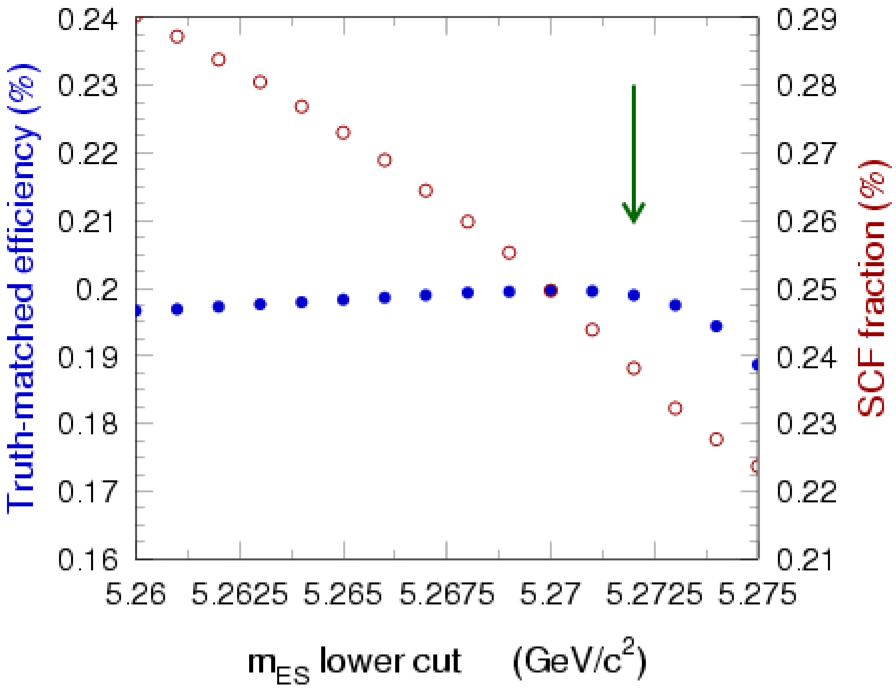
Analysis Overview

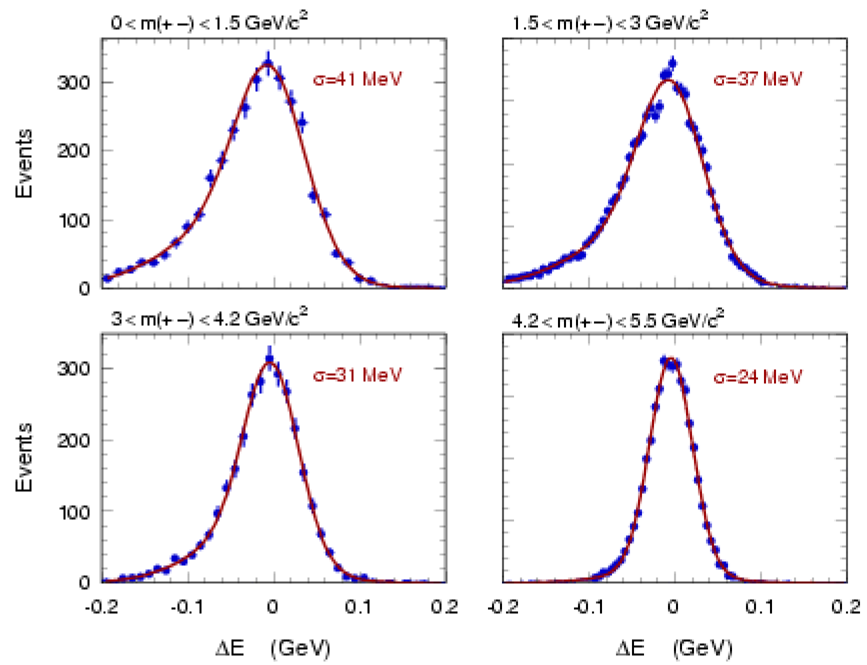
- Why is this analysis so difficult?

1. *Rare B decays with branching fraction $2x10^{-5}$ and tagging effectively reducing the efficiency by a factor of two*
2. *~80% of the sample are continuum events even after rather tight pre-selection criteria*
3. *Three body B decays with neutral particles in the final states, suffer large cross-feed from other B decays*
4. *B dalitz plots are difficult to model*
5. *Significant amount of signal events are mis-reconstructed and create dilution in CP measurement and bad "resolution" on the dalitz plot*
6. *Signal efficiency drops to zero in the corners of the dalitz plot which are the place where the interference is expected to happen*
7. *Many variables (both kinematic and event shape) are correlated to the dalitz plot which makes the maximum likelihood fit difficult*
8. *……*

Selection

- *Tight selection:* $5.272 < m_{ES} < 5.288$ GeV/c^2 , $-1 < \Delta E' < 1$
- *Remove uninteresting regions of the dalitz plot:*
  $m(\pi^+\pi^-) > 0.53$ GeV/c^2, $m(\pi\pi) < 1.5$ GeV/c^2





* *Signals are separated into truth matched signal and mis-reconstructed signal (SCF)*

Extended Maximum Likelihood Fit

For signal events:

Category (c)	Lepton	KPiorK	KorPI	Inclusive	UnTagged
Efficiency(%) ($\rho\pi$), ε_c	1.9	3.3	3.9	4.0	6.9
Mis-tag rate (%)	3.8	9.3	19.6	31.4	50.0
f_{SCF}(%) ($\rho\pi$)	14.1	19.9	23.8	22.4	24.2

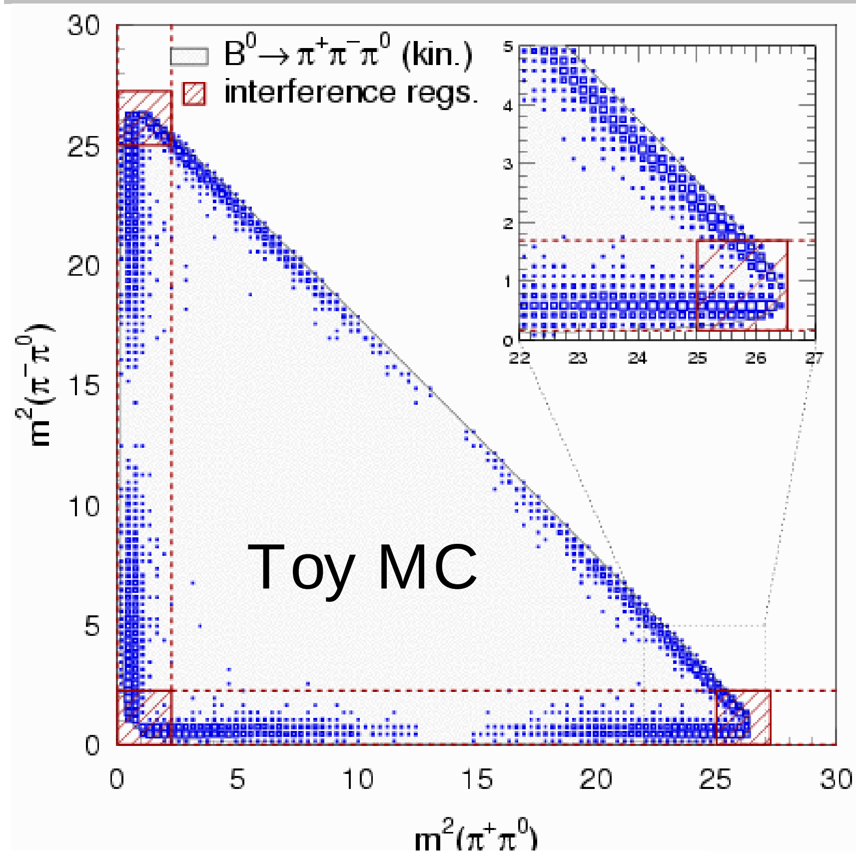
Each event is classified in one of five categories (c=5) and tested for the four hypotheses (j=4) in the likelihood:*

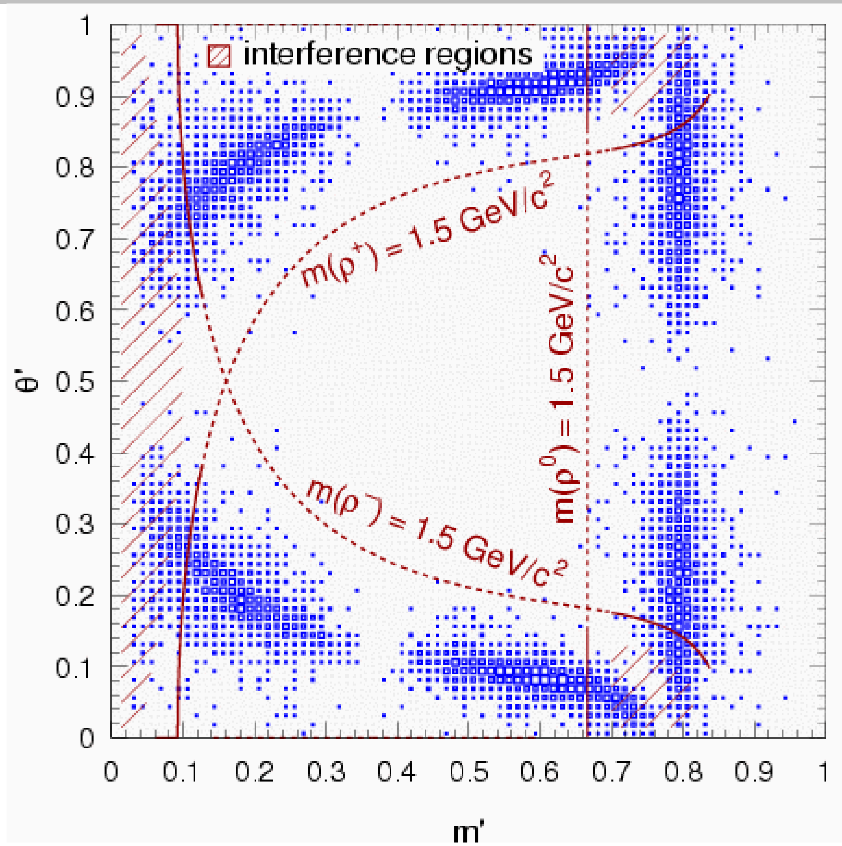
$$L = \prod_{c=1}^{5} e^{-N_c'} \prod_{i=1}^{N_c} \left(N_S \varepsilon_c \left(1 - f_{SCF,c}\right) \mathrm{P}_{S,c}^{TM} + N_S \varepsilon_c f_{SCF,c} \mathrm{P}_{S,c}^{SCF} + N_{q\bar{q}} \mathrm{P}_{q\bar{q},c} + \sum_{j=1}^{N_{class}^{B}} N_{B,j} \varepsilon_{B,c} \mathrm{P}_{B,c} \right) \left(\vec{x}_i\right)$$

where: $\vec{x}_i = \left(m_{ES}, \Delta E, xNN, Btag, \Delta t, DP\right)$

* *Truth matched signal, mis-reconstructed signal, continuum events, and events from other B decay*

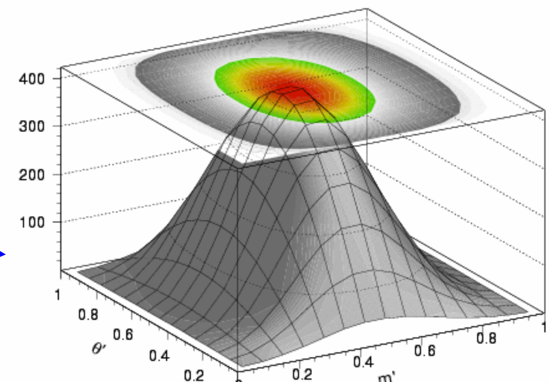
The Square Dalitz Plot





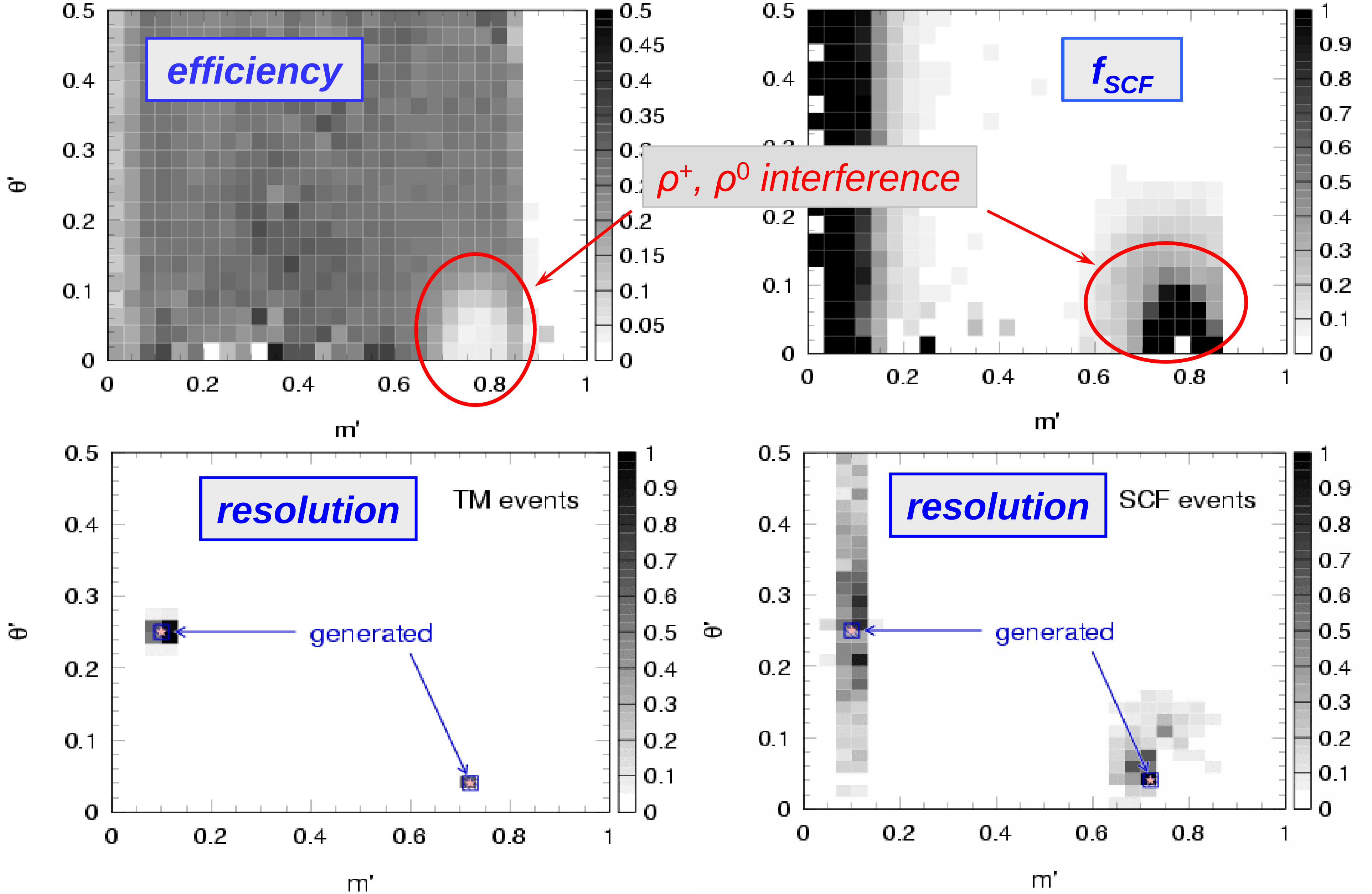
$$\theta' \equiv \frac{1}{\pi}\cos^{-1}\left(\cos\theta_{+-}\right)$$

$$m' \equiv \frac{1}{\pi}\cos^{-1}\left(\frac{m_{+-} - m_{+-}^{\min}}{m_{+-}^{\max} - m_{+-}^{\min}}\right)$$



det|J|

The Signal Dalitz Plot Treatment

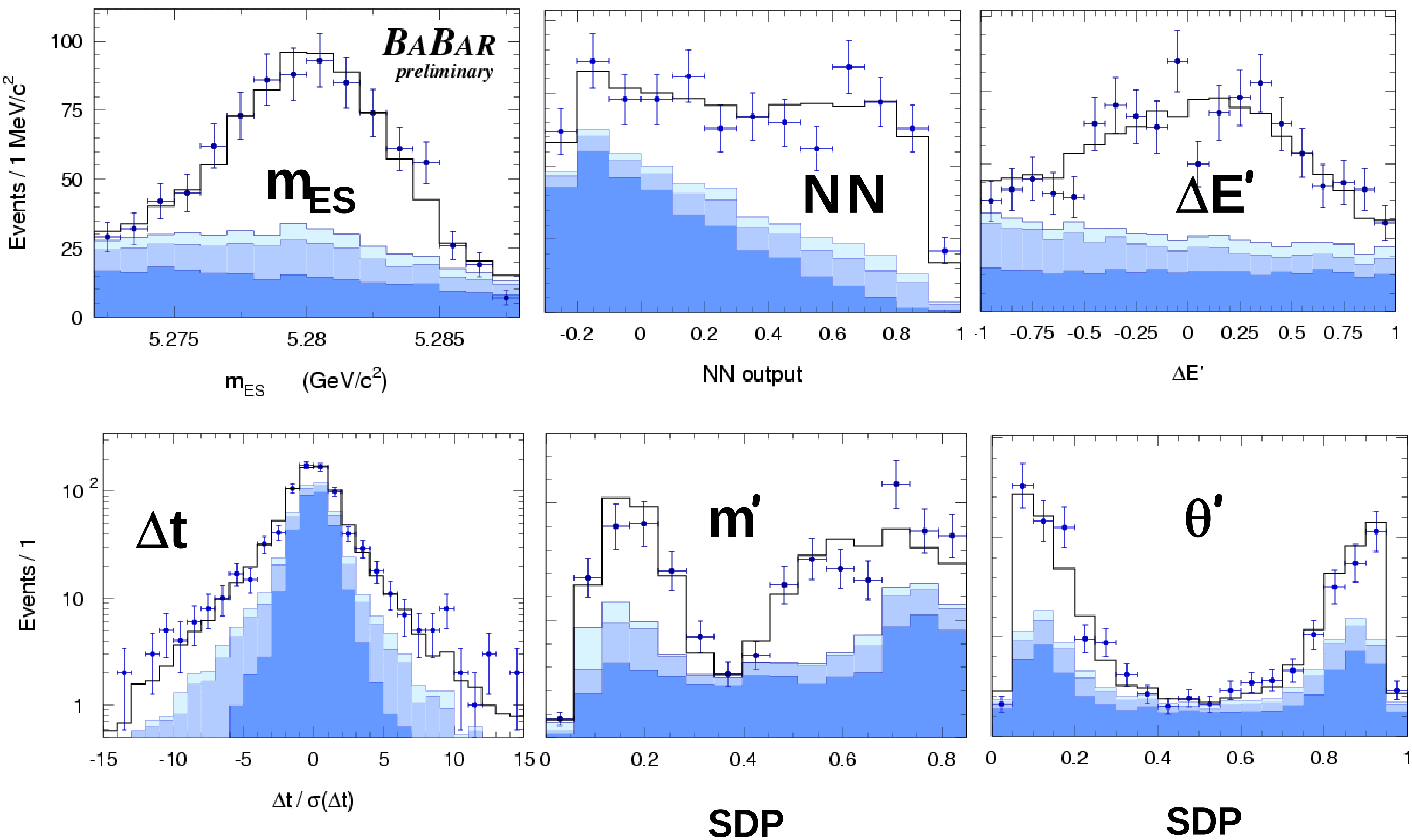


Systematic Uncertainties

- Δm, τ_B: within the uncertainties on the world average
- Signal PDF parameters: within statistical uncertainties
- Average SCF fractions: by 25% from $B \to D\rho$ control sample
- Tagging eff., dilutions, biases: within stat. Uncertainties
- Contribution from non-resonance: by adding MC in data
- *B* background tagging parameters, Δt resolution parameters
- Continuum DP extrapolation from m_{ES} sideband: from data
- Continuum DP parameterization: adding protection classes
- *B* background yields, *CP* parameters: allowed ranges
- Floating 16 UsIs instead of 27: from toy study
- ρ masses and widths: *doubled* uncertainties from e^+e^- and τ fits
- ρ(1450) amplitude and phase: 0, free in the fit
- ρ(1700) amplitude and phase: toy plus data fit
- Fit bias from fitting on fully simulated MC samples

Statistical errors dominant

Dalitz Plot Analysis: Fit Projection plots



Dalitz Plot Analysis: Direct Fit Results

BABAR Preliminary

U_-^+	*Coeff. of* $\lvert f(\rho^-)\rvert^2$	$1.19 \pm 0.12 \pm 0.03$
I_-	*Coeff. of* $\lvert f(\rho^-)\rvert^2 \sin(\Delta m \Delta t)$	$-0.19 \pm 0.11 \pm 0.02$
I_+	*Coeff. of* $\lvert f(\rho^+)\rvert^2 \sin(\Delta m \Delta t)$	$0.06 \pm 0.11 \pm 0.02$
U_-^-	*Coeff. of* $\lvert f(\rho^-)\rvert^2 \cos(\Delta m \Delta t)$	$0.22 \pm 0.16 \pm 0.05$
U_+^-	*Coeff. of* $\lvert f(\rho^+)\rvert^2 \cos(\Delta m \Delta t)$	$0.50 \pm 0.17 \pm 0.05$
$U_{+-}^{-,Im}$	*Coeff. of* $Im[f(\rho^+)\, f(\rho^-)^*] \cos(\Delta m \Delta t)$	$0.25 \pm 1.4 \pm 0.3$
$U_{+-}^{-,Re}$	*Coeff. of* $Re[f(\rho^+)\, f(\rho^-)^*] \cos(\Delta m \Delta t)$	$2.0 \pm 1.2 \pm 0.2$
$U_{+-}^{+,Im}$	*Coeff. of* $Im[f(\rho^+)\, f(\rho^-)^*]$	$0.16 \pm 0.70 \pm 0.14$
$U_{+-}^{+,Re}$	*Coeff. of* $Re[f(\rho^+)\, f(\rho^-)^*]$	$-0.26 \pm 0.65 \pm 0.17$
I_{+-}^{Im}	*Coeff. of* $Im[f(\rho^+)\, f(\rho^-)^*] \sin(\Delta m \Delta t)$	$-5.2 \pm 1.9 \pm 0.7$
I_{+-}^{Re}	*Coeff. of* $Re[f(\rho^+)\, f(\rho^-)^*] \sin(\Delta m \Delta t)$	$-0.3 \pm 2.0 \pm 0.5$
U_0^+	*Coeff. of* $\lvert f(\rho^0)\rvert^2$	$0.16 \pm 0.05 \pm 0.05$
$U_{+0}^{+,Im}$	*Coeff. of* $Im[f(\rho^+)\, f(\rho^0)^*]$	$0.25 \pm 0.35 \pm 0.18$
$U_{+0}^{+,Re}$	*Coeff. of* $Re[f(\rho^+)\, f(\rho^0)^*]$	$-0.34 \pm 0.39 \pm 0.15$
$U_{-0}^{+,Im}$	*Coeff. of* $Im[f(\rho^-)\, f(\rho^0)^*]$	$0.34 \pm 0.43 \pm 0.17$
$U_{-0}^{+,Re}$	*Coeff. of* $Re[f(\rho^-)\, f(\rho^0)^*]$	$-0.98 \pm 0.44 \pm 0.18$

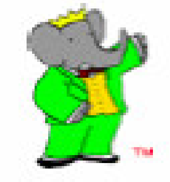
BABAR Preliminary

Q2B
$U_+^+ = 1$

interfering terms
Less sensitive

$\rho^0\pi^0$ terms
Significant

Extract physics parameters



- Tree amplitudes, penguin amplitudes and trigonometrical functions of α – such as its ambiguities – are 'hidden' in the *U*s and *I*s coefficients
- Extract physics parameters using *U*s and *I*s fit results

$$C = \frac{1}{2}\left(\frac{U_+^-}{U_+^+} + \frac{U_-^-}{U_-^+}\right) \quad \Delta C = \frac{1}{2}\left(\frac{U_+^-}{U_+^+} - \frac{U_-^-}{U_-^+}\right) \quad S = \frac{I_+}{U_+^+} + \frac{I_-}{U_-^+} \quad \Delta S = \frac{I_+}{U_+^+} - \frac{I_-}{U_-^+} \quad A_{CP} = \frac{U_+^+ - U_-^+}{U_+^+ + U_-^+}$$

		Q2B, LP2003	Dalitz Plot Analysis
$A_{\rho\pi}$	*Direct CPV*	–0.114 ± 0.062 ± 0.027	–0.088 ± 0.049 ± 0.013
C	*Direct CPV*	0.35 ± 0.14 ± 0.05	0.34 ± 0.11 ± 0.05
ΔC	*Dilution*	0.20 ± 0.14 ± 0.05	0.15 ± 0.11 ± 0.03
S	*Mixing-induced CPV*	–0.13 ± 0.18 ± 0.04	–0.10 ± 0.14 ± 0.04
ΔS	*Strong phase difference*	0.33 ± 0.18 ± 0.03	0.22 ± 0.15 ± 0.03

hep-ex/0408099

* *Using a Q2B approach and 144fb^{-1} data, BELLE measured*:

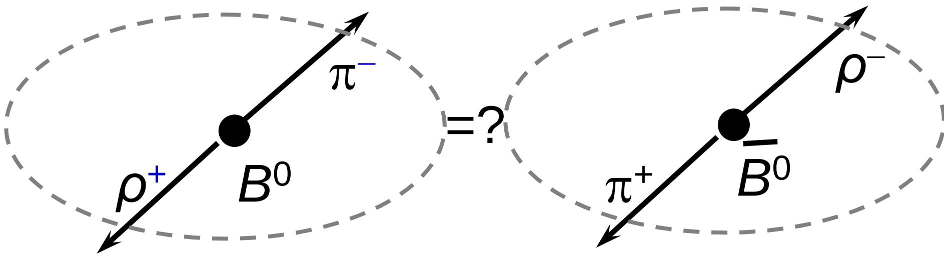
$$A_{CP} = -0.16 \pm 0.10, C = 0.25 \pm 0.17, \Delta C = 0.38 \pm 0.18, S = -0.28 \pm 0.24, \Delta S = 0.33 \pm 0.18$$

Probing Direct CP Violation

Define physically more intuitive quantities:

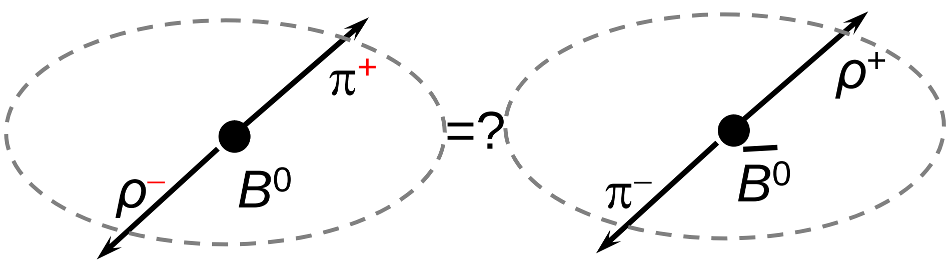
$$A_{\rho\pi}^{+-} \equiv \frac{\left|\bar{A}^{-+}\right|^2 - \left|A^{+-}\right|^2}{\left|\bar{A}^{-+}\right|^2 + \left|A^{+-}\right|^2} = \frac{A_{\rho\pi} + C + A_{\rho\pi}\Delta C}{1 + \Delta C + A_{\rho\pi} C}$$

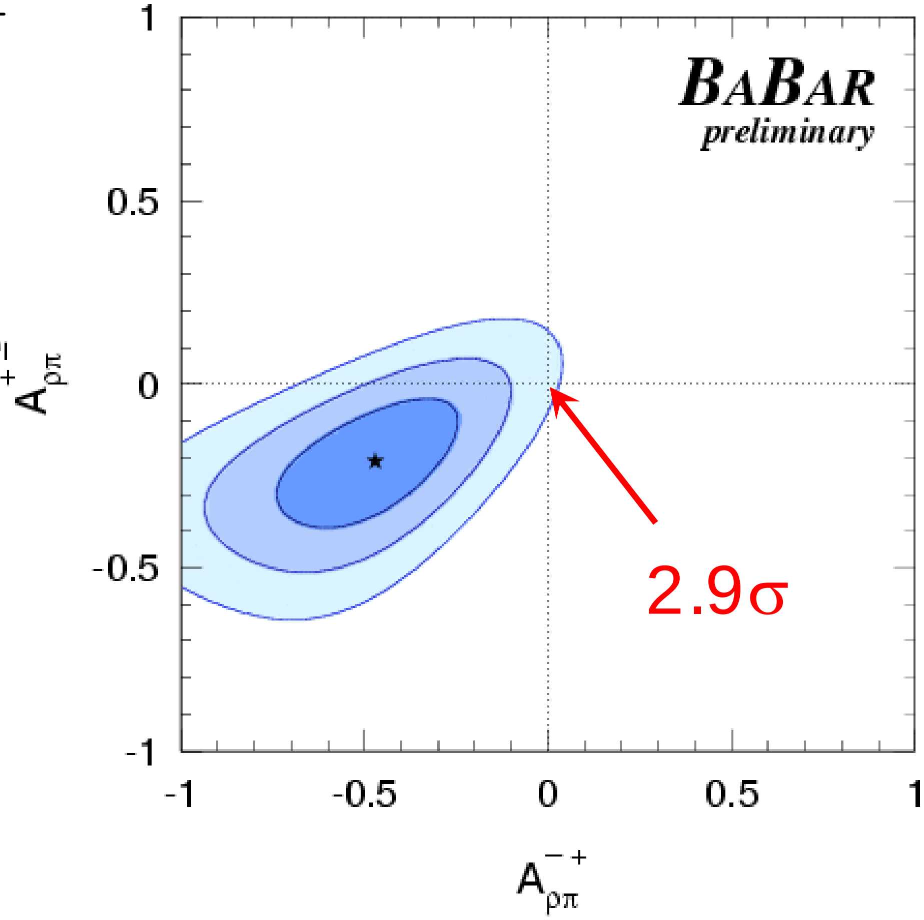
$$= -0.21 \pm 0.11 \pm 0.04$$



$$A_{\rho\pi}^{-+} \equiv \frac{\left|\bar{A}^{+-}\right|^2 - \left|A^{-+}\right|^2}{\left|\bar{A}^{+-}\right|^2 + \left|A^{-+}\right|^2} = \frac{A_{\rho\pi} - C - A_{\rho\pi}\Delta C}{1 - C - A_{\rho\pi}\Delta C}$$

$$= -0.47^{+0.14}_{-0.15} \pm 0.06$$





Probing Direct CP Violation

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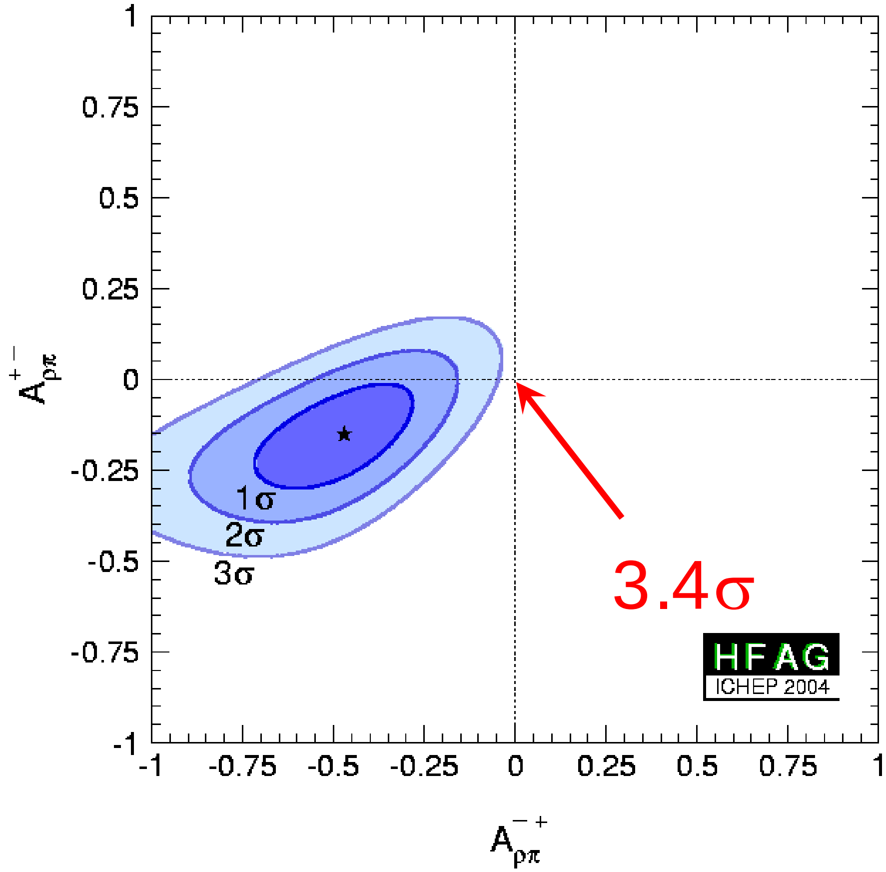
$$A_{\rho\pi}^{+-} \equiv \frac{\left|\bar{A}^{-+}\right|^2 - \left|A^{+-}\right|^2}{\left|\bar{A}^{-+}\right|^2 + \left|A^{+-}\right|^2} = \frac{A_{\rho\pi} + C + A_{\rho\pi}\Delta C}{1 + \Delta C + A_{\rho\pi} C}$$

$$= -0.15 \pm 0.09$$

$$A_{\rho\pi}^{-+} \equiv \frac{\left|\bar{A}^{+-}\right|^2 - \left|A^{-+}\right|^2}{\left|\bar{A}^{+-}\right|^2 + \left|A^{-+}\right|^2} = \frac{A_{\rho\pi} - C - A_{\rho\pi}\Delta C}{1 - C - A_{\rho\pi}\Delta C}$$

$$= -0.47^{+0.13}_{-0.14}$$



Large Direct CPV not expected…

A_{CP} = 0.01±0.10
C = 0.00±0.02

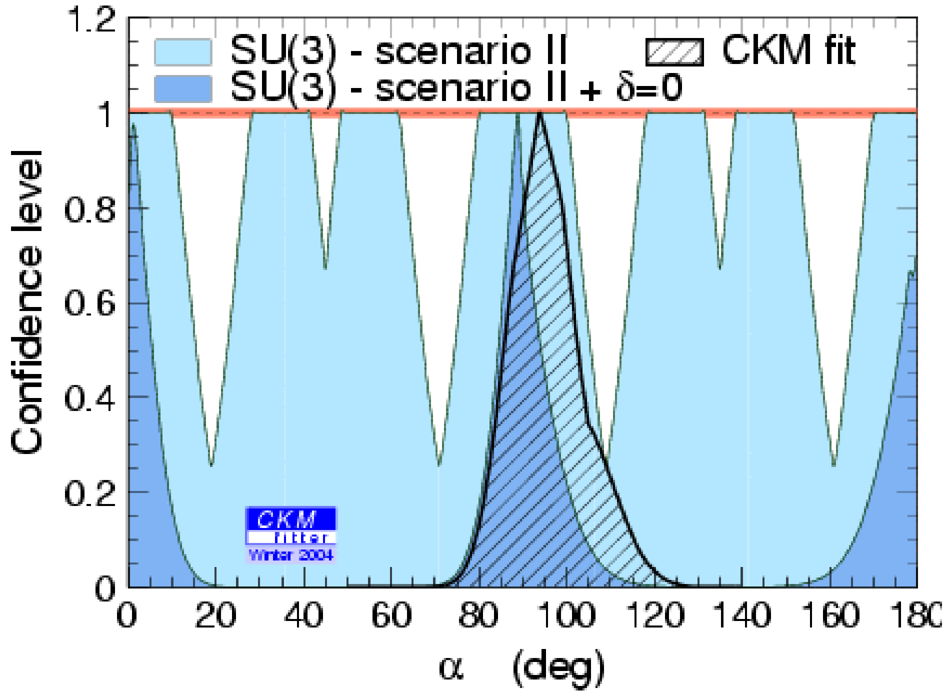
QCD FA, Beneke & Neubert
Nucl. Phys. B675(2003) 333

Road to α: the Strong Phase

- What is the strong phase between $B^0 \to \rho^- \pi^+$ and $B^0 \to \rho^+ \pi^-$?

Method 1:

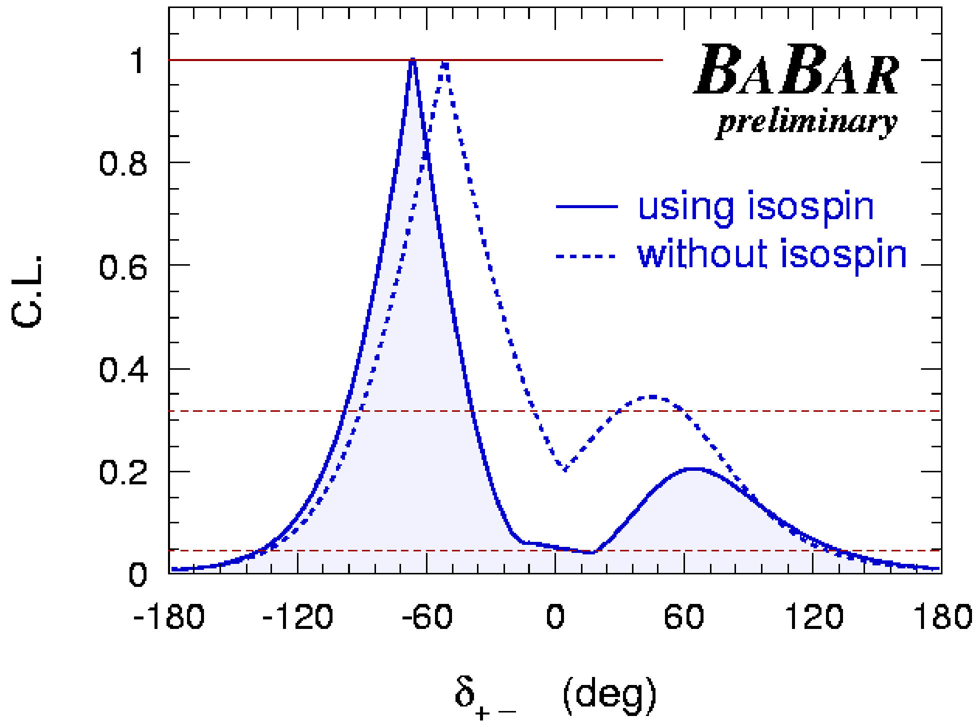
$$\delta = -\arctan\left(\frac{U_{+-}^{+,\mathrm{Im}} + U_{+-}^{-,\mathrm{Im}}}{U_{+-}^{+,\mathrm{Re}} + U_{+-}^{-,\mathrm{Re}}}\right)$$

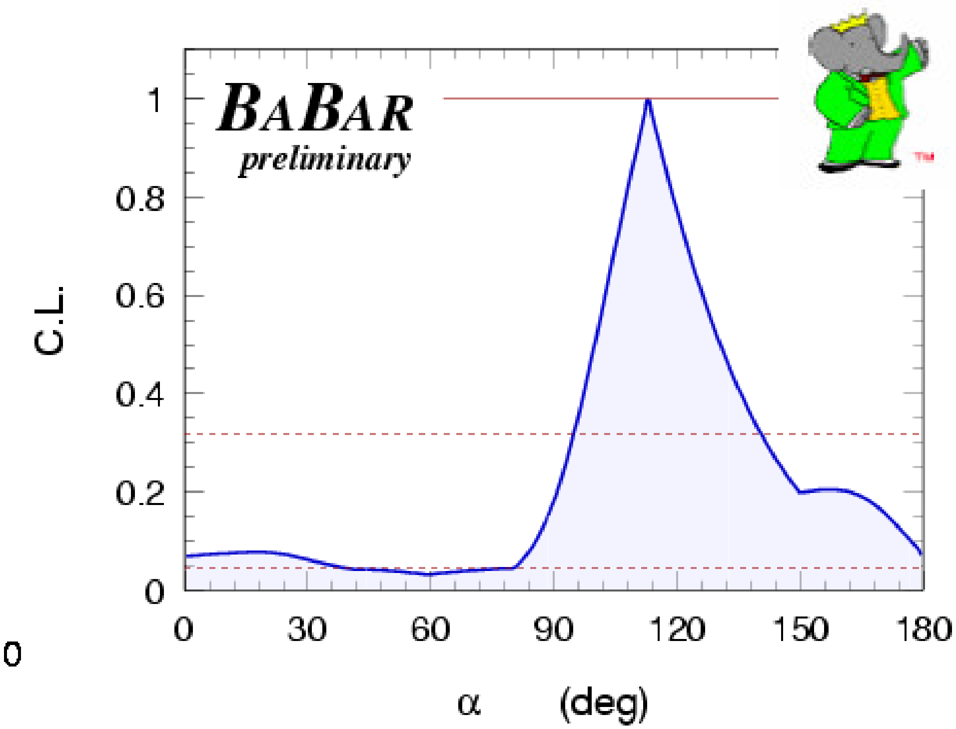


Method 2:

$$\chi^2_{\mathrm{scan}} = \sum_{i,j} \left(UI_i^{data} - UI_i^{scan}\right)\left(C^{data}\right)^{-1}\left(UI_j^{data} - UI_j^{scan}\right)$$

The Scans





Systematic uncertainties included

$\delta = \left(-67^{+28}_{-31} \pm 7\right)^{\circ}$ *and <u>weak</u> constraint at two standard deviation*

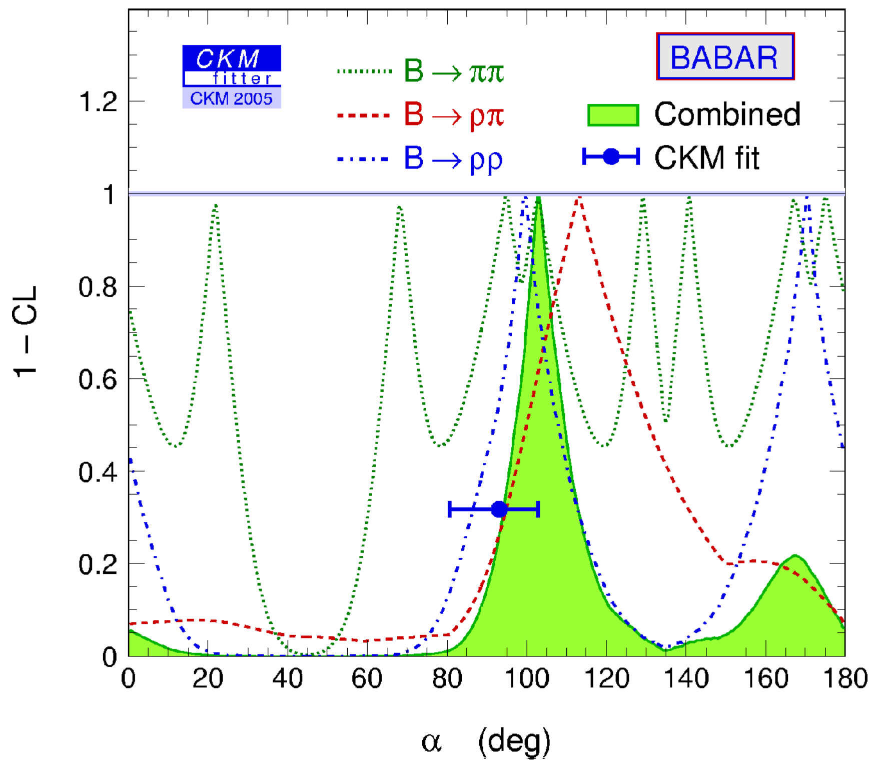
$\alpha = \left(113^{+27}_{-17} \pm 6\right)^{\circ}$ *and <u>weak</u> constraint at two standard deviation*

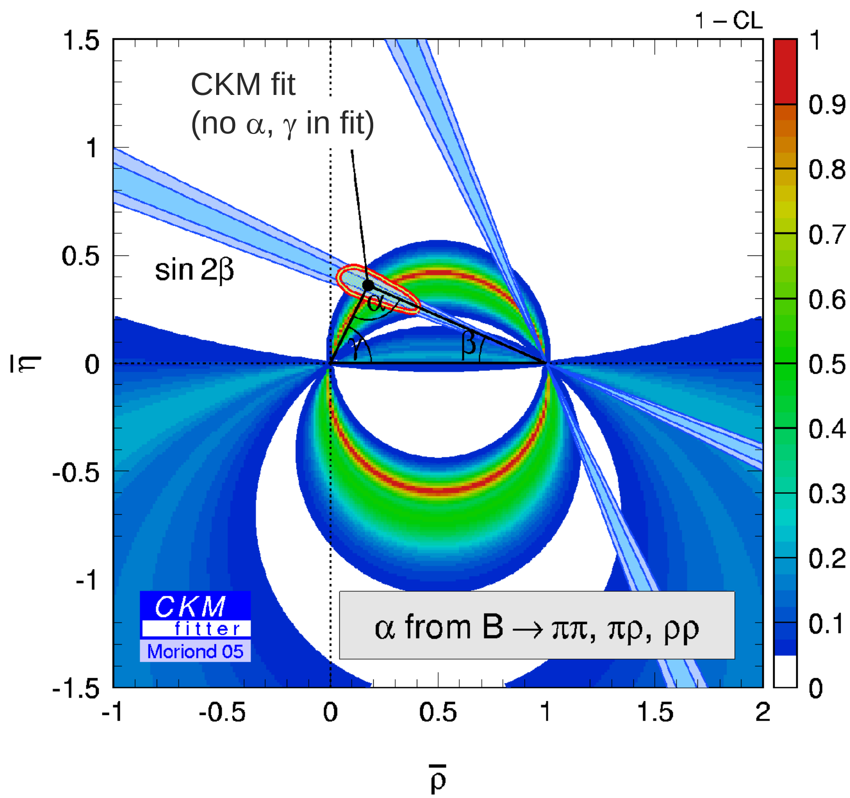
Combination of $\pi\pi$, $\rho\pi$, $\rho\rho$: First Measurement of α

Combining the three analyses ($B \to \rho\rho$ best single measurement) :

- similar precision as CKM fit :

$$\alpha_{\pi\pi\text{-BABAR}} = \left[103\,^{+10}_{-9}\right]^\circ \qquad \alpha_{CKM} = \left[93\,^{+10}_{-13}\right]^\circ$$





Conclusion

- Shown two methods of extracting α from $B \to \rho\pi$
  - *The isospin analysis appears hopeless for the near future*
  - *There is hope for the Dalitz plot analysis although it's technically difficult. We have overcome most of the these difficulties, demonstrated the feasibility and already achieved a weak constraint on α!*

- Limitation of the Dalitz plot analysis
  - *Biggest limitation is now luminosity!*
  - *Eventually, ρ line shape, other content on the Dalitz Plot will become important. But knowledge of these will also improve with statistics.*